

General Fundamentals of Electrical Grounding Techniques

(Les Bases Générales de la Technique des Mises à la
Terre Dans les Installations Électriques)

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I. Classification and General Performance of Earth Electrodes

I-1. The Earth Electrode Problem

The earthing of electric installations plays an important part as regards the behavior of the network and personal safety when there are disturbances on the lines. There are, however, few fields of electrical engineering in which empirical methods are more largely employed.

In spite of important and impartial study devoted to the subject, the opinion is still sometimes expressed that "earthed", even if only by the use of a few inches of rod pushed into the ground, immediately becomes incapable of being the seat of a dangerous voltage. Several people lose their lives every year as a result of this simple belief. In one place the same type of earthing will be used on all cases, although its effectiveness may vary in the ratio of 100 to 1, depending on the type of ground. In another place considerable sums will be spent on earth circuits, whereas the foundations themselves, or the buried links between the different structures forming the plant, would be quite sufficient to give an excellent connection with the ground. The methods used for checking earth connections are, too, often used without the correct conditions for carrying out satisfactory measurements being properly appreciated.

It must be recognized that earthing problems are often of a very complex nature for a number of reasons. The ground is a poor conductor (the conductivity

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of a mass of good soil with a cross-section of 2.47 acres is comparable to that of a copper wire of one square millimeter; it is not homogeneous and has characteristics of which little knowledge is available). Conductors and electrodes buried in the soil are out of sight, often have a complicated shape and can only be examined, and their properties measured, with difficulty. Personal safety introduces questions of the probability of contact which are difficult to formulate in equations. Finally, installing good earthing systems often involves serious financial difficulties, especially under poor ground conditions.

The aim of the present paper is to lay down, in the simplest manner possible, the basis governing the determination and behavior of earthing systems. It is divided into five sections. The first deals with earthing arrangements in general; the second, with soil characteristics; the third, with the most usual types of earth electrodes; the fourth, with multielement connections and interconnected earths; and the fifth, with the special problems of large substations and currents through the human body.

Our chief purpose has been to give the basic principles. The more complex equations are put forward for reference purposes only or for illustration, and the working has been omitted except in very simple cases. A number of these equations have been taken from the excellent paper by Dwight in the December 1946 issue of "Electrical Engineering".

We have not considered the question of the behavior of earth electrodes when subjected to very steep waves, as this would require a special paper.

I-2. General Purpose and Classification of Earth Electrodes

The purposes of earthing electric installations are protection of the installation, improvement in the quality of the service and safety of personnel.

These general aims by no means determine the methods to be used, and widely different techniques can lead to the same average degree of perfection. Absolute perfection in these fields is impossible, since electricity, however much it may be controlled, remains intrinsically the source of incidents to plant, breakdowns in service, and accidents to people, which it is impossible to avoid completely without condemning the principle of application altogether.

It will be seen to what extent earthing of certain units can contribute to better operation and greater safety on networks. Distinction should first be made between two types of earth electrodes — viz. the "system" earth and the "protective" earth.

The system earth forms an integral part of the network and forms links, permanent or otherwise, between the conductors either directly or through suitable impedances, the effect of which is to influence the behavior of the network in case of an earth fault in the plant or on the line.

The protective earth originates rather with the consumer than with the supplier. It is not connected to the conductors of the network, but to objects which risk accidental contact with the conductors and which would be dangerous if maintained at a high potential over a period.

System earths are, for instance, the neutral earths or voltage limiter earths. Protective earths apply to housing and frames or line supports. (Figure 1).

It may happen that the same earth electrode fulfills both functions. Further, the impedance of the earth electrode through which the fault current flows obviously has an effect on the electrical behavior of the network similar to that of the system earth. The distinction between the two is thus not always completely clear.

It should further be mentioned that earth faults can occur at points which have not intentionally been provided with protective earths, for instance, in the case of a wire which has broken and fallen to earth, or a person with his feet on the ground who makes direct contact with a live bare conductor. In this case, it is a quite accidental resistance, with a value difficult to determine, which plays the part of protective earth (Figure 2).

Fig 1
System Earths (A) and Protective Earths (B)

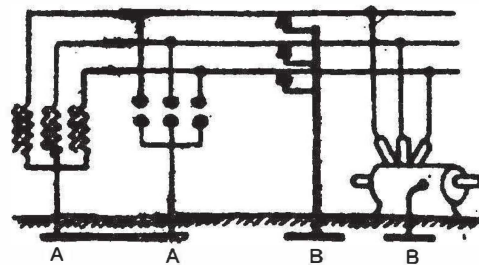
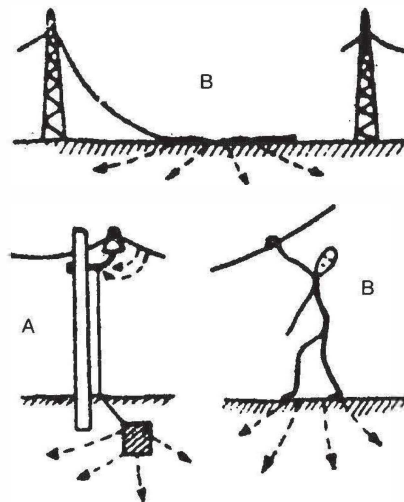


Fig 2
Protective Earths (A) and Accidental Earths (B)



I-3. Function of the Earth Electrode in High-Voltage Networks

We do not propose here to go into the question of the different high-voltage systems, that is, with neutral either insulated or more or less directly connected to earth. It will be sufficient to mention the different ways in which these systems contribute to the three aims set out above, that is, protection of plant, continuity of service and safety of personnel.

On high-voltage networks with a sufficiently short length and an insulated neutral or with Petersen coils, the absence of a direct earth contributes to good performance of the plant by limiting earth fault current to low values, so that the thermal and electrodynamic effects are less pronounced. This low fault current is of help in ensuring reliable service and in favoring self-extinction of arcs to earth. It also reduces the discharge voltage of these arcs through the protective earth and thus contributes to human safety.

On the other hand, a network with insulated neutral or with quenching coils induces relatively high dielectric stresses in the plant. Further, advantage is often taken of the low fault currents to allow discharge to earth to continue over a considerable period and this may give rise to a serious risk of accident when the protective earths or contacts taking their place have a certain resistance.

A high-voltage network with earthed neutral protects the plant at the cost of high overcurrents, first by reducing the overvoltages and then by limiting the duration of the faults, thanks to high-speed interruption circuit breaking caused by the overcurrent. This system improves the quality of the service by making it easier to cut out defective sectors selectively. Finally, it helps in ensuring human safety by high-speed clearing of faults.

On the other hand, the earthed neutral, unless combined with high-speed reclosing, does not have the advantage of self-extinction of transient faults. Further, automatic disconnection of defective units is compromised when the resistance of the protective earth or contact is so high that the fault current falls below the sensitivity limit of the protective devices.

This rough comparison only serves to show the two widely different ways in which protective earths act, either by reducing the fault current discharge voltages to earth to safe values, or by favoring the rapid disconnection of defective sectors.

Assume a potential of 125 volts as the danger limit and imagine that a person simultaneously touching the structure connected to the earth lead and the ground in proximity is subjected to half the total voltage on the earth connection. Assume further that the network has an insulated neutral and that the capacitive fault current to earth has a value of 10 amperes. Danger will arise if the earth electrode resistance exceeds 25 ohms, unless the sensitivity of the protective devices or the instructions given to the staff are such that a fault discharging at 10 amperes is rapidly eliminated.

Assume now a 15-kilovolt network with earthed neutral and a fault current reaching 1,000 amperes at the place where the fault occurs, and a sensitivity limit, for the earth current relay arrangements, of 50 amperes. Reduction of the local potential to a safe value of 125 volts would necessitate an earth resistance of 0.25 ohm, a value which is obviously impossible to attain. It is then necessary to

count exclusively on high-speed clearing in order to ensure safety, and this requires that the earth resistance should not exceed 173 ohms for the fault current to be higher than 50 amperes.

Adequate protection is thus not only a question of earths, but of coordination between these and other means of protection. The sensitivity of the relays should, in principle, be the greater the more difficult it is to make good earth connections on the network in question. Serious technical difficulties will often be met within the attempt.

It is necessary to point out that one cannot always expect protective earths to limit the voltages at the fault to safe values, especially on high-voltage networks with earthed neutral. Human safety is thus almost exclusively ensured on these networks by high-speed circuit-breaking, which reduces to an infinitesimal value the chances of a person finding himself at the spot where a fault occurs at the precise instant and in a hazardous attitude. The object of protective earths is simply to allow the passage of a sufficiently high current to ensure high-speed protection. Practice shows the safety thus offered is satisfactory. Accidents to people are practically non-existent on high-voltage networks with earth neutral and suitable protection. They are, indeed, certainly much rarer than on networks with insulated neutral, where the fault currents to earth are much smaller, but where they are allowed to continue for a considerable time.

I-4. Functions of Earths on Low-Voltage Networks

We will only deal briefly here with the familiar problem of the relationship between earthing systems and safety on low-voltage networks. In the case of low voltages, all public power distribution networks have the neutrals more or less well earthed. The effect of this earthing should be to maintain the neutral, in all circumstances, at a low voltage and the phase conductors at a voltage close to the rated phase-to-earth voltage, since this reduces the gravity of direct contact with them without altogether eliminating it. Actually, the neutral earthing of a low-voltage network is often poor and sometimes worse than that of the intentional or accidental protective earths which discharge the fault currents. In such cases it is the protective earth which imposes a low potential on the conductor concerned and, by a balancing effect, the voltages at the sound conductors are raised to the neighborhood of the voltage between phases. In this way to a certain extent, although somewhat erratically, we gain in safety as regards contact with an object which is the subject of a fault what we lose as regards direct contact with the neutral and the unaffected conductors.

It will be noted that it is very difficult to ensure simultaneously a very low voltage on the neutral conductor and on the earthed frame of the defective apparatus as long as a connection exists between the latter and the source. For this to be achieved it will be necessary for both the neutral earth and the frame earth to have sufficiently low resistance to practically form a short circuit. Further, in actual circumstances it is scarcely possible to count on automatic clearance ensured by the apparatus damaged, as is the case with high voltages. The faults are often of too high a resistance to blow the fuses which, in practice, is the only form of protection used in France on the majority of equipment, since improved devices are considered rightly or wrongly as too costly and too delicate.

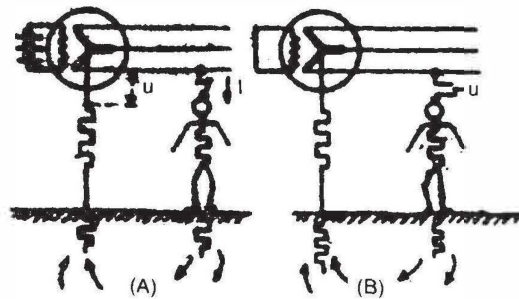


Fig 3
Application of Thevenin's Theorem to Currents Flowing in the Ground:
A, Actual Circuit; B, Equivalent Circuit

One method used in Switzerland consists in connecting the frame of consumer apparatus to protective earths, which are intentionally made always better than neutral earth. Protection against contact with the earthed frame is then completely ensured and good insulation of the conductors is relied on to prevent direct contact with the latter. This system should give excellent results, but it means that all consumers must have very high-quality earths, the arrangement of which is often costly.

It will be seen that the problem of earthing on low-voltage networks is delicate. Earthing the neutral remains, however, desirable on public supply networks, particularly because of the risk of mixing with high voltages. The same is not necessarily the case with private networks, especially when it is possible to maintain permanent supervision of the insulation of the network in relation to earth. Every simple fault, or all direct contacts with the conductor, are then immediately signalled and can be eliminated before a double fault can occur. The simple fault is harmless because it only causes the passage of a very low leakage or capacitive current.

I-5. Current Path Through the Ground

Before considering each earth electrode separately, and in order to justify the liberty often taken of separating it from the complex circuit in which it is inserted, it is of advantage to scan all the elements met with successively by the current discharging into the ground at the fault. We would mention straight away that the current which will appear when there is contact between a conductor and the ground close to it is most usually, according to Thevenin's theorem, identical with that caused by a single-phase potential equal to the pre-existing voltage between these two points (that is to say, equal to the normal phase voltage of the network), injected into the accidental connection which joins them, and flowing between the earth and the network assumed to be completely de-energized (Figure 3).

For a current to penetrate from the conductor into the ground, it must obviously have a return circuit to the network. This return might take place either

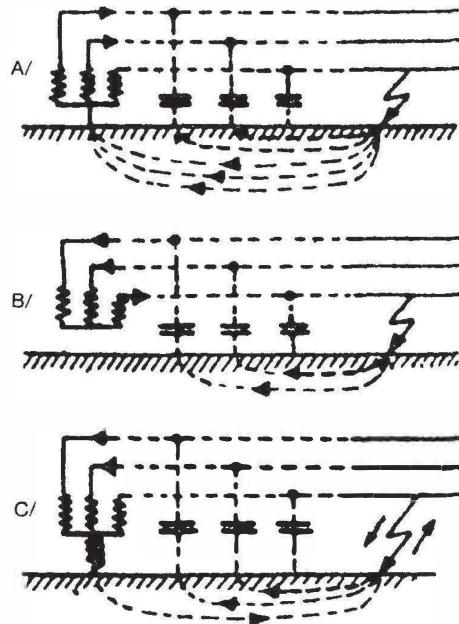


Fig 4

Return Path of a Fault Current to Earth:

A, Earthed Neutral System; B, Insulated Neutral System; C, Quenching Coil System

by one or more neutrals arranged for this purpose, or through the distributed capacitance between the ground and the unaffected phases of the network. The impedance of the return system is added to that of the input earth connection and to the internal longitudinal impedances of the network to limit the current discharged into the ground.

It often happens that the current has several parallel return paths available, some concentrated and some distributed. We know how this fact is taken advantage of in earthing through Petersen coils, which provide an inductive return circuit tuned to the capacitance of the lines in such a manner that the whole forms a very-high-impedance choke, limiting the current to a low value (Figure 4). It is also well to consider the case where the current discharged into the ground does not come from the ordinary sources on the network, but from a lightning stroke. The return of the current to the thundercloud must then take place by way of the distributed capacitance between the latter and the area of the ground surface covered by it. (Figure 5).

Current which enters the ground through a local earth electrode begins by spreading out around the latter. Then each thread of current bends toward the earth return connection or towards the zone which takes its place if the return operates through a distributed capacitance.

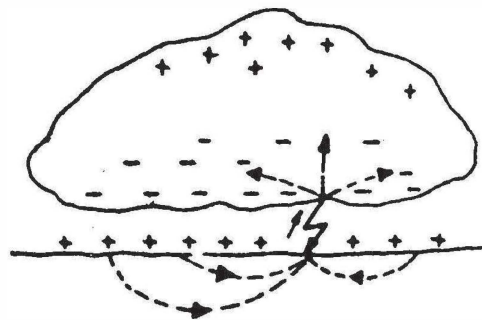


Fig 5
Lightning-Current Circuit

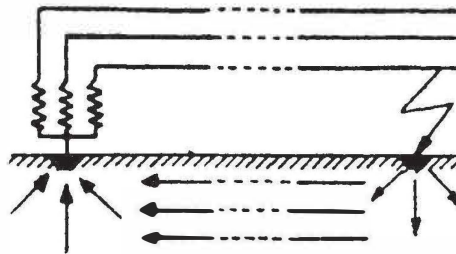


Fig 6
Path of a Fault Current in the Ground

In principle, the range of spread of such a current extends to the whole planet. In practice, the current densities and the potential gradients are obviously only perceptible within a restricted area. As a general rule, and this constitutes an important simplification, we can consider that the entry and exit earth electrodes for the current are sufficiently far apart not to affect each other. The current entering by an earth electrode starts by spreading vaguely in all directions up to a considerable distance, as compared with the dimensions of the earth electrode, and it can be assumed that the latter provides the same impedance as if the return earth electrode was uniformly distributed to infinity in all directions. The second phase of the path consists of circulation in the ground and the current thus diluted from the area of entry up to the area of exit. A last phase corresponds to drainage of the current from the return earth electrode to the network if this earth electrode is itself concentrated (Figure 6). It is further obvious that the terms "entry" and "exit", "discharge" and "drainage" are only used as simple references and that the two earth connections operate in a completely symmetrical manner for low-frequency phenomena where the propagation time can be ignored.

