

EQUATIONS OF MOTION OF THE UNIVERSE :

TWO GALAXIES

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The Hubble-LeMaitre (H/L) Law: $dR/dt = H_0 R$,

leads to an exponential expression: $R = R_0 e^{H_0 t}$. (as shown previously)

Instantaneous position, velocity, and acceleration of “proper” galaxies are derived from this simple exponential growth equation.

In the Hubble Flow, the galaxies differ in their properties by the proper time interval separating them. It may be imagined that a distant galaxy occupies a position that the observer’s galaxy will occupy after some time interval. By comparing two proper galaxies, an expression for all proper galaxies may be formulated. It must be realized that all expressions are instantaneous, and constantly changing.

An important concept is “distance”. To find the instantaneous distance between 2 proper galaxies, the position of one galaxy must be subtracted from the position of the other. Both are expanding exponentially. Because the H/L law mandates that all proper galaxies are moving at light speed for their position, this procedure is simple:

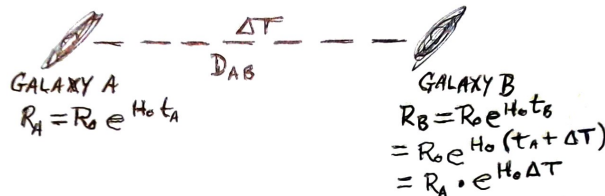
Figure 31-1. shows two proper galaxies, A and B, occupying two instantaneous positions:

$$\begin{aligned}
 R_A &= R_0 e^{H_0 t_A} & R_B &= R_0 e^{H_0 t_B}, \\
 & & &= R_0 e^{H_0 (t_A + \Delta T)} \\
 & & \text{where } t_B > t_A & \text{ and } \Delta T = t_B - t_A = \text{Light time interval}
 \end{aligned}$$

The distance between the two proper galaxies = D_{AB}

$$\begin{aligned}
 D_{AB} &= R_B - R_A = R_0 e^{H_0 t_B} - R_0 e^{H_0 t_A} \\
 &= R_0 e^{H_0 (t_A + \Delta T)} - R_0 e^{H_0 t_A} \\
 D_{AB} &= R_0 e^{H_0 t_A} (e^{H_0 \Delta T} - 1)
 \end{aligned}$$

Figure 31- 1. Galaxy Distance



If R_A is the location of the observer, then $R_A = R_0$,

$$D_{AB} = R_0 (e^{H_0 \Delta T} - 1)$$

A MacLaurin expansion of this expression approximates:

$$D_{AB} = H_0 R_0 \Delta T \sim C_0 \Delta T$$

This example demonstrates that the distance from the observer to a galaxy (in the standard model), is an approximation.