These considerations lead to the convenient idea of the natural impedance of an earth electrode, considered independently of the return circuit, as if this latter occurred to infinity in all directions.

As to the passage of the current in the ground between the entry and exit areas, we will confine ourselves here to mentioning that it tends to take place by following the feeder lines at an average depth of some hundreds of meters. The resistance and the reactance along the line of this return path are dependent on the nature of the ground and are generally in the neighborhood of a rather small fraction of an ohm per kilometer.

In any case, there is usually no need to take this impedance into account because this has already been done in the calculations for determining the zero-sequence line impedance of the feeder lines. It should merely be noted that the tendency of return currents in the ground to follow the lines can be more or less falsified if they find other paths which are good conductors, such as metal conduits, cable sheathing, etc. Certain inconveniences may result from this, especially when the electrical continuity of these paths or their links with neighboring earth electrodes is badly arranged.

II. Electrical Characteristics of the Ground

II-1. Ground Resistance; Effect of Temperature; Humidity and Grain Size

The effect of the ground as regards the behavior of earth electrodes is based largely on its resistivity. Now the resistivity of natural ground has the following peculiarities:

(a) It is extremely variable from one spot to another, according to the nature of the rocks and their degree of humidity. The extreme values met with in practice may vary in the ratio of 1000 to 1 and more.

(b) The ground at a given spot is often very nonuniform, both horizontally and vertically.

(c) The resistivity of the superficial layers of the ground shows considerable seasonal variations, due to the effect of frost and drought, which increase resistivity, or to humidity, which reduces resistivity. This effect is found to have a depth of approximately one or two meters. It is well to count on resistance variations of at least 1 or 2 and, exceptionally, 1 to 5 or more, for electrodes of average dimensions buried at a depth of 1 meter.

(d) It is rare for the resistivity of the ground at the surface, and still less at a depth, to be known before the installation of the plant, and it is often only the installation of the earth electrodes which gives the first indications as to the properties of the ground.

We will give a few values, which are not very accurate, of resistivity in ohm-meters. Resistivity in ohm-meters is mathematically equal to the resistance in ohms of a cube with one-meter edges. It is expressed by a figure one-hundredth the resistivity in ohm-centimeters. A resistivity of 100 ohm-meters corresponds to a conductivity of 10^{-13} cgs, a figure which is sometimes assumed for calculations relating to propagation in the ground.

Heavy soils, clays, marls, and damp chalky sands: Some multiples of 10 ohm-meters which may reach 100 to 200 for very compact clays.

Stony soils, covered with poor herbage: Often 300 to 400 ohm-meters.
Siliceous sands: 200 to 500 ohm-meters when damp; thousands when dry.
Damp peats, low dissolved-salt content: In certain mountainous areas up to 150 to 300 ohm-meters.
Calcareous rocks: Some multiples of 10 ohm-meters in cases of impregnation by underground water; 1000 to 3000 in a dry state.
Old rocks (granite, gneiss, schists, basalts): The older and the more compact the rock, the greater the resistivity. It may reach several tens of thousands ohm-meters for very healthy and very dry granites, but may often drop to values between 1000 and 5000. In the case of rocks which are greatly decomposed and mixed with clay, it may fall to around 100 ohm-meters.

The resistivity of *river waters* varies according to the place and season. It is 16 ohm-meters for the Seine in Paris, 40 for the Garonne in St. Gaudens, 20 to 40 for the Rhine, 30 to 60 for the Rhone, 80 to 300 for the Dordogne, and 5 to 80 for the Gave de Pau. It will be seen that it is approximately the same as for good soils.

Ballasts and dry concentrates are very good insulators. On the other hand concrete buried in a damp soil has a resistivity which scarcely exceeds some hundreds of ohm-meters. It would not be far wrong to assume that the resistivity of buried concrete is approximately the same as that of the ground around it.

The conductivity of the soil is essentially of an electrolytic nature. It is thus very low in a dry state and increases with humidity for a constant temperature, and increases with the temperature for a given degree of humidity.

Trials carried out in England on a light mould gave the following results:⁹

<u>Temperature, C</u>	<u>Resistivity</u> ohm-meters
- 15	3300
- 5	790
0 (Ice)	300
0 (Water)	138
+10	99
+20	75
<u>Degree of humidity</u> (%)	<u>Resistivity</u> ohm-meters
0	More than 10 ⁷
2.5	1500
5	430
10	185
15	105
20	63
30	42

Another important element in the ground formation is its grain size, which influences both its porosity and its capacity for retaining humidity and the degree of contact with the electrodes. Large-grain earths such as gravel, pebbles, etc.

make it difficult to establish satisfactory earth electrodes, and it is often necessary to remedy this state of affairs by surrounding the surface of the electrodes with a thickness of fine rich soil or with some other material which is a relatively good conductor.

II-2. Variation in Earth Electrode Resistance with Current

As a general rule the resistance of earth electrodes varies little in relation to the current. The values measured with the usual portable apparatus which involve relatively small currents, are often quite representative of those obtained by injecting heavy currents, if account is taken of the inaccuracies from which these instruments often suffer. The resistance frequently has a tendency, however, to fall as the current increases under the action of somewhat complex factors.

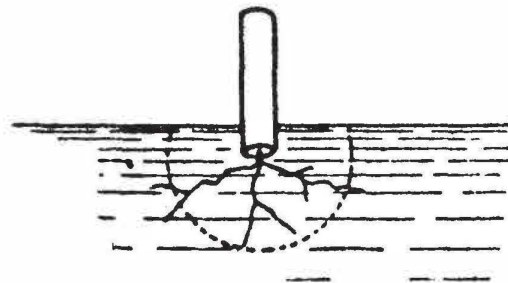
A first reason for this drop may be heating of the earth in the neighborhood of the electrodes, due to the passage of current. The heating, as we have seen, tends to reduce the resistivity of the ground.

A second factor which acts in the same sense is the sometimes defective contact between the metal of the electrode and the surrounding earth. In cases of earth shrinkage there may be a thin air space which offers a high resistance to small currents, but which is readily traversed by arcs caused by a higher voltage applied to the electrode. Similar bad contacts may also occur at the joints of the connections.

Again, with the very heavy currents, another phenomenon is encountered. The ground, considered an imperfect insulator, has a breakdown gradient in the neighborhood of 200 kilovolts per meter¹⁰ for clayey soils, for instance. The result is that a higher gradient cannot exist in the soil in the neighborhood of the electrode without giving rise to internal discharges which increase the equivalent dimensions of the electrode and return the gradients to this limiting value.

This phenomenon is clearly demonstrated if a high voltage is applied to a very small electrode plunged into water for a short distance. Branching sparks will be seen to appear on the electrode, which they make longer in effect, their own length increasing with the voltage (Figure 7). The result is that, however small the original dimensions of the electrode, the resistance cannot exceed a certain value

Fig 7
Flow of a Heavy Current Through a Small Electrode Immersed in Water
(Piece of Fine Insulated Wire); Equivalent Hemispherical Electrode



which is the lower, the higher the voltage. The resistance has an approximate value of

$$R = \frac{K}{U}$$

The constant K is in the neighborhood of 3000 ohm-kilovolts for a resistivity of 100 ohm-meters.

This result is easily interpreted from equations given later for hemispherical electrodes. It will be seen that the gradient at the surface of a hemispherical volume discharging current is equal to the quotient of the applied voltage over the distance to the center. Under these conditions the limitation of the gradient to 200 kilovolts per meter implies that the internal discharges in the ground bring the dimensions of the earth electrode (if it has not already exceeded them) up to approximately those of a hemispherical conducting medium, having a radius equal to the quotient by 200 of the voltage applied in kilovolts. The expression above for resistance is deduced if a value of some 100 ohm-meters is taken for the resistivity.

The table below gives the minimum radius r_m to which the electrode is thus enlarged when the full phase voltage is applied to it, as well as the maximum resistance R_M deduced from it and the corresponding fault current I , for a soil of this nature and for different network voltages:

U (kilovolts)	r_m (meters)	R_M (ohms)	I (amperes)
15	0.043	350	250
60	0.17	87	1000
150	0.43	35	2500
220	0.65	23	5600

A comparison of the figures in the second column with the usual dimensions of the earthing arrangements of overhead line supports shows that soil breakdown will rarely occur in good ground at normal operating voltages. It will occur more frequently when lightning strokes are being discharged, since these involve much heavier currents.

Data are lacking regarding the limiting gradients for the ground in the case of poor soils, but the passage of arcs certainly results more frequently.

II-3. Effect of Soil Capacitance

It is easy to verify the fact that the equipotential surfaces caused by the flow of a current through an electrode in a homogeneous conducting medium have the same form as the equipotential surfaces of the electrostatic field developed by the same electrode in a homogeneous insulating medium (ure 8).

In the case of a spherical electrode, for instance, the equipotential surfaces are concentric spheres in both cases. A very narrow and very simple relationship thus results between any electrode in a homogeneous conducting medium and the capacitance of the same electrode in a homogeneous insulating medium. This relationship can be obtained readily from any elementary case, for instance, by

considering a medium limited by two surfaces S, parallel and removed by L, between which flows a current or an electrostatic field develops.

If ρ is the resistivity of the conducting medium in ohm-meters and K is the dielectric constant of the insulating medium referred to vacuum, the resistance in the first case and the capacitance in the second, respectively, are expressed by:

$$R = \frac{\rho L}{S} \text{ ohms} \quad C = \frac{KS}{4\pi L} \text{ meters}$$

from which, noting that a microfarad represents 9000 meters, we can deduce directly:

$$CR = \frac{K\rho}{36000 \pi} \text{ ohm-microfarads}$$

or approximately:

$$CR = \frac{K\rho}{100,000} \text{ ohm-microfarads}$$

It will be noted further that the product CR is homogeneous at a time expressed in microseconds in the preceding equation. These considerations allow of linking calculations for the resistance of earth electrodes to those for conductor capacity in an insulating medium. Another important consequence can be deduced from this.

No substance is a perfect conductor or insulator. Every material can be considered as possessing both a finite resistivity ρ and a dielectric constant K, so that each small volume between two sections of equipotential surface can be considered as a combination of a resistance and of a capacitance in parallel.

For a highly conductive material, the resistance will be very small and will completely shunt the capacity of the element. For a good insulating material on the contrary, the effect of the capacity will be preponderant.

In all cases the product $CR = \frac{K\rho}{100,000}$ will represent (in microseconds) the time constant for the charging and discharging of the capacitance of the elementary volume. The following rule can be directly deduced from this:

A material in which $\frac{K\rho}{100,000}$ is very small as compared with the duration of the variation in the applied voltages will behave as a pure resistance with regard to these variations. The capacitance charge will not materially restrict the establishment of the voltages. On the other hand, a material in which $\frac{K\rho}{100,000}$ is large in relation to the duration mentioned will behave like a low-loss capacitance.

With alternating currents, the capacitance does not begin to intervene to an appreciable extent until a certain frequency threshold has been passed, this threshold being very high in practice.

Assume 10 and 4 as average dielectric constants of slightly humid and very dry grounds. The charge and discharge time constants for the first ground will only be 0.01 microsecond for a resistivity of 100 ohm-meters, and that of the second

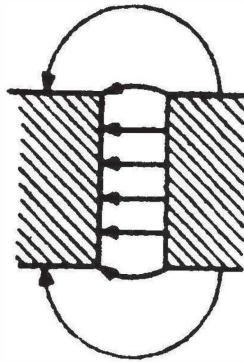


Fig 8
Flow of Current Between Two Electrodes, by Displacement in an Insulating Medium or by Conduction in a Conducting Medium

0.4 microsecond for a resistivity of 10,000 ohm-meters. We can thus check the fact that the ground behaves like a quasi-pure resistance in regard to nearly all electric phenomena. It is only when there is a combination of an extremely high resistance with a very-high frequency or with extremely steep-fronted surges that, if our argument is accurate, the capacitance should begin to have an effect.

III. Review of Earthing Electrodes According to Shape

III-1. Inaccuracy of Practical Earthing Electrode Calculations

Uncertainties as to the resistivity of the ground and its seasonal variations, as the heterogeneous nature of the ground and the more or less good contacts provided, result in it being impossible to claim in practice that earth electrodes can be calculated. It is only possible at the most, when there is an idea as to what the resistivity is, to predetermine the order of magnitude of their resistance and of the potential gradients in their vicinity, or inversely, to estimate the dimensions they should be given for the purpose of maintaining resistances and gradients within the limits desired.

The apparent accuracy of the equations we are going to give should not, therefore, give rise to any illusions. These equations are of interest because they permit comparisons and because of the information they give as to the relative effect of one factor or other. Strict accuracy must not be expected in the final mathematical result. It is thus permissible to simplify without inconvenience and to reduce them to a few broad rules which can be remembered readily and which are quite sufficient in practice, and so avoid the need for referring to curves and nomographs.

This is the essential purpose of this chapter.

III-2. Resistance, Potentials and Local Gradients

The resistance of an earth electrode is not sufficient in itself to characterize performance. It only defines the over-all potential $U_0 = RI$, which the passage of a

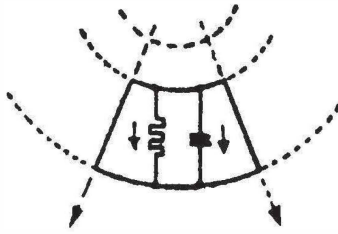


Fig 9
Resistance and Capacitance of a Small Unit of Ground

current I produces in an electrode in relation to the earth at a great distance. The potential U_0 is the integral of all the elemental increases which are met with when following any path from infinity to the electrode. Practical interest attaches to the potential between specific points along this path which may not be the electrode and infinity. In order to characterize an electrode completely, it is necessary in practice to know the distribution, or the “map”, of the potential throughout the whole of the surrounding ground, taking into account its natural and artificial lack of homogeneity.

It should be noticed that potentials which have to be considered in nearly all practical problems are between points which are physically close. For instance, the flow of a current through the earth of a substation raises the earthed metal frames of machines to a certain potential. Does this potential risk cause a “reverse flashover” between these frames and another circuit in the substation? This will depend on the potential between the top and bottom of the insulators which carry the circuit. Or again, the danger for anyone passing will be a function of the potential which he picks up across the feet (some 75 centimeters apart), or between feet and hand resting on a structure connected to the frame (Figure 10).

This applies when checking earths after the installation has been completed. To investigate the danger presented (both to persons and to equipment) by an earth electrode, it is advisable to pass a current and to carry a suitable voltmeter for measuring the potentials at suspected points close together, such as insulator terminals, places where people pass frequently and neighboring earthed frames, etc. Data are thus obtained which are much more concrete than those provided when simply measuring the earthed frame potential with respect to a distant reference point.

Another interesting point is that these physically close points, as shown in Figure 10, may be at very different distances electrically. In the case of contact between the feet, the potential U_2 picked up will usually be fairly closely related to the local potential gradient at the ground surface. With contact between feet and hand, on the other hand, the potential U_3 picked up may integrate the gradients covering a path which may be fairly long or complex because the structure touched will occasionally be earthed at a considerable distance from the place of contact.

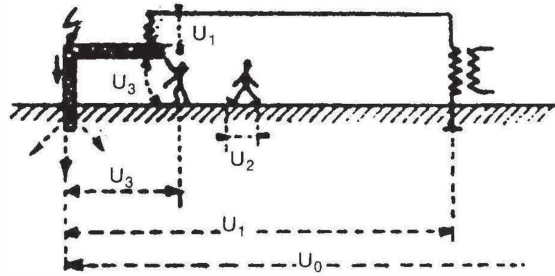


Fig 10
Voltages Which May Result from the Flow of a Current in the Ground

If, for instance, we consider the case of an electrode buried at a certain depth and connected to the ground by an insulated connection, it is obvious that increasing the depth at which it is buried will lower the potential gradients on the surface and will thus reduce the step potential. Although the result is thus to reduce the absolute potential of the surrounding earth, it will increase to some extent the potential between the latter and the protruding structures connected to earth, as well as the danger of placing the hand on them. Burying an earth electrode is not, therefore, a panacea against danger to life.

If we now examine the danger of return flashovers between the earthed frames carrying a current and a safe electric circuit entering the substation, we shall have to ask in what manner the average potential of this circuit is determined. In many cases it will be determined by a very distant earth electrode, or by the distributed capacities of lines which are also at a distance. The circuit will then bring into the substation the potential of the distant ground, and the overall potential $U_0 = RI$, corresponding to the resistance of the frame earths, will appear at U_1 across the ends of the insulators which separate this circuit from the earthed frames.

There will, on the contrary, be only a fraction of this total potential if the earth electrode for the circuit is within the bounds of the substation and shares more or less, as a result, the potential of the earthed frames. At the limit, the potential would be nil if this earth is common with the frame earth. On the other hand, in this latter case, the potential RI at the earthed frames will be propagated by the circuit to distant areas and it will be found there between the terminals of the insulators. Under intermediate conditions, the potential RI will be distributed, in proportions which are a function of the coupling of circuit earth and earthed frames, between the insulators in the substation and those at a distance. This question of coupling will be defined later.

It will be seen that an estimate of the quality of an actual earth electrode introduces local considerations which escape general investigation. The simple factors which will be retained below are, in addition to the over-all resistance R , the potential U_x and the gradient G_x at a distance X from the electrode.

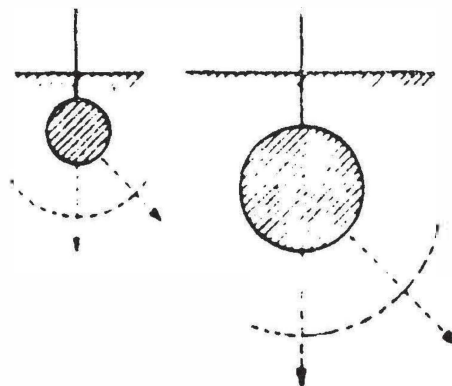


Fig 11
Alternation in Resistance According to Size of Electrode

III-3. General Remarks

(a) Consider a given earth electrode in homogeneous ground and assume that the resistivity of the ground is doubled. The path of the threads of current will not be modified, but all the potential drops will be doubled:

The resistance of an earth electrode of given shape and dimensions is proportional to the resistivity of the ground.

(b) Assume, now, that we expand all the dimensions concerned with the electrode to twice the original figure without altering its form or the resistivity of the ground. The equipotential surfaces will have their surface area quadrupled, but the path of the current between two equipotential surfaces will be doubled. The resistance will be consequently reduced by a half (Figure 11):

The resistance of an earth electrode of given shape in homogeneous ground is inversely proportional to its linear dimensions.

(c) A consequence of this expansion of the whole of the equipotential surfaces along with the dimensions of the electrode is that the resistance of the earthing arrangements, when these are very large, brings in the effect of the distant layers of ground to a relatively greater extent.

This is so because to the resistance corresponding to the distant layers (the only resistance involved in the case of large electrodes), there is added (in the case of small electrodes) the resistance introduced by the intermediate layers, which is greater and thus reduces the relative importance of the former.

The potential on an earth plate or rod has already been absorbed to the extent of approximately half at a distance of one pace. On the other hand, the rise in potential of the ground area of a large substation has a substantial effect up to several hundred meters away.

(d) It would result from the two first laws that, in order to obtain a given resistance in ground with a resistivity 100 times greater, an installation with a

volume one million times larger would be required. Luckily, it is possible to escape this consequence, as we shall explain later by showing that the “small dimensions” of very elongated electrodes only come into the equations giving the resistance as logarithmic terms, which variables change slowly.

The resistance of earth electrodes of very elongated form is only slightly dependent on the “small dimensions” and varies approximately in inverse proportion to the “large dimension”.

With very elongated electrodes a ground resistivity 100 times greater only increases by some 100 times the volume of installation work necessary for obtaining a given resistance.

(e) The metal is thus used to better advantage in electrodes of elongated shape and, further, the earthing installations become more costly as the earth resistance rises. The result is that:

The poorer the conductivity of the ground, the more desirable it is to use electrodes of elongated form.

(f) We will now consider the effect of the depth to which the electrode is buried, assuming that the connection linking it with the surface is insulated.

If the electrode is symmetrical, it is obvious that the current has half the area for discharge when only half buried, as when buried at infinite depth (Figure 12):

The resistance of an electrode with an insulated connection thus varies in the ratio of 2 to 1 for half and complete immersion respectively.

At intermediate depths an evaluation by rule of thumb, based on these two limits, will usually allow a sufficiently close estimate without further calculation.

III-4. Hemispherical Earth Electrode in Homogeneous Ground

In homogeneous ground the simplest electrode consists of a conducting hemisphere with the flat side level with the surface. The current paths are then extensions of the radii of the electrode and the equipotential surfaces are concentric half-spheres (Figure 13).

Between two equipotential surfaces, separated by X and $X + dX$ from the center of the electrode, the current I has a cross-section $2 \pi X^2$ through which to pass.

The corresponding element of resistance is thus $\frac{\rho dX}{2 \pi X^2}$.

The voltage drop in the element is $\frac{\rho I dX}{2 \pi X^2}$, and the local gradient $G_x = \frac{\rho I}{2 \pi X^2}$

The resistance R and the potential U_0 at the surface of the electrode are readily deduced from this by integration, together with the potential U_x at a distance X from the center (Figure 14).

$$R = \frac{\rho}{2 \pi r} = \rho \frac{\frac{r}{2}}{\pi r^2}, \quad U_0 = \frac{\rho I}{2 \pi r}, \quad U_x = \frac{\rho I}{2 \pi X} = U_0 \frac{r}{X} = r G_x$$

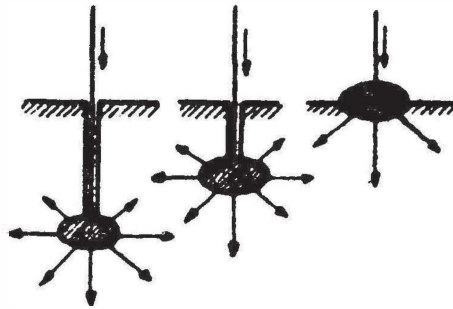


Fig 12
Effect of Depth of Immersion

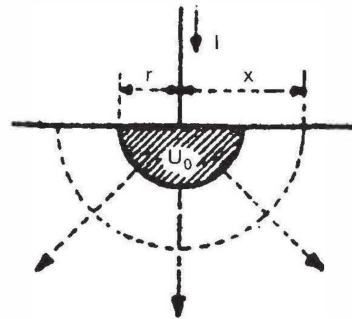


Fig 13
Hemispherical Earth Electrode in Homogenous Ground

It will be seen that:

The resistance offered by the ground to the flow of current from a hemispherical electrode is equal to that of the cylindrical unit of ground having the diameter cross-section of the electrode with a length half the radius. The resistance varies as the resistivity of the ground and as the inverse of the radius of the electrode.

The absolute potential and the gradient at any external point are dependent on the distance of the point from the center of the electrode, but not on the dimensions of the latter. The potential varies as the inverse of the distance to the center and the gradient as the inverse of the square of this distance. The gradient falls relatively quickly. Half the total potential is absorbed at a distance from the periphery of the electrode equal to its radius.

Hemispherical electrodes are scarcely ever used in practice because the concentrated shape results in bad utilization of the metal. It is preferable to spread this over a large stretch of ground.

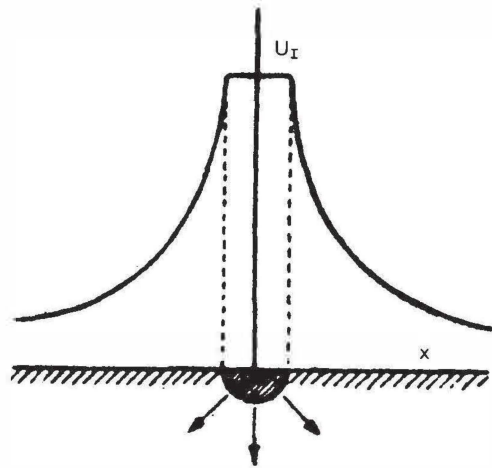


Fig 14
Potential Around a Hemispherical Earth Electrode

It is important to mention that, however irregular the shape of an earth electrode, the equipotential surfaces always tend to approach a hemispherical shape, with increasing distance from the electrode. The absolute potential and the gradient at even a short distance, by comparison with the dimensions of the electrode, are thus dependent neither on the shape nor on the dimensions of the latter and are expressed by the equations given above as a function of the distance and of the current.

Further, as an equivalent hemispherical electrode, which has the same resistance and which the flow of the same current would thus raise to the same potential, can be made to correspond to any kind of electrode.

The radius of this equivalent hemispherical electrode will evidently be intermediate between the three dimensions of the actual electrode. We shall see, for instance, that a vertical rod is equivalent to a half-sphere of radius something like a sixth of the length and that a superficial plate to a half-sphere of radius some 65 percent that of the plate, etc. It is also clear that the resistance of any electrode will be higher than that of any circumscribed hemispherical electrode.

The simple rules given above are thus valid for all earth electrodes, even when not hemispherical, insofar as the irregularities are not too great or as the distance is sufficient for these local effects and irregularities to be inappreciable.

III-5. Buried Spherical Earth Electrode with Insulated Connection

In the case of a spherical electrode of radius r buried at a great depth in homogeneous ground, the surface offered for the discharge of current at a distance X is $4\pi X^2$, that is, twice as large as with a hemispherical electrode with its flat top level with the ground. Whence resistance, potentials and gradients are halved:

$$U_x = \frac{\rho I}{4 \pi X} \quad G_x = \frac{\rho I}{4 \pi X^2}$$

$$U_0 = \frac{\rho I}{4 \pi r} \quad R = \frac{\rho}{4 \pi r}$$

When the depth h to which the center of the electrode is buried is finite while remaining large in relation to the radius (Figure 15), the non-uniformity due to the surface of the ground can be eliminated by super-imposing on the electrode and ground systems their image system in relation to this surface and by making a similar current I flow from the image electrode. Considerations of symmetry would readily show that the field in the ground remains unchanged. Further, the individual field of one of the electrodes is not noticeably modified by the presence of the second electrode which is assumed to be of fairly small dimensions. The result is that the field in the over-all area is practically the pure and simple superposition of the individual effects of the two electrodes (Figure 16).

For the potential at a point, distant by X and X' from the centers of the two electrodes, we can immediately deduce from this

$$U_x = \frac{\rho I}{4 \pi X} + \frac{\rho I}{4 \pi X'}$$

In particular, if we choose the point at the surface of one of the electrodes, and insofar as the distance $2h$ is large in relation to the radius r :

$$U_0 = \frac{\rho I}{4 \pi} \left(\frac{1}{r} + \frac{1}{2h} \right),$$

whence the resistance

$$R = \frac{\rho}{4 \pi r} \left(1 + \frac{r}{2h} \right).$$

The resistance is thus raised in the ratio $\frac{r}{2h}$ in relation to an infinite buried depth.

It will be noticed that the magnification factor is only 25 percent for a depth equal to the diameter. It would be 100 percent as we have seen for half emergence, the previous equation then ceasing to apply.

The potential and the gradient at the surface of the ground at a (horizontal) distance X from a point directly above the sphere are (Figure 17):

$$U_x = \frac{\rho I}{4 \pi} \frac{2}{\sqrt{X^2 + h^2}}, \quad G_x = \frac{\rho I}{4 \pi} \frac{2 X}{(X^2 + h^2)^{\frac{3}{2}}},$$

In particular, vertically above the center

$$U_v = \frac{\rho I}{2 \pi h}, \quad G_v = 0, \quad U_0 - U_v = \frac{\rho I}{4 \pi} \left(\frac{1}{r} - \frac{3}{2h} \right).$$

The potential vertically above the center is the lower, the greater the buried depth. On the other hand, the difference of potential between this area of the

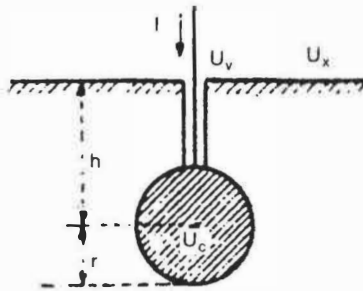


Fig 15
Buried Spherical Electrode

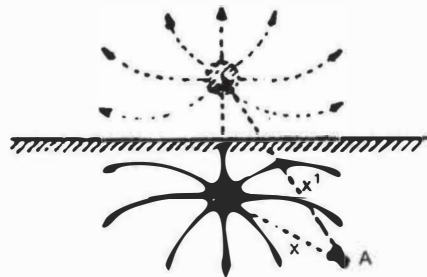


Fig 16
Introduction of Electrode "Image" in Relation to Earth

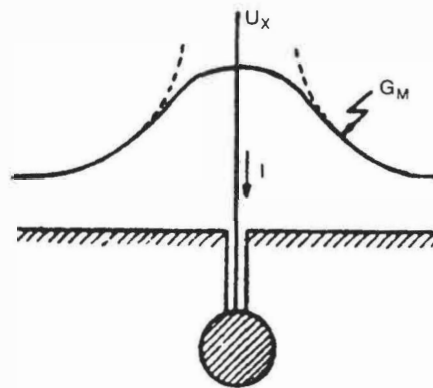


Fig 17
Potential Around a Buried Spherical Electrode

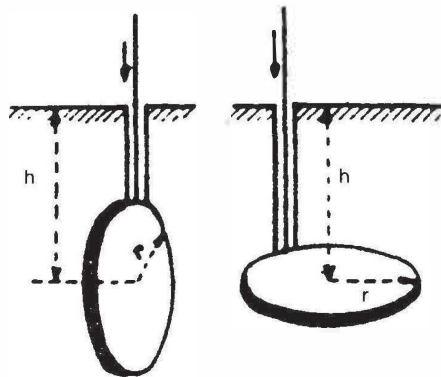


Fig 18
Plate Electrode

ground and the metal structures connected to the electrode increases somewhat with the depth. Burying an electrode is thus no protection against contact with a structure connected to it.

The potential gradient on the ground surface, which is zero vertically above the center, passes through a maximum at a distance from this point equal to 0.7 times the buried depth. The maximum gradient can be written:

$$G_M = \frac{\rho I}{2\pi(1.6h)^2}$$

Comparing this expression with that for a gradient around a hemispherical earth at the surface of the ground, it is found that burying a more or less spherical electrode with an insulated connection to a depth h reduces the maximum gradient to the same extent as an enclosure surrounding it at a distance $1.6h$ from the center of the surface-electrode.

III-6. Plate Electrode

A horizontal or vertical circular plate of radius r , the center of which is buried to a depth h , with the connection insulated, has the approximate resistance (Figure 18):

$$R = \frac{\rho}{8r} \left(1 + \frac{r}{2.5h + r} \right).$$

This purely empirical equation gives results which diverge very little from the theory. In practice, it is sufficient to remember that

$$R = \frac{\rho}{8r} \text{ for infinite depth,}$$

$$R = \frac{\rho}{4r} \text{ for zero depth,}$$

applying a suitable interpolation factor empirically.

Comparison of these values with those corresponding to spherical electrodes shows that a plate electrode has substantially the same resistance as a sphere of radius 1.5 times smaller.

The thickness of the plate does not seriously affect its resistance. Further, it will be seen later that a plate electrode is not much more effective than a simple ring of the same diameter. Finally, it will be noted that a square plate is comparable to a circular one with a radius equal to 0.6 times the side, or again that a plate one meter by one meter buried at a depth of one m represents a resistance 0.25ρ , which is close to that of a four-meter rod.

The distribution of the potential on the ground near the plate, buried at a considerable depth, differs only slightly, to a first approximation, from what it would be with an equivalent spherical electrode.

III-7. Loop-Shaped Electrode

The resistance of a horizontal circular loop of radius r , buried at a depth h and consisting of a conductor of diameter d can be expressed, according to Dwight by:

$$R = 0.366 \frac{\rho}{2\pi r} \left(\log \frac{16r}{d} + \log \frac{4r}{h} \right).$$

The second term is approximate and does not apply to extreme values of h . It should be made to tend towards zero for infinite depth and towards the first term for a depth of nil.

As an example, the resistance of a ring with a radius of 50 centimeters, formed of a conductor 1 centimeter diameter and buried 1 meter down, is 0.374ρ . The resistance of a solid disc with the same diameter would be 0.288ρ , which is only 23 percent less than that of the ring.

A solid plate is not much more effective than a simple ring of the same diameter.

The distribution of earth potential in the neighborhood of a loop of small dimensions buried at an appreciable depth is approximately the same as with a plate of the same diameter or the equivalent sphere. If, on the other hand, it is no longer a question of a compact electrode, but of a loop with a diameter which is large as compared with the buried depth, the ground potential in the neighborhood of the electrode should be calculated by use of the same equation as in the case (investigated below) of a buried rectilinear electrode.

It is of interest to note that the absolute potential at the center of a loop, not buried or only sunk to a slight depth, and carrying a current i per unit length, has the very simple expression $U_c = \rho i$

A circular loop buried at a shallow depth and carrying one ampere per meter in homogeneous ground has its center at a potential numerically equal to the resistivity of the ground.

III-8. Vertical Rods in Homogeneous Ground

(a) *Note on the calculation of rectilinear electrodes* The characteristics of earth electrodes of more or less rectilinear shape can be quite simply calculated by breaking them down into small sectional elements between which it can be assumed, to a first approximation, that the current is equally distributed. The

discontinuity introduced by the surface of the ground is also avoided by superimposing its image system on the original system, as was done in connection with buried spheres.

To arrive at the potential at a given point, the sum of the effects of all the sectional elements at this point is found (considered as a chain of small separate spheres). To obtain the potential of the electrode and its resistance, it is sufficient to move the reference point to the surface of one of the sections.

This is the same method that will be given later for calculating multiple electrodes. The approximations come, on the one hand, from the assumption of a uniform distribution of the currents carried by the electrode, as if the different sections did not react on each other; and on the other hand from taking a circular section as being equivalent to a sphere, a process which is valid if the reference point is only a short distance away, but which becomes risky when it is actually brought into contact with the electrode. Some authors, such as Dwight, have investigated the problem more closely, but the approximation of the elementary equation remains sufficient in practice.

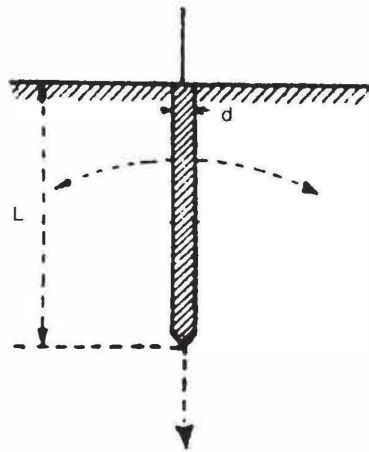
(b) The *resistance* of a vertical rod with a length L and a diameter d has the following fairly accurate expression (Figure 19):

$$R = 0.366 \frac{\rho}{L} \log \frac{3L}{d} \text{ ohms.}$$

The flow of a current $I = L_i$ thus raises the rod to a potential

$$U_0 = 0.366 \rho \frac{I}{L} \log \frac{3L}{d} = 0.366 \rho i \log \frac{3L}{d} \text{ volts.}$$

Fig 19
Vertical Rod in Homogenous Soil



The simplified method of calculation referred to above would give the coefficient 4, instead of 3, for the logarithmic term, and this coefficient 4 has been adopted by many authors. The resultant error is usually of no importance and we have ourselves kept to the rough form of equation for the simplified calculations of the gradients and potentials which will be gone into later.

A particularly interesting value is the earthing resistance RL per each meter of rod, which is also the potential $\frac{U_0}{i}$ to which the rod is raised when it carries one ampere per meter.

$$RL = \frac{U_0}{i} = 0.366 \rho \log \frac{3L}{d} \text{ ohms or volts per ampere per meter.}$$

The resistance per meter increases a little with the length, and as a consequence long rods are a little less effectively used than short ones; resistance falls a little as the diameter increases and enlarging the diameter is thus not without some advantage.

But it should be mentioned that L and d appear in this expression in a logarithmic term which only changes very slowly when L and d have the usual values. This term is almost always comprised between 2 and 3, and doubling the length or diameter only alters it to the extent of 0.3, that is, merely 10 to 15 percent.

The following simple rules can be deduced from this:

The resistance per meter of rod, or the voltage of the rod when it carries one ampere per meter, is only slightly dependent on the dimensions of the rod and is approximately equal to the resistivity of the soil.

For instance, in a soil with a resistivity of 100 ohm-meters, a one meter rod will have a resistance in the neighborhood of 100 ohms, and will be raised to some 100 volts by the flow of one ampere. A 20 meter rod, again, will have a resistance close to 5 ohms and will be raised to some 100 volts by a flow of 20 amperes.

Doubling the length of a rod reduces the effectiveness of the meter length by 10 to 15 percent; doubling its diameter increases it by the same amount

It is therefore pointless to attempt to reduce the resistance of an earthing arrangement by increasing the size of the rods. It is more effective to increase their length or number.

As an example, here are overall resistances and the resistances per meter (or potentials for 1 ampere per meter) of 3 centimeters diameter rods of various lengths:

<u>L</u> (meters)	<u>R</u> (ohms)	<u>RL</u> (ohm-meter or volts per ampere per meter)
1	0.73ρ	0.73ρ
2	0.42	0.85
4	0.24	0.95
8	0.13	1.07
16	0.07	1.17
32	0.04	1.28

We would mention again that a hemispherical earth, equivalent to a rod, has a radius close to a sixth of the length of the rod. A hemisphere with a resistance of 0.24ρ has, for instance, a radius of 0.66 meters as compared with a length of 4 meters for the corresponding rod.

(c) *The potential on the ground* at a distance X from the rod (Figure 20) has the expression (which is slightly inaccurate for distances approximating the diameter of the rod):

$$U_x = 0.366 \rho \frac{I}{L} \log \left(\frac{L}{X} + \sqrt{1 + \frac{L^2}{X^2}} \right).$$

The gradient on the ground at a distance X is:

$$G_x = \frac{\rho I}{(2 \pi X \sqrt{L^2 + X^2})}.$$

These equations can be simplified for distances X which are long or short as compared with L .

For long distances, the gradient evidently tends towards the general law $\frac{\rho I}{2 \pi X^2}$ which corresponds to hemispherical flow.

For distances which are short as compared with the length of the rod, we get:

$$U_x = 0.366 \rho \frac{I}{L} \log \frac{2L}{X} = 0.366 \rho i \log \frac{2L}{X},$$

$$G_x = \frac{\rho I}{2 \pi XL} = \frac{\rho i}{2 \pi X}.$$

The potential found from these equations for the surface of the rod is:

$$U_0 = 0.366 \rho \frac{I}{L} \log \frac{4L}{d}$$

(a value which is slightly inaccurate as mentioned above in section (b)). The error is, however, so slight that this expression can be retained in making practical calculations. The difference of potential between the rod and the ground at a distance X , which is short as compared with L , is then:

$$U_0 - U_x = 0.366 \rho \frac{I}{L} \log \frac{2X}{d}$$

The mathematical application of this equation shows that:

The voltage between a rod of the usual dimensions and the ground at the distance of a pace is of the order of 50 to 80 percent of the total voltage on the rod, that is, 50 percent for very long rods and 80 percent for very short rods (Figure 20).

III-9. Vertical Rod with Top Buried in Homogeneous Soil

Burying the top of the rod reduces the potentials and gradients at the surface of the ground. For the same bare useful length, burying up to a depth h gives

approximately the same reduction in the maximum gradient as an enclosure surrounding the non-insulated rod at a distance of 3 or 4 h (Figure 21).

The resistance of the rod for a given bare length is not affected by burying the head for all practical purposes.

III-10. Rectilinear Electrode Buried Horizontally

(a) *The resistance* of a rectilinear electrode with a length L and a diameter d, buried at a depth h, is expressed by:

$$R = 0.366 \frac{\rho}{L} \left(\log \frac{3L}{2d} + \log \frac{3L}{8h} \right)$$

The second term in brackets is suitable for moderate depths, but should be replaced by zero for infinite depth and by the value of the first term for semi-emergence.

The flow of a current $I = L_i$ raises the electrode, under the same conditions, to a potential

$$U_0 = 0.366 \rho i \left(\log \frac{3L}{2d} + \log \frac{3L}{8h} \right)$$

The resistance per unit length RL, which is equal to the voltage for one ampere flowing per meter, is usually a little larger for horizontal electrodes than for vertical rods because of usually greater lengths, smaller diameters and shallower depths.

With an electrode one centimeter diameter buried at a depth of one meter or 25 centimeters, the transverse resistance per unit length and the voltage taken by the electrode for one ampere carried per meter have, as common value:

L (meters)	RL (Ohm-meters or volts per ampere per meter)	
	(h = 1 meter)	(h = 25 centimeters)
10	1.37 ρ	1.59 ρ
20	1.59	1.81
50	1.89	2.11
100	2.11	2.33

It is sufficient to remember that:

The earthing resistance per meter of rectilinear electrodes buried horizontally is of the order of 1.5 to 2 ρ ohm-meters and the flow of one ampere per meter raises the electrode to about 1.5 to 2 ρ volts. The buried depth has no great effect in practice on the resistance.

(b) *The potential difference* between the electrode carrying one ampere per meter and a point on the ground situated at a distance X (which is small with respect to L) from the vertical plane of the electrode is expressed by (Figure 23):

$$U_0 - U_x = 0.366 \rho i \log \frac{X^2 + h^2}{dh}$$

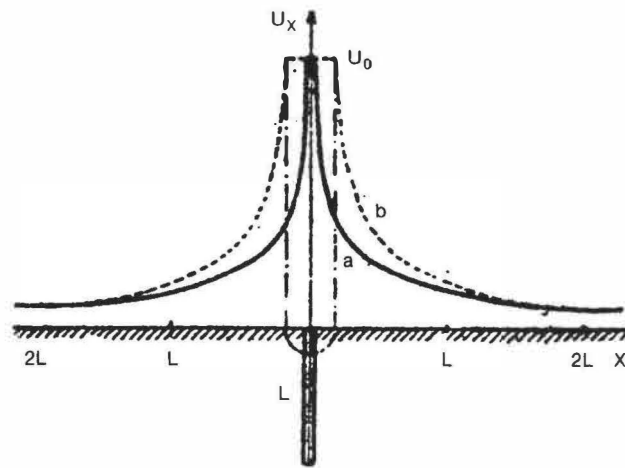


Fig 20
Potential Around a Rod (A) and
Around the Equivalent Hemispherical Electrode (B)

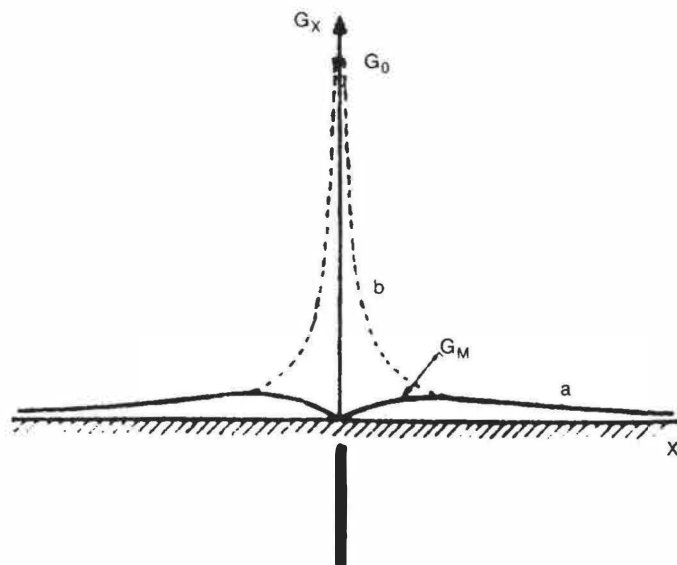


Fig 21
Earth Gradient Around an Earth Rod with Head Buried (A),
or Level with Ground (B)

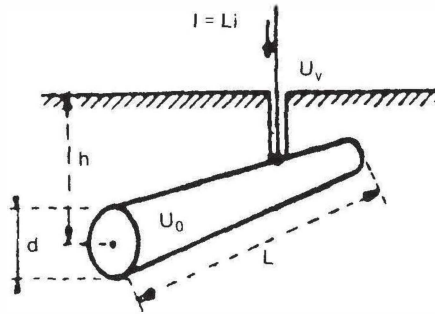


Fig 22
Straight Conductor Buried Horizontally

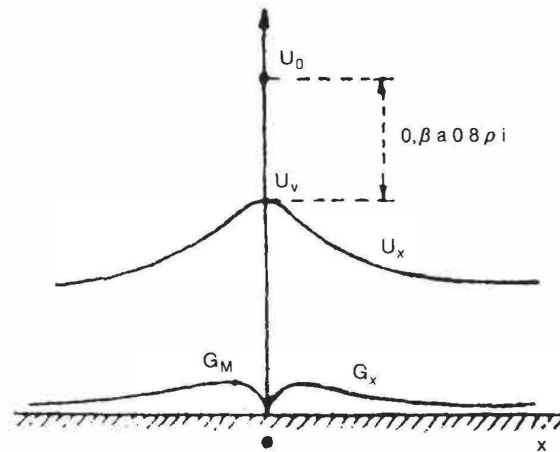


Fig 23
Potential and Gradient in Neighborhood of a Buried Horizontal Conductor

The maximum gradient on the ground is found at a horizontal distance from a point vertically above the electrode equal to the buried depth and has a value of

$$G_M = 0.16 \frac{\rho i}{h}$$

The application of these principles to usual conditions leads to the following rules:
The potential difference between the buried electrode and the soil around it is of the order of 60 to 80 percent of the product of the resistivity of the soil times the current carried away per meter. It increases slightly with the buried depth.

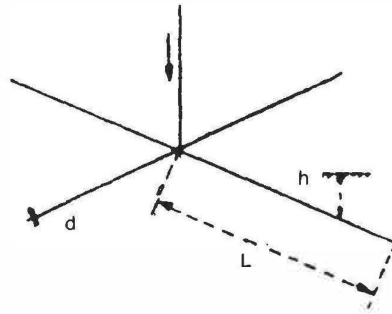


Fig 24
Earth Electrode with Radiating Arms

The potential gradient on the ground falls as the buried depth increases. For depth of the order of a pace, the maximum step voltage is about 10 to 15 percent of the product of the resistivity times the current carried away per meter.

III-11. Earthing Electrode with Radiating Arms

We can apply the following equations given by Dwight, in which L is the length of each arm, d the diameter of the electrode and h the depth at which it is buried (Figure 24):

Two arms at 180 degrees:

$$R = 0.366 \frac{\rho}{2L} \left(\log \frac{4L}{d} + \log \frac{L}{h} - 0.250 \right),$$

Two arms at 90 degrees:

$$R = 0.366 \frac{\rho}{2L} \left(\log \frac{4L}{d} + \log \frac{L}{h} - 0.103 + 0.19 \frac{h}{L} \right),$$

Three-branched star:

$$R = 0.366 \frac{\rho}{3L} \left(\log \frac{4L}{d} + \log \frac{L}{h} + 0.465 - 0.18 \frac{h}{L} \right),$$

Four-branched star:

$$R = 0.366 \frac{\rho}{4L} \left(\log \frac{4L}{d} + \log \frac{L}{h} + 1.265 - 0.93 \frac{h}{L} \right),$$

Six-branched star:

$$R = 0.366 \frac{\rho}{6L} \left(\log \frac{4L}{d} + \log \frac{L}{h} + 2.98 - 1.36 \frac{h}{L} \right).$$

These equations are only suitable for average buried depths. For zero depth the second logarithmic term should be made equal to the first, and for an infinite depth the resistance should be reduced by half as compared with zero depth.

These equations bring out the fact that the effectiveness of radiating arms decreases when their number is increased.

Assuming arms 10 meters long and one centimeter diameter buried at a depth of one meter, we have the following results:

Number of branches	Resistance (ohm)	Increase of length (percentage)	Gain in conductivity (percentage)
1	0.137ρ	—	—
2	0.082	100	68
3	0.062	50	32
4	0.053	33	17
6	0.046	50	15
Ring, $r = 10$ meters	0.034	4.7	35
Plate, $r = 10$ meters	0.023	—	48

The gains would be slightly better with longer arms. The star with several arms has the further advantage over the simple type in that it facilitates the discharge of the very steep-fronted surges. However, it is usually completely useless to exceed three or four arms.

III-12. Effect of Non-Insulated Connections

Bare buried connections linking the earth electrode with the structures to be earthed behave like extensions of the electrodes. Their diameter is generally small, but it has been seen that this fact is of little importance provided that contact with the soil is well arranged. On the other hand, account must be taken of possible drying out of the superficial layers of the ground in which they are set.

The effect of bare connections on resistance may be of secondary or predominant importance, according to their lengths as compared with that of the electrodes.

Consider a plate with an area one square meter buried at a depth of 1.5 meters in homogeneous ground and connected by a bare vertical connection. The plate alone would have a resistance of approximately 0.25ρ and the connection alone, considered as a rod, a distinctly higher resistance of about 0.70ρ . Further, the plate will interfere with the diffusion of the current by the connection. In actual fact, the presence of the connection will not modify the overall resistance to any great extent.

Consider now four 3-meter rods at the angles of a 6-meter square and linked by cross-shaped connections. The resistance of the 4 rods alone will be in the neighborhood of 0.10ρ and that of the connections some 0.11ρ . The over-all resistance will be of the order of 0.08ρ . The placing of the rods, provided the ground is homogeneous and that no drying-out effect of the superficial layers is introduced,

will not change the value of the resistance by comparison with that which would correspond to the connections along, to any great extent.

It will be seen later that, for substations covering a large area, the connections between the structures to be earthed are usually sufficient to form an excellent earth electrode by themselves.

III-13. Use of Non-Circular Section Earthing Conductors

The circular form of earth conductor gives the smallest perimeter for a given section and thus has the lowest capacity for discharging current.

It will be noted in particular that a long thin strip with a width W is approximately equivalent to a round conductor with diameter $\frac{W}{2}$.

The resistance of a long electrode depends only to a slight extent on its diameter, to such an extent that the gain to be expected from non-circular conductors is generally illusory. Compare, for instance, a one centimeter round conductor and a strip 4 centimeters wide and 0.2 centimeters thick. They have approximately the same cross-section. The strip is equivalent to a two centimeter diameter round conductor, double that of the first one, but for a same buried length of ten meters, the resistance will only differ by 6 percent. The difference would be even more insignificant for longer buried lengths.

It is therefore useless to depart from the circular type of conductor which is the sturdiest mechanically.

III-14. Earth Electrode in a Heterogeneous Subsoil

We will only discuss here the simple case in which the ground consists of a superficial horizontal layer with a thickness H and a resistivity ρ_1 and of an homogeneous subsoil with a different resistivity ρ_2 .

We will confine ourselves to indicating briefly how much non-uniformity of the soil is able to react on the resistance of earth electrodes and the gradients in their vicinity without going into the question of methods used for exploring the resistivity of the soil in depth.

(a) *Resistance of a small electrode on non-uniform subsoil.*

If the electrode can be compared to a hemispherical electrode having a radius r , sufficiently small as compared with the thickness H of the superficial layer, its resistance has the approximate value:

$$R = \frac{\rho_1}{2\pi r} + 0.366 \frac{\rho_1}{H} \log \frac{\rho_1 + \rho_2}{2\rho_1} .$$

The first term will be the resistance of the hemisphere in a homogeneous medium having a resistivity ρ_1 . The second term which is independent of the radius r , provided that it is small, is additive or subtractive according as ρ_2 is higher or lower than ρ_1 . It tends toward infinity for a subsoil with infinite resistance.

To give an example, for an electrode with a radius half that of the thickness H of the superficial layer, the resistance takes (as a function of the relationship

$\frac{\rho_2}{\rho_1}$), the following relative values:

$\frac{\rho_2}{\rho_1}$	0	0.1	1	10	100	1000
R	0.65	0.7	1	1.85	3.1	4.1

Figure 25 gives the general course of the flow of current in the case of a point electrode for different values of $\frac{\rho_1}{\rho_2}$. The same paths would be suitable for all electrodes which conform approximately to the contour of an equipotential surface, and in particular for small hemispherical electrodes.

The potential gradients are practically the same, in the close vicinity of small electrodes, as if the soil were homogeneous and had a resistivity ρ_1 . The effect of the subsoil becomes more and more pronounced the greater the distance. It tends to reduce the gradients if the subsoil is more conductive than the surface and to increase them under the opposite conditions by bringing them, very quickly in the first case and more slowly in the second, close to the value corresponding to a homogeneous soil with a resistivity ρ_2 .

(b) *Subsoil more conductive than the surface*

The equation given above allows of calculating the resistance of small electrodes. It is generally possible to get an idea of the behavior of more complex electrodes without complicated calculations. If, for instance, the electrode is a long vertical rod having a length H in soil of resistivity ρ_1 , and a length $L-H$ in subsoil or resistivity ρ_2 , it can be assumed that the ground is homogeneous and has a resistivity ρ_2 , by simply reducing the length H situated in the resistant layer in the proportion $\frac{\rho_2}{\rho_1}$.

If the electrode can be compared to a superficial plate of larger dimensions as compared with the thickness H , it may be assumed that the plate is placed directly on subsoil having a resistivity ρ_2 , and to the resistance thus found should be added that of a cylinder having a resistivity ρ_2 , and to the resistance thus found should be added that of cylinder having a resistivity ρ_1 with a height H and a cross-section equal to the surface of the plate.

As mentioned above, the potential and gradient round an electrode implanted on a subsoil which is more conductive than the surface, very quickly reach (at a distance of approximately the thickness of the superficial resistant layer) values which would correspond to homogeneous ground with the same resistivity as that of the subsoil.

(c) *Subsoil more resistant than the surface*

With subsoil having a very high resistivity ρ_2 , the threads of current have difficulty in spreading in depth and run superficially to distances which become all the greater as ρ_2 gets higher. The result, as we have seen, is an increase in the resistance and a less rapid fall in the potential gradients around the electrode as compared with homogeneous soil with a resistivity ρ_1 .

A particularly simple limiting case is that of a subsoil which is a perfect insulator. The equation above would then give the resistance an infinite value. An

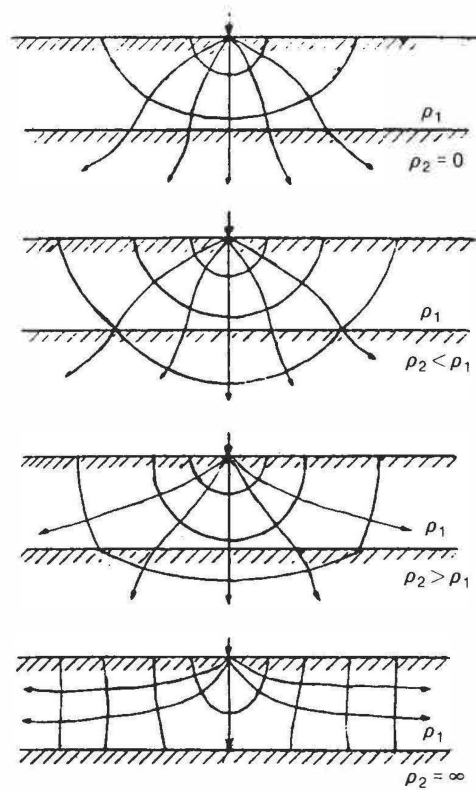


Fig 25
Field Around a Point Electrode, in Nonuniform Ground,
for Different Values of Subsoil Resistivity

infinite potential is, in fact, necessary to force the current to flow as far as infinity when it cannot spread out in depth. But it would still be possible to define the potential difference between the electrode having a small radius r and a point situated at a distance X . It will be approximately

$$U_0 - U_x = \frac{\rho_1 I}{2\pi} \left(\frac{1}{r} - \frac{1}{X} \right) + 0.366 \rho_1 I \log \frac{1 + \sqrt{1 + \frac{X^2}{H^2}}}{1 + \sqrt{1 + \frac{r^2}{H^2}}}$$

Further, in such a case the most simple electrode is no longer hemispherical, but a vertical cylinder having height H and diameter d , and passing through the whole of the conducting layer (Figure 26). The threads of current then spread radially in horizontal planes and the equipotential surfaces are cylinders concen-

tric with the electrode. The current $I = Hi$ has available for its passage, at a distance X from the electrode, a cross-section $2 \pi HX$, whence a resulting local gradient:

$$G_x = - \frac{\rho I}{2 \pi H X} = - \frac{\rho i}{2 \pi X}$$

and, by integration, a potential difference between two points X and X' :

$$U'_x - U_x = 0.366 \rho i \log \frac{X'}{X},$$

whence, by taking one of the points at the surface of the electrode

$$U_0 - U_x = 0.366 \rho i \log \frac{2X}{d}.$$

We thus find again the same expression as for gradient and potential in the vicinity of a rod in homogeneous ground. This was to be expected because, in this case, too, the current lines extend radially before spreading in depth.

Each multiplication by 10 of the distance X increases the voltage drop $U_0 - U_x$ by an identical amount $0.366 \rho i$.

These simple equations can be applied to electrodes of any shape on condition that they are made to correspond to equivalent cylinders having a length H and that the point X is chosen sufficiently far from the irregularities of the surface.

They also apply for moderate distances from the electrode in the usual case in which the soil has not an infinite resistance. At greater distances account must be taken of the diminution in current which continues to follow the superficial layer, since an ever larger fraction of the total current leaves it progressively and spreads in depth.

The mean path of the current in the superficial layer, that is, the distance from the electrode at which half the current has left this layer, can be defined. This distance is approximately

$$X = \frac{\rho_2}{\rho_1} H$$

in the case of a small electrode.

Thus, with a subsoil having ten times the resistance of the surface, the average path of the currents in the more conductive superficial layer is of the order of 10 times the thickness of this layer. The current which discharges through the superficial layer at this distance is only half the total current. The gradient is half what would be computed, assuming the subsoil to have an infinite resistance, or, again, half that which would correspond to a homogeneous soil with the same resistivity ρ_2 as the subsoil (Figure 27).

At definitely greater distances the superficial layer contributes less and less to the current discharge and the potential and gradient can be calculated in practice as if the soil were homogeneous with a resistivity ρ_2 .

With an earth electrode of very large dimensions as compared with the thickness of the superficial conducting layer, the latter has practically no effect on the

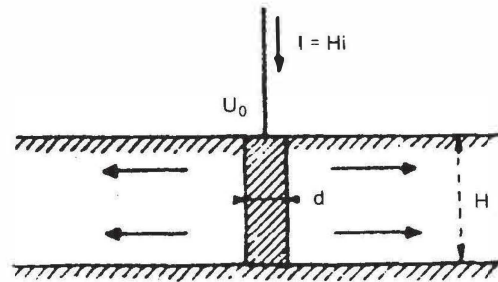


Fig 26
Cylindrical Electrode with Insulating Subsoil

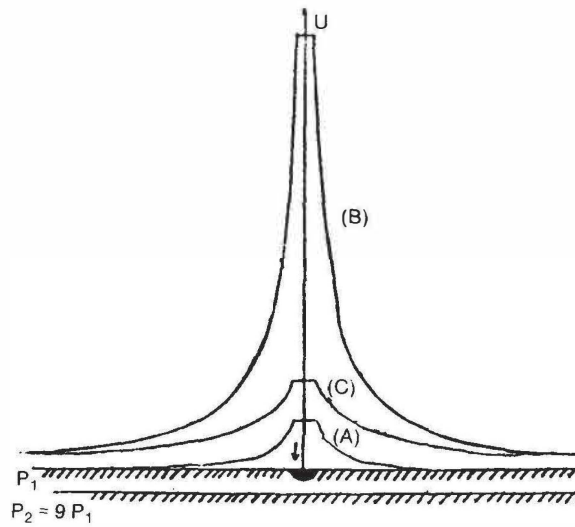


Fig 27
Potential Around an Earth Electrode Having a Radius r :
A, Homogenous Medium with Resistivity P_1 ;
B, Homogenous Medium with Resistivity $P_2 = 9 P_1$;
C, Layer with Resistivity P_1 and Thickness $2r$ on Subsoil with Resistivity $P_2 = 9 P_1$

value of the resistance. Its only effect is to reduce the local potential gradients in the close vicinity of the boundaries of the electrode.

IV. Multiple Earth Electrodes. Interaction Between Electrodes.

IV-1. Electrodes in Parallel in Homogeneous Soil

(a) General considerations

A multiplicity of electrodes in the ground interferes with the diffusion of their partial currents in the soil and thus increases their individual resistances. (Figure 28).

The central electrodes are more affected than the peripheral ones and the current will tend to be displaced towards the latter.

These effects are small when there is a small number of electrodes and the separations are large compared with their dimensions. They are considerable when the electrodes are numerous and close together.

Above a certain degree of occupation of a ground area of a given size there is practically no gain, as regards resistance, by adding additional electrodes. However, even beyond this limit, increasing the number of electrodes reduces the current discharged by each and thus reduces the local potential gradients.

A basic statement, linked with the fact that the gradient decreases rapidly as the distance from the electrode increases, runs as follows:

The potential gradients in close proximity to a particular electrode depend solely on the current discharged by this electrode and are only very slightly affected by the more distant electrodes.

In consequence, in the neighborhood of a horizontally buried electrode carrying IA/m , the potential between electrode and ground and the maximum gradient will be approximately the same as if the electrode were single, that 0.50 to 0.80 ρI and 0.10 to 0.15 ρI respectively, however complex may be the whole of the buried circuits to which the electrode considered belongs.

(b) Review of methods of computation

The computation of multiple or complex electrodes is based on two statements made above regarding hemispherical and spherical earth electrodes.

First, the effect of irregularities in the shape of an electrode becomes less as the distance from it increases, and the ground potential at a distance X from the electrode by the flow of a current I tends to approach the simple law

$$U_x = \frac{\rho I}{2 \pi X} .$$

In the second place, the presence of electrodes which are of small dimensions as compared with the distances causes only a slight degree of non-uniformity in the ground and thus produces little change in the field of an individual electrode with the consequence that the resulting potential, at a point situated at distances $X_1, X_2 \dots$ from electrodes discharging currents, $I_1, I_2 \dots$ in a homogenous medium, is substantially the sum of the individual effects of each electrode assumed to exist singly (Figure 29).

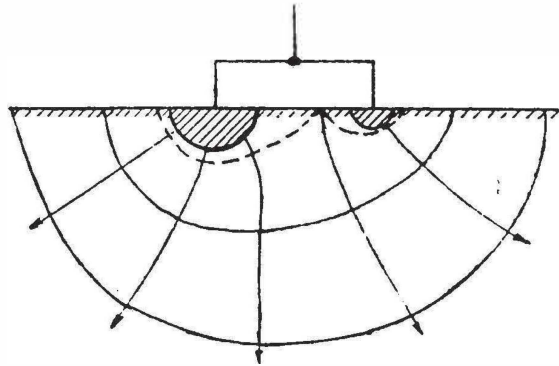


Fig 28
Distribution of Ground Between Two Electrodes in Parallel

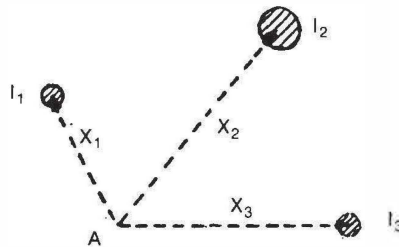


Fig 29
Point A in Relation to Several Electrodes

$$U = \frac{\rho I_1}{2 \pi X_1} + \frac{\rho I_2}{2 \pi X_2} + \dots$$

This equation, in which neither the shape nor the dimensions of the electrodes are introduced, is sufficient for taking into account all the electrodes distant from the point considered. When this point is very close to some particular electrode, the more accurate equations given in preceding paragraphs should naturally be used for the term corresponding to this electrode.

This method permits treatment of problems of multiple or complex electrodes with fairly close approximation and we have already referred to it in connection with the calculation of rods and other rectilinear electrodes.

The question arises as to how the total current is distributed between individual electrodes in parallel. The calculation can be carried out by giving arbitrary values $I_1, I_2 \dots$ to the individual currents and examining the potentials resulting at the surface of each electrode and stating that all the potentials are equal. This is tedious when the number of electrodes is large.

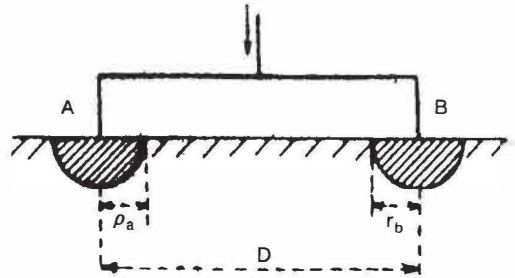


Fig 30

Two Hemispherical Electrodes in Parallel

As a rule, only permissible errors occur with regard to the over-all resistance when assuming that the current is distributed between the electrodes as the inverse of the individual resistances. When calculating the local gradients, the results obtained can be made more accurate by somewhat increasing the contribution of the peripheral electrodes to the disadvantage of the central ones, or in the case of a buried electrode, by raising towards the extremities the current flowing per unit of length. Such corrections can, without objection, be made empirically in view of the low accuracy of practical data as to the nature of soils.

(c) *Two electrodes not overlapping*

As an example, in the very simple case in which two earth electrodes A and B are sufficiently far apart to be considered as two equivalent hemispheres of radii r_a and r_b spaced D apart, we get

$$U_0 = \frac{\rho}{2\pi} \left(\frac{I_a}{r_a} + \frac{I_b}{D} \right)$$

$$U_0 = \frac{\rho}{2\pi} \left(\frac{I_a}{D} + \frac{I_b}{r_b} \right),$$

by taking the reference point successively on A and B (Figure 30), and further

$$I_a + I_b = I$$

The currents I_a and I_b can readily be deduced from these equations as well as the overall resistance $\frac{U_0}{I}$.

In particular, if the two electrodes are identical:

$$I_a = I_b = \frac{I}{2}$$

$$R = \frac{1}{2} \cdot \frac{\rho}{2\pi r} \left(1 + \frac{r}{D} \right).$$

The coefficient of increase in resistance due to proximity of the electrodes is $\frac{r}{D}$.

We will come back to these calculations in paragraph 4, which deals with coupling between neighboring earth electrodes.

(d) *Vertical rods forming a regular pattern of squares*

The rate of increase of the individual resistances is about:

15% for	2 rods spaced at once their length
15% for	3 rods spaced at twice their length
15% for	6 rods spaced at four times their length
50% for	6 rods spaced at once their length
50% for	13 rods spaced at twice their length
50% for	40 rods spaced at four times their length
100% for	10 rods spaced at once their length
100% for	28 rods spaced at twice their length
100% for	100 rods spaced at four times their length

This table allows for rapid interpolation so that intermediate cases can be dealt with to a considerable degree of accuracy without the use of nomographs. For electrodes other than rods it should be noted that a four-meter rod is approximately equivalent to a plate with one-meter sides or to a hemisphere with a radius of 0.65 meters.

From another standpoint, if R is the resistance of a solid plate covering 1 hectare (100 meters · 100 meters), 30 three-meter rods distributed over this surface will have a resistance $3R$, 56 rods a resistance $2R$ and 100 rods a resistance $1.5R$. It will be seen that the ground is almost saturated with a total of 300 meter length of rods.

These evaluations assume that the rods are linked by non-buried connections, an arrangement in considerable favor in certain foreign countries, combined with small-diameter rods driven in with a pneumatic hammer. In this way the work of digging and boring is greatly reduced. If, on the other hand, the connections are buried they will, since their length is usually considerably greater than that of the rods, often be sufficient to saturate the ground without it being necessary to use the rods.

(e) *Electrodes forming horizontal meshes or loops*

As an example we will compare the relative values of conductances and local gradients obtained by burying two or three regularly spaced concentric loops formed from a one-centimeter-diameter conductor in a circular area of ground 80 meters in diameter (Figure 31).

The gradients in the vicinity of the loops, which are proportionate to the current per meter, are designated, starting from the peripheral to the center loop:

Number of Loops	Buried Length, Percent	Conductance, Percent	Local Gradients, Percent		
1	100	100	100	—	—
2	150	123	70	59	—
3	200	135	56	44	41
Solid plate	—	148	—	—	—

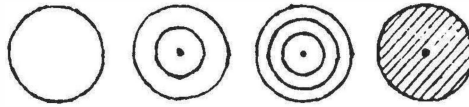


Fig 31
Concentric Rings and Solid Plate

It will be seen that one or two loops are almost sufficient to saturate the ground as far as resistance is concerned. As regards the gradients, on the other hand, increasing the lengths buried secures a gain approximately proportional to the lengths. The fact is confirmed that the currents discharged per meter are greater at the periphery than at the center, but this effect is not very pronounced. Regular distribution would have given a uniform gradient of 50 percent for the three loops, which is not very far removed from the extreme value of 56 percent found for the outside loop.

It will be noted, too, that obtaining a given resistance requires greater length in continuous conductors than in spaced rods. But the cost per meter may be different and each case needs special investigation from the financial viewpoint.

The case for earthing arrangements for very large stations where the length of buried conductors becomes very great will be considered in chapter V.

IV-2. Multiple Electrodes on a Non-Uniform Subsoil

(a) Neighboring electrodes in the superficial layer of the ground interfere with each other relatively less than the subsoil as a better conductor than the surface. The mutual interference between the current paths is confined more especially to the portion situated in the more conductive subsoil and thus only influences a relatively small part of the resistance. If, however, the electrodes penetrate far into the conducting subsoil, the superficial layer can be more or less ignored and the disturbance is approximately the same as with homogeneous ground.

(b) When the subsoil, on the other hand, has a much higher resistance than the surface, it has been seen that the current has difficulty in spreading out in depth and that its effects show much further away from the electrodes.

The obvious result is that electrodes in parallel cause more interference and fewer electrodes are needed to saturate a given area.

In the extreme case of a subsoil which is a perfect insulator, the sum of the individual effects of the electrodes would immediately give

$$U - U' = 0.366 \rho_1 \left(i_1 \log \frac{X'_1}{X_1} + i_2 \log \frac{X'_2}{X_2} + \dots \right)$$

for the potential difference between any two points at distances $X_1, X_2, \dots$ and $X'_1, X'_2, \dots$ from electrodes discharging currents $i_1, i_2 \dots$ per meter of conductive thickness.

The distribution of the total current between electrodes in parallel would be obtained by taking the two points of reference at the surface of different pairs of electrodes and by stating that all the corresponding left-hand sides of the equations are zero.

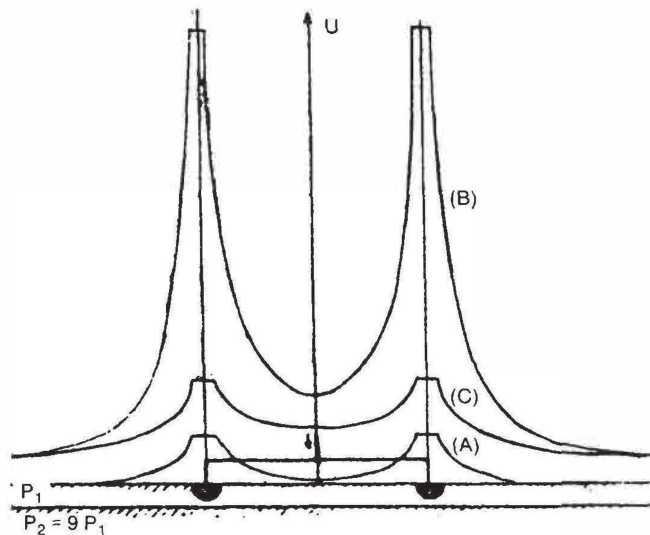


Fig 32

Potential Around Two Earth Electrodes Having a Radius r and Spaced $20r$:

A, Homogenous Medium with Resistivity P_1 ,

B, Homogenous Medium with Resistivity $P_2 = 9 P_1$,

C, Layer with Resistivity P_1 and Thickness $2r$ on Subsoil with Resistivity $P_2 = 9 P_1$

This equation also applies approximately to cases in which the resistivity of the subsoil is finite, on condition that the electrodes are not too far from each other. It gives lower values for the local potential differences than if the high resistivity of the subsoil prevailed throughout the whole ground area. These local potential differences depend essentially on the resistivity of the surface, whereas the overall potential of the area occupied by the earth electrodes depends rather on that of the subsoil (Figure 32).

On a low-voltage rural network run over a granite subsoil and earthed at a number of points separated by several hundred meters, it was found that the discharge of current through the neutral conductor raised the whole of the area supplied to a fairly high fraction of the potential of this conductor.

IV-3. Effect of Proximity of Earth Electrodes Carrying Return Current; Application to Measurement of Earth Electrode Resistances.

The general equations so far given for calculating the earth potential in the vicinity of multiple electrodes, in homogeneous soil or with a subsoil which is an insulator, obviously apply also to cases in which one or more of the electrodes act as return current paths. It is only necessary to change the sign of the currents they carry.

The case of return-current electrodes at a short distance is met with, more especially, in the measurement of earthing resistances when the distance of the

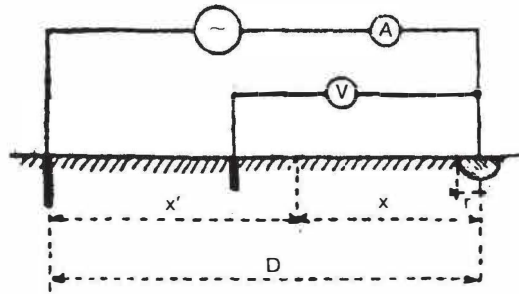


Fig 33
Resistance Measurement of an Earth Electrode

auxiliary electrodes is not great as compared with the dimensions of the earth to be investigated. The error resulting may then be large. Consider the case of an earth to be measured and a return-current earth separated by D in homogeneous ground (Figure 33). The potential of a point at a distance X and X' from the two earth electrodes will be

$$U = \frac{\rho I}{2\pi X} - \frac{\rho I}{2\pi X'} = \frac{\rho I}{2\pi} \left(\frac{1}{X} - \frac{1}{X'} \right)$$

to the extent that irregularities in the shape of the electrodes have no effect as far as this point. The potential is zero when $X = X'$, and the additional electrode for measuring the potential should be placed at an equal distance from the centers of the two electrodes whatever the dimensions of the electrodes.

Assume that the electrode to be measured is equivalent to a hemisphere electrode with radius r . Its potential can be deduced from the equation above by taking the reference point on the surface of the hemisphere:

$$U_0 = \frac{\rho I}{2\pi} \left(\frac{1}{r} - \frac{1}{D} \right).$$

Assuming that the connection for measuring potential has been properly attached at a point of zero potential, it is this voltage U_0 which measurement will show, and

$$R' = \frac{\rho}{2\pi r} \left(1 - \frac{r}{D} \right)$$

will be deduced for the resistance of the earth electrode instead of the theoretical value with the return-current earth distributed to infinity:

$$R = \frac{\rho}{2\pi r}$$

The relative error due to the proximity of the return-current electrode is thus $\frac{r}{D}$, that is 10 percent if the distance from the earth electrodes is ten times the

equivalent radius of the tested earth and 25 percent if it is only four times this radius, etc. Since high accuracy is not essential, distances of 10 to 30 meters are sufficient for measuring the resistance of a plate or rod of average value.

On the other hand, in case of a station 200 by 200 meters, equivalent to a half sphere of some 75 meters radius, the return-current earth must be placed at more than 300 meters from the center to ensure that the error does not exceed 25 percent.

Of course the error thus committed is added to that caused by the fact that the connection for ascertaining potential is never placed exactly at a point of zero potential. In the calculations, too, the ground is assumed to be homogeneous and fairly large errors may otherwise be introduced, in particular, when buried cables or ducts pass close to auxiliary earth electrodes.

We would mention, however, that there are cases in which the true value of the resistance has only a platonic interest, as regards safety, and in which it is much more important to measure the potential differences in reference to the different points at which one may stand while touching structures connected to the earth electrode.

IV-4. Coupling Effect Between Neighboring Earth Electrodes

Consider an elementary volume of ground B close to an earth electrode A, which the flow of current raises to a potential U_o . The ground unit B will itself be raised to a potential U equal to a fraction of U_o , and this fraction $\frac{U}{U_o}$ may be called the "coefficient of coupling" of B with A.

In cases where the unit B is small and distant by D from the center of the hemisphere of radius r_a , equivalent to the electrode A, the coefficient of coupling of B with A in homogeneous soil has the simple expression:

$$k_{ba} = \frac{r_a}{D} .$$

When B is also an earth electrode (Figure 30), it is possible similarly to define a coefficient of coupling of A with B, this being equal to the fraction of the potential of B to which the flow of a current through the latter raises A. If B is equivalent to a hemisphere of radius r_b , we now have:

$$k_{ab} = \frac{r_b}{D} .$$

Again, it may be necessary to deal with the ratio K of the potentials which one of the electrodes A or B acquires according as it is inactive or as it is placed in parallel with the other electrode to discharge the current.

The calculation of the over-all resistance of the two electrodes has been mentioned in IV-1. The new coefficient K is deduced from this without difficulty and the fact that it has the same value for A and B can be checked.

It is preferable to deal with the problem in a more general way in noting that the unit comprising any two earth electrodes A and B and the return-current electrode (consisting here of the distant ground) forms a passive system with three terminals. But it is known that the exchanges between the terminals of such a system, however complicated this latter may be, are completely defined by three

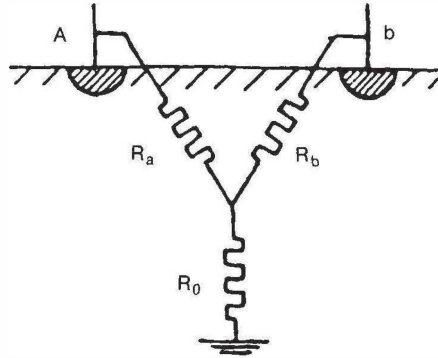


Fig 34

Equivalent Diagram of Two Coupled Earth Electrodes

internal parameters only, for which we can choose, for instance, three resistances R_a , R_b , and R_o joining the three terminals in a Y-circuit (Figure 34).

$R_o + R_a$ is the total resistance offered to the flow of current via A if B is not concerned.

$R_o + R_b$ is again the resistance which determines the potential of B when A is not concerned.

R_o is the part common to these resistances and it thus determines the potential at one of the terminals when the other is the one which discharges the current.

In particular, if it is possible to consider the earth electrodes as equivalent hemispherical electrodes, we have:

$$R_o + R_a = \frac{\rho}{2\pi r_a}, \quad R_o + R_b = \frac{\rho}{2\pi r_b}, \quad R_o = \frac{\rho}{2\pi D}.$$

The three coefficients of coupling defined above can be expressed as a function of the three resistances by the general relationships:

$$k_{ba} = \frac{R_o}{R_o + R_a}, \quad k_{ab} = \frac{R_o}{R_o + R_b}$$

$$K = \frac{R_o}{R_o + \frac{R_a R_b}{R_a + R_b}}$$

Further, the three coefficients are linked by the relationship

$$K = \frac{k_{ba} + k_{ab} - 2k_{ba}k_{ab}}{1 - k_{ba}k_{ab}},$$

which becomes

$$K = k_{ba} + k_{ab} = \frac{r_a + r_b}{D}$$

when the coupling coefficients are small, that is, when the distance from the electrodes is great as compared with their equivalent diameters.

From these expressions it is possible to immediately deduce the following rules are valid in homogeneous ground:

If the potential of a passive electrode is to be less than 10 percent of that of the active (live) electrode, the former must be spaced from the latter by at least ten times the radius of the active element. If the potential of a passive electrode is to be less than 10 percent of the potential it would acquire if placed in parallel with the active electrode, the two electrodes must be spaced by at least ten times the sum of their equivalent radii.

A subsoil which is a better conductor than the superficial layers tends to reduce the coupling and a more resistant subsoil to increase it.

In cases in which one of the earth electrodes is of very large dimensions, it is practically impossible to obtain low values for the coupling. This point will be taken up again in the next chapter in connection with the question of earthing arrangements for large stations.

IV-5. Advantages and Limitations of Electrode Interconnection

Interconnection of all the safety earthing arrangements within an installation is always desirable. It is the best method of obtaining an earth having the lowest resistance at the minimum cost. It is also the method of reducing the potential differences within the installation to the greatest extent possible.

Interconnecting the earths is also the simplest system to realize, as well as the one which requires the minimum of investigation and thought. Apart from the fact that effective separation of the earths may be, as we have seen, difficult to effect it is often arduous to provide for all the phenomena to which the appearance of considerable local potential differences might give rise in complex installations, the subsoil of which is often crossed by many conducting wires and ducts laid with no general layout in view by workmen from different sections of the undertaking.

Although the same arguments apply to system earths, the solution is not so obvious for the neutrals of high or low potential circuits which extend outside the boundary of the plant.

(a) Where there is a high-potential circuit leaving the station, every earth fault *on the line* causes a flow of current, part of which at least re-enters by the local neutral and raises its potential. If the protective earth is also used for earthing the neutral, the whole of earthed frames and structures of the station is raised to this potential. A separate neutral earth situated in an inaccessible spot avoids in principle inconveniences which might result for personnel. (Figure 35, a and c).

But it may be objected that stations in which the neutral of high-potential networks is earthed are often large ones where the earth for the frames is frequently good enough to carry heavy currents without excessive potential gradients and where an effective separation of the earth electrodes would be very difficult to carry out. It was seen, too, at the beginning of this paper that safety is primarily assured on these networks by high-speed circuit-breaking and not by limiting the potentials on the ground.

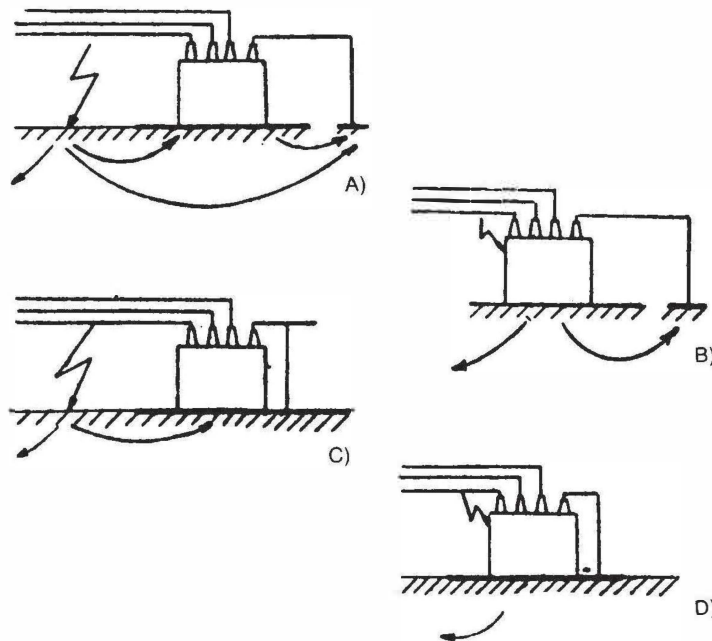


Fig 35

**A, Separate Earths: Line Fault;
B, Separate Earths: Fault In Station;
C, Interconnected Earths: Fault On The Line;
D, Interconnected Earths: Fault In Station**

Again, if the fault occurs in the plant itself and not on the line, separating the earths compels the frame earthing arrangement to carry the whole of the fault current, whereas their interconnection would offer a metal path to the part of the current which completes the circuit through the local neutral (Figure 35, b and d). The circulation in this case of large currents from earth to earth in a restricted area of ground is, too, not without various risks.

It can be considered that separation of the earthing arrangements for high-potential neutrals is not to be recommended except in exceptional cases.

The question should then be the subject of careful investigation into the positioning of the separate electrodes and the course of the buried conduits and cables.

(b) With neutrals of low-potential circuits extending outside the station, the problem is no longer to protect the staff and the material in the plant, but to protect distant consumers against chance rises in potential of the station earth, a potential which low-potential overhead lines connected to this earth may propagate to localities in which the local potential is greatly different.

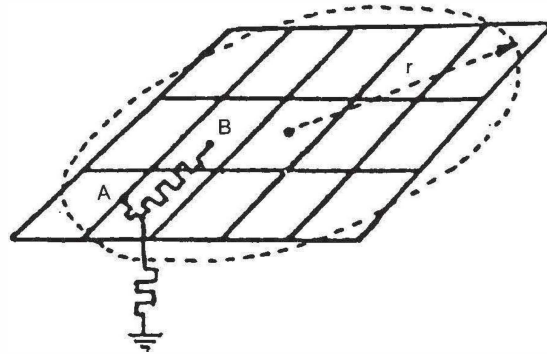


Fig 36
Earth Circuits A of a Large Station; Equivalent Plate (Shown Dotted)
and Coupling With a Small Local Electrode B

It may be argued against separation that the earthing arrangements for the low-potential neutral conductor are often very good and that the fault currents to be discharged by the station earth are sometimes very small, for instance, when high-potential networks have an insulated neutral or Petersen coils. Again, potentials will still exist in the station between circuits and frames, from which the low-potential circuit is to be safeguarded. In the case of these potentials being high, flashover may result (apart from the danger to personnel), thus making violent connection.

All these arguments need to be weighed impartially in each particular case. It must be recognized, however, that with the poor earth electrodes still to be found in some small substations, the separation of the low-potential earth is often justified there.

V. Review of Some Special Problems

V-1. Earth Resistance and Potential Gradients in Large Stations

The length of the conductors it is necessary to bury at large stations in order to effect the required connections between frames, structures, etc., is so great that the ground can be considered to be practically saturated and almost equivalent to a solid plate from the point of view of resistance, if not always as regards local gradients.

(a) *Homogeneous ground.* The resistance of a substation with an average radius r , erected on homogeneous ground and having a length L of buried wiring (Figure 36) can be expressed approximately by the following equation:

$$R = \frac{\rho}{4r} + \frac{\rho}{L},$$

or still more simply:

$$R = \frac{2\rho}{\text{perimeter}}$$

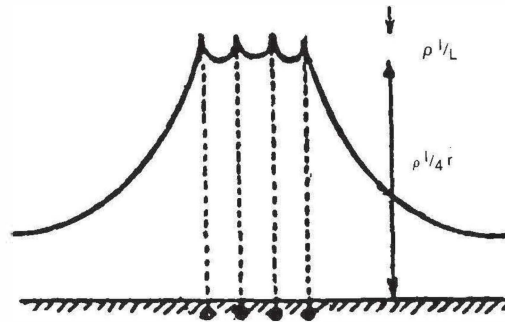


Fig 37
Potential in Neighborhood of Earth Circuits of a Large Station

The resistance of the earth circuits of a large station in homogeneous ground differs only slightly from the quotient of the resistivity of the soil over the half-perimeter of the installation.

In the first expression for R , the first term is the resistance of a superficial plate having a radius r . The second term, which is usually much smaller, expresses the divergence of the real earth as compared with this solid plate. The corresponding voltage $\frac{\rho I}{L}$, which is proportional to the average current flowing per meter of

conductor, represents to some extent the potential drop between the buried electrodes and the central area of the more or less regular meshes which they form. (Figure 37).

The difference in potential between the soil in the middle of the meshwork and the structures connected to the earth circuits will thus be of the order of ρi .

The step-potential in the neighborhood of the buried electrodes will be close to 0.10 to 0.15 ρi , being somewhat higher towards the periphery and somewhat lower toward the center.

These rough equations yield only approximations, since they take no account of the size of the meshwork, but the approximate interpretation they provide for the phenomena involved is not without interest.

Consider a substation 250 meters $\cdot$ 150 meters, erected on ground with a resistivity of 200 ohm-meters and having 4000 meters of buried electrodes which have to discharge 4000 amperes in case of fault in the installation. The resistance of the earth circuits will be of the order of:

$$R = \frac{200}{250 + 150} = 0.50 \text{ ohm.}$$

The discharge of 4000 amperes will raise the earth circuits of the station to some 2000 volts in relation to the distant ground.

The part of the resistance which is a function of the buried length will be about

$$\frac{\rho}{L} = \frac{200}{4000} = 0.05 \text{ ohm.}$$

The differences in local potentials in the substation may reach approximately:

$$\rho_i = \frac{\rho I}{L} = 200 \text{ volts}$$

and the step potentials in the neighborhood of the buried electrodes will be approximately

$$0.15 \rho_i = 30 \text{ volts}$$

This calculation, though approximate, shows to what extent large stations are usually favorably placed as regards the potentials which may appear, in the case of an earth fault, within the area covered by the earth circuits.

(b) *When the subsoil is a better conductor than the surface*

Assume that the subsoil has a resistivity ρ_2 which is less than the resistivity ρ_1 at the depth at which the earth circuits are buried.

In the resistance equation, ρ_2 should be introduced for the term corresponding to the effect of the plate, and ρ_1 for that corresponding to the local potential drops in the neighborhood of the electrodes. The equation then becomes:

$$R = \frac{\rho_2}{4r} + \frac{\rho_1}{L}$$

In the previous example, if drying of the ground in summer raises ρ_1 from 200 to 1000 ohm-meters, the second term will change from 0.05 to 0.25 ohm, and the total resistance will only vary from 0.67 to 0.87 ohm.

The earths of stations covering a large area will, as a rule, only show small seasonal variations.

The gradients at the surface will be increased to a greater extent than the resistance by drying of the soil, but the very high resistivity of the latter in contact with the feet will then, by itself, often constitute a form of protection for the staff.

The very rudimentary form of the equation above may be criticized since it does not take into account the thickness of the resistive layer. It assumes by implication that this thickness is about half the dimension of the meshes at most. But it may easily be seen that 10 meters additional thickness of a soil with a resistivity of 1000, for a station covering two hectares (20,000 square meters), only introduce a resistance of some 0.5 ohm. It is also easy to adapt the equation to particular cases when measurements of resistivity at different depths are available. It is only given as a guide and does not dispense with the need for a minimum of inference in each specific case.

Similar considerations show that it is generally useless to add deep earthing "wells" to the earth circuits of large stations, even when the subsoil is a better conductor than the surface. A very large total length of rods would be necessary if their effectiveness were to be made greater than the normal earth circuits of the station by themselves.

These conclusions, however, only apply to large stations and it would be unwise to apply them to substations erected on confined areas.

c) *Highly resistive subsoil*

With a subsoil having a resistivity ρ_2 , which is much higher than the superficial resistivity ρ_1 , the term $\frac{\rho_2}{4r}$ corresponding to the effect of the solid plate becomes

much larger and the term $\frac{\rho_1}{L}$, corresponding to the total voltage drops in the neighborhood of the buried conductors, is even more negligible than before as a relative value.

These local potential drops, however, retain approximately the same values in the absolute sense as if the ground were homogeneous throughout and had a resistivity ρ_1 . The resistance itself is, practically speaking, dependent on the resistivity ρ_2 only.

It is even more useless to carry out deep boring in such cases. The buried connections are amply sufficient to saturate the ground covered by the station, and any substantial reduction in resistance could only be obtained by a considerable increase in the superficial dimensions of the area occupied by the earth circuits.

On a subsoil with a resistivity of 1000 ohm-meters, a station with a more or less square-shaped area will have a resistance of some four ohms for one hectare (10,000 square meters) covered and about three ohms for two hectares.

V-2. Coupling Between Earth Electrodes in Large Stations

We have seen that the resistance of the earthing network at a large station is some $\frac{2\rho_2}{P}$, where P is the perimeter of the installation and ρ_2 the resistivity of the

subsoil. Again, the part of this resistance which corresponds to a potential drop between the earth circuits and the center of the mesh is some $\frac{\rho_1 I}{L}$, where L is the total length of buried conductors and ρ_1 the superficial resistivity.

The consequence is that a small independent earth electrode B, placed in the center of a mesh (Figure 36), will have a coupling of some

$$k_{BA} = 1 - \frac{\frac{\rho_1}{L}}{\frac{2\rho_2}{P}},$$

with the frame earthing system A, or in homogeneous ground:

$$k_{BA} = 1 - \frac{P}{2L}.$$

As placing the small electrode B in parallel with A would not substantially modify the latter's resistance, this coefficient also represents the ratio of the potential assumed by one of the electrodes depending on whether it is separated from the active electrode or connected to it.

With homogeneous ground and a station having a half-parameter of 400 meters embracing 4000 meters of buried conductors, the coefficient of coupling will be some 90 percent, and the separation of the electrodes will only reduce the potential on the inactive electrode by 10 percent.

When the subsoil is a better conductor than the superficial layers, the coefficient of coupling will be less, but will remain considerable.

The above calculations only give a rough value, but one confirmed by experience. It is generally found that the coupling between earths situated within the area of large stations is of the order of 50 to 100 percent.

To obtain reduced coupling factors, the electrodes to be separated must be removed well outside the area of the earthing circuits.

V-3. Flow of Earth Along Overhead Earth Wires

An earth wire connected to earth at many points behaves as a conductor having a certain longitudinal line impedance Z_1 and a certain transverse line conductance $\frac{1}{R_2}$. If the line is sufficiently long, calculation shows that it behaves

towards the source like an equivalent impedance independent of this length having the value:

$$Z = \sqrt{Z_1 R_2}$$

A 70 square millimeter steel earth wire of the type generally used on overhead lines has a longitudinal impedance of four ohms per kilometer. Assume that it has three earth connections per kilometer with an impedance of 30 ohms each. The transverse resistance per kilometer R_2 will be some 10 ohms and the wire, seen from the source, will appear as an impedance of 6.3 ohms, which is slightly inductive and will act in parallel with the earth resistance proper of the station if the wire is connected to the structural framework of the latter.

If several lines terminating at a station have earth wires connected to structural frameworks, the conductance of the earth wires of the different lines will add up more or less arithmetically.

It will be noticed that the equivalent impedance of the earth wire in the calculation above is roughly equal to the longitudinal impedance of 1500 meters of cable only. This shows that the currents do not travel far along these very resistive wires and flow rapidly to earth (Figure 38).

V-4. Discharge of Earth Currents Through Buried Cables

The sheathing of a large cross-section armored cable, if it is connected to the frame earthing of a station, and leaves the area of the latter, constitutes an excellent earth electrode which tends to shunt an important part of the currents to be discharged into the ground.

The impedance of such a discharge path, viewed from the station end, can be calculated in the same way as is done for an earth wire by estimating the longitudinal line impedance and the transverse line conductance. This method of calculation is very inaccurate because neither is the longitudinal inductance (which is a function of the distance from the buried cable to the feeder lines) precisely known, nor is the transverse resistance, which is dependent on the resistivity of the ground, the quality of the contacts and the useful equivalent length of the sheath. However, as an approximate value is sufficient, the first of these factors can be estimated at 0.5 ohm per kilometer and the second at 4ρ ohms for one meter, that is, 0.004ρ ohm-kilometer. The longitudinal resistance can be readily calculated from the cross-section and resistivity of the sheath.

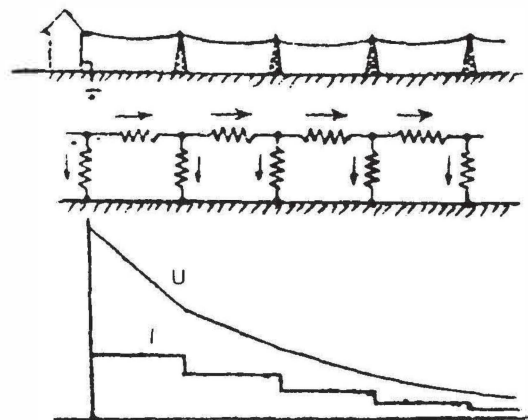


Fig 38
Flow of Earth Current Through an Earth Wire

The resulting impedance thus obtained will generally be about a fraction of one ohm, or of a small number of ohms, according to the cross-section of the sheath and the quality of the ground. In most cases the duration of the faults will be too short for heating of the lead covering by the diverted current to constitute a problem. In critical cases, the sheathing could have an alternative path provided over a certain distance by the use of a bare copper conductor connected to the junction boxes.

If the cable is sufficiently long the potential on the sheath will fall progressively from the station as a result of *longitudinal* potential drops until it blends with that of the distant ground. It is of interest to note that inductive potential drops have their full effect on the internal conductors in the cable, whose magnetic coupling with the sheath is perfect. In consequence, an internal conductor, if connected at one of its ends to the frame earthing of the station, will also arrive on its part at the other end of the cable with a potential close to the local potential, insofar as it is possible to ignore the longitudinal ohmic drops as compared with the inductive.

This self-adaptation of the potential of the internal cable conductors to the local potentials of the territory traversed is of particular interest as regards telephone cables. It obviously requires that the resistance of the sheath should be made low, as compared with the reactance, by appropriate means.

Apart from the cable sheathing all metal conduits (wiring, piping, etc.) leaving the station contribute towards discharging earth currents. This may cause incidents or constitute a danger, especially when the metallic continuity of the conduits is not perfect. If, however, they are properly arranged, they can play a useful part. Without stressing the point, it will merely be mentioned that it is generally much easier and safer to guide the currents by increasing the number of conducting connections than to attempt to bar their path by means of increased distances and barriers, the insulating value of which is often doubtful or insufficient.

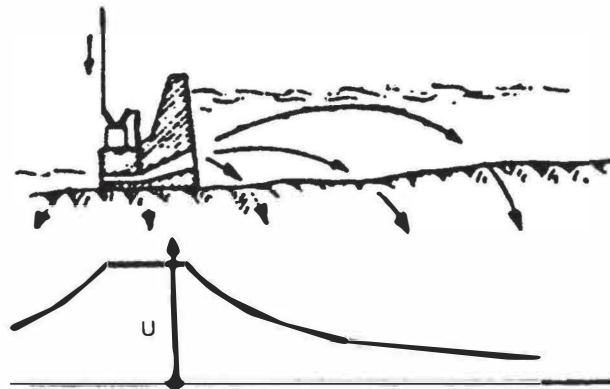


Fig 39
Flow of Earth Current Through Headwater at a Dam

V-5. Discharge of Earth Current by Sheets of Water

Geological faults having good conductivity and considerable thickness and sheets of surface or underground water may play the part of channels draining or discharging earth currents in cases in which the general ground is very resistant.

A case of particular interest is that of power stations adjoining dams built on granite. These frequently have a step-up substation, the earth electrodes of which often have to discharge heavy currents. The installation of good earthing systems in such ground is very difficult.

On the other hand, the earth circuits of the substation and power station are in good contact with the headwater via the metal conduits passing through the dam, and the latter's own conductivity which is high because of the dampness of the concrete.

The resistivity of the water in the most unfavorable cases rarely exceeds 100 to 200 ohm-meters and may be very much lower than that of the adjacent rocks. The headwater then behaves to a certain extent as a conducting channel which will divert a portion of the fault currents and finally diffuse them into the ground (Figure 39).

It is true that such a conductor has a very high longitudinal resistance which is about the same for a hectare section of water as for a one square millimeter copper wire. Calculations show, however, that the impedance of a large volume of headwater, seen from the dam, is moderate and in the neighborhood of only a few ohms. The portion of the fault currents flowing through the water may thus be considerable.

As a result of measurements made by M. Bodier at the Valentine station on a shallow canal, we have suggested improving the discharging capacity of the headwaters by burying them in considerable lengths of copper conductors connected to the frame earthing of the power station with the aim of reducing the longitudinal resistance to a very low value. It is often possible, even in very poor quality ground, to limit the resistance of the ground to values less than one ohm.

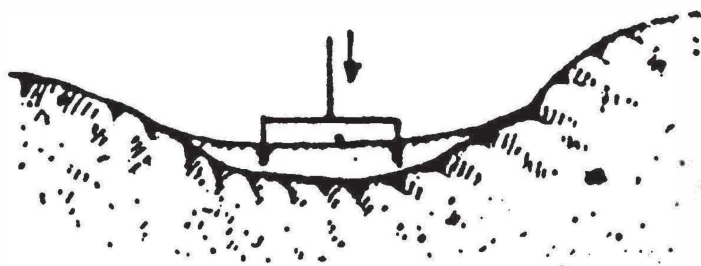


Fig 40
Restricted Area of Conductive Soil In a Resistant Mass

However, it may be of interest to check the fact that the flow of current does not raise the potential gradient at the surface of the water to excessive values at locations liable to be frequented.

V-6. Restrictive Area of Conductive Ground at a Very Resistant Site

It sometimes happens that a hollow is available for installing an earthing system which contains high conductivity sediment, isolated in the middle of a highly resistant mass.

The value of such an earthing arrangement should not be over-estimated. Its resistance comprises two terms, one corresponding to the potential drop across the conductive ground and the other to the potential drop in the resistant medium. (Figure 40).

It is possible to modify the first term, but not the second, which is dependent on the dimensions of the restricted spot of good conductivity only. Let r be the average radius for the conducting zone. In comparing this to an electrode with a shape intermediate between plate and hemisphere we arrive for the second term at a resistance of about $\frac{\rho}{5r}$, which represents the minimum resistance that can be

obtained when using the conducting isolated zone.

If the resistivity of the rock is, say, 2000 ohm-meters, and if the conducting zone has an average radius of 20 meters, the resistance of the earthing arrangement could not fall below 20 ohms.

It remains to design the electrodes in such a way that the potential drop in the conducting soil does not increase this figure unduly. If we allot, say, 5 ohms for the passage through the conducting soil and if the resistivity is 100 ohm-meters, it would be necessary to bury about 20 meters of vertical earthing rods or 40 meters of horizontal electrodes.

V-7. Effect of Relatively High Conductivity Geological Faults

The appearance of such conductive faults is sometimes counted on for reducing the resistance of earth electrodes in rocky ground. Their importance should not, however, be over-valued when they are narrow and only spread in area after having reached a considerable distance.

All that has been said regarding superficial layers which are better conductors than the subsoil also applies to geological faults. They behave as diffusers, making

it possible to connect up to the ground through electrodes of smaller sizes, but it is not wise to count on them to carry current to great distances while maintaining moderate voltage drops. It results from what has been said in paragraph III-14, that half the current will already have left the fault and spread out in the media which, though more resistant, are more extensive, at a distance from the electrode 10 to 100 times the half-thickness of the fault, according as this latter is 10 to 100 times more conductive than the neighboring media.

V-8. Improving Earth Electrodes by the Use of Conducting Salts, Coke or Wood Charcoal

It is possible to improve the conductivity of soil around earth electrodes locally by the injection of electrolytes such as common salt or sodium carbonate. The resistance of the connection first begins to fall as the salt diffuses into the ground and then finishes by increasing again after water has carried the salt away from the electrode.

The time the treatment is effective is variable according to the permeability of the soil. In many cases it may attain or exceed ten years. On the other hand, it is almost zero when there is an appreciable flow of underlying water or when the ground is very porous.

The diffusion of the salt can, however, be retarded by various means, for instance, by using insoluble gels as electrolytes (a system tested by M. Ducrot in 1944 and taken up again in Sweden by M. Sanick).

For the injection of salts to have an appreciable effect on the resistance, the area of ground treated must be large as compared with the principal dimension of the electrodes. Reduction in resistance will be considerable with a rod of small size or a plate, but the method will be of no benefit for improving the earthing arrangements of large stations.

Common salt accelerates the corrosion of electrodes, but to an extent which is not serious if copper or soft steel are used. The latter should not be employed with salt, however, in chalky soils.

Another means of improving earths consists of surrounding the electrodes with a bed of coke, or better still with wood charcoal, which is distinctly less corrosive.

It should be mentioned that these conducting beds do not, as a rule, increase the largest dimension of the electrodes to a great extent, so that their effect on resistance is often poor. They make it necessary, too, to increase the diameter of the excavations, when using rods, and it is then less expensive to increase the number of small cross-section rods. On the other hand, they have the advantage (according to experiments made in England) of reducing seasonal variations in resistance at the same time as they increase the current that the electrodes can carry without the ground being heated dangerously.

V-9. Currents Diverted Through the Human Body Between Two Points of Contact

The risk of electrocution is a function of many factors such as the value of the current, its duration and path. The parts of the body particularly sensitive to the passage of currents are: the heart, the beats of which may be stopped or disarranged; the ganglion controlling respiratory movements; and the hands, because

of the muscular contraction which prevents release of a live conductor carrying alternating current. The contraction threshold is usually 10 to 15 milliamperes. The threshold of cardiac fibrillation appears to be higher than 50 milliamperes for periods of contact of one second or more. We have no accurate information as to currents through the ganglion causing respiration to cease.

The most dangerous contacts are, in fact, between head and feet and between hands and feet. Contacts between foot and foot are of little danger except from resultant falls which may of course change them into more dangerous contacts. Further, it is impossible to disassociate the idea of danger due to a current from that of the probability of contact. As seen at the beginning of this paper, supports for lines may be raised to a potential of thousands of volts without any appreciable danger resulting for the public, if they are situated in a not-too-frequented spot and if the damage to insulators is sufficiently rare and sufficiently, quickly eliminated to restrict danger to, say, a fraction of a second over several decades.

As, however, it is necessary to adopt a figure for purposes of comparison, we will assume that a current of 50 milliamperes through the body is a dangerous value which should not be reached, insofar as it is desired to ensure safety by limitation of potential only. The rules and equations given in preceding chapters permit calculation of the potential U , which separates two points A and B between which the body will establish a contact, and we must now determine the resultant current through the body. For this purpose we will apply (as in Chapter I-5) Thevenin's theorem (Figure 3):

The current which will appear in the body between the point of contact A and B will be equal to that which the pre-existing potential U will cause to flow in the circuit constituted by the body closing on the impedance of the external network viewed between points A and B.

If the impedance of the external system, seen from AB, is low compared with the resistance R_c of the body, the current shunted by the latter will be substantially $\frac{U}{R_c}$. In another extreme case in which the impedance of the circuit external to the body is high compared with the resistance of the latter, the potential drops in this external impedance will determine the current diverted.

We can distinguish two conditions, according as to whether it is a question of direct contact with a live conductor, or simple shunting through the body of the whole or part of the path to earth of a fault current.

(a) *Direct contact with a live conductor*

In this case, the pre-existing potential is generally that of the network. The impedance of the external system, viewed between the hand and feet of the victim, comprises the impedance of the small earth electrode formed by shoes resting on the ground, then, in series, the impedance of the return circuit to the network which is a function of the position of the neutral (Figure 4), and finally the longitudinal impedance of the circuit.

These latter terms (except with low-voltage networks having an insulated neutral) will always be low as compared with the resistance of the body. On the other hand, the resistance of the earth electrode consisting of the feet may be high if

they are resting on material which is a poor conductor. The resistance of the body is a value which is not clearly defined. Roughly, the internal tissues may be compared to an electrolyte having a resistivity of some 4 to 6 ohm-meters. These tissues are protected by the skin wall, which offers a resistance of several tens of thousands of ohms per square centimeter in a dry state to penetration of the current, but much lower when the skin is damp, in which state it is fairly sensitive to the value of the potential applied. The skin may also show gaps where there are wounds or scars, and is perforated by potentials of a few hundreds of volts only.

Actually, the resistance measured between a hand and the soles is, in the majority of cases, higher than 3000 ohms, but it may fall to values less than 1000 ohms, when the skin loses all protective power and the shoes are good conductors. Here again, we must confine ourselves to a typical value and will choose 3000 ohms for this purpose. The variables and uncertainties in these problems are so many that it is hopeless to attempt to deal with them more fully in this brief paper. The resistance constituted by the feet is calculated by considering them as two small neighboring earth plates. On ground with a resistivity ρ , each foot, flat down, will have a resistance of some 2.5ρ , and the two feet in parallel a resistance of about 1.5ρ . When there is direct contact with live conductor U ; the current shunted through the body between a hand and the two feet resting on the ground will be (on these assumptions):

$$i = \frac{U}{3000 + 1.5\rho} \text{ amperes.}$$

For resistivities up to some 1000 ohm-meters, the body resistance will almost solely determine the current flowing. For resistivities of 10,000 ohm-meters or over, on the contrary, the body will only be subjected to a small part of the total potential. This fact shows the value of insulating coverings, such as wood or very dry concrete, ballast, bitumen, etc., in places where there is considerable risk of contact with live objects.

(b) *Diversion by the body of the path of a fault current*

We will now examine the case in which the body simply shunts the path to earth of a fault current (Figure 10). If the shunting occurs between one foot and the other when walking, with one of the feet not completely flat down, the impedance of the ground from one foot to the other will be about 6ρ and the shunted current will have the approximate value:

$$i = \frac{U}{3000 + 6\rho} \text{ amperes.}$$

U being the pre-existing potential.

If, on the other hand, the shunt takes place between the feet resting on the ground and a hand touching an object connected to the earthing system, the resistance of the return path as far as the hand must be introduced into the denominator in series with the resistance 1.5ρ at the entrance into the ground through the two feet. This return may take place through two paths in parallel, one direct, constituted by the earth electrode and its connection to the object touched, and the other via the network return-circuit system and the accidental

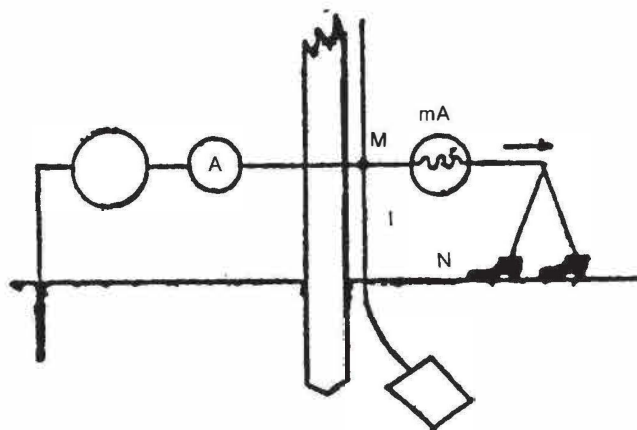


Fig 41
Measurement of Current Shunted by a Human Body Between M and N

contact between this and the object. In many cases, this additional term will be negligible and we shall get approximately:

$$i = \frac{U}{3000 + 1.5\rho} \text{ amperes.}$$

It will be seen that, in all cases, in order to determine directly the current shunted, without having to measure separately U and ρ , it is sufficient to substitute for the person a voltmeter having an internal resistance approximately that of the human body and in contact with the ground through soles representing the feet, and to read off the current i flowing through this voltmeter when a fault current I is made to flow to the earth electrode (Figure 41).

c) Examples

1) Assume a one-meter bare earthing rod carrying a current of 5 amperes in ground with a resistivity of 100 ohm-meters. A person touches the top of the rod. What current will flow through him? The potential at the rod in relation to distant earth will be about 400 volts. The potential between the top of the rod and the ground at a distance of one pace will be some 300 volts. The resistance of the two feet resting on the ground will be about 150 ohms. The resistance of the circuit comprising the two feet in series with the earth rod will be about 200 ohms. The current shunted will thus be approximately:

$$i = \frac{300}{3000 + 200} = 0.094 \text{ ampere}$$

assuming a resistance of 3000 ohms between hand and soles. This contact may be fatal if conditions are unfavorable.

2) Assume an electrode buried horizontally, and carrying one ampere per meter, in ground with a resistivity of 200 ohm-meters. What will be the current

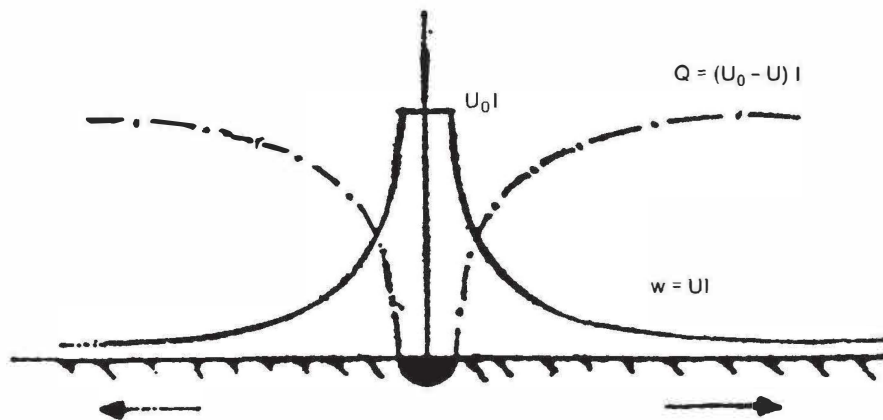


Fig 42
Radial Flow of Electric Energy and Heat

shunted from foot to foot or between neighboring ground and a structure connected to the electrode? The maximum voltage from one foot to the other in the neighborhood of the conductor will be 0.10 to $0.15 \rho i$, that is, 20 to 30 volts. The current from one foot to the other will have the maximum value:

$$i = \frac{30}{3000 + 1200} \quad 0.007 \text{ ampere.}$$

This is quite harmless.

The potential between the feet and hand might reach 0.6 to $0.8 \rho i$, that is, 120 to 160 volts. The corresponding current will be about

$$i = \frac{120 \text{ to } 160}{3000 + 150} \quad 0.038 \text{ to } 0.050 \text{ ampere.}$$

Contact in this case may occasionally be dangerous.

The instances met with in practice vary far too much for it to be possible to give general rules. The simplest method is often, as mentioned in paragraph III-2, to cause a current to flow to the earth electrode, and to check the potentials between points of the installation which are liable to be touched simultaneously. Local improvements can then be effected at the points considered the most dangerous.