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The Cover: Transmission towers are symbols of electric power, essential to the many industries in and around New York, N. Y., where the AIEE winter technical meeting will be held January 22-26, 1945.

Grounding Principles and Practice—I Reinhold Rüdenberg . . . 1

The Engineer and His Future Charles A. Powell . . . 14

Lighting of Budd Field J. L. Kilpatrick, L. N. Blugerman . . . 17

Recent Electron-Tube Developments S. B. Ingram . . . 22

Electrical Engineering in the Postwar World—IX A. W. Hull . . . 25

Miscellaneous Short Item: High Soldering Temperatures Destroy Wire, 24

Institute Activities . . . 28

Of Current Interest . . . 43

TRANSACTIONS SECTION

(Follows EE page 46; a preprint of pages 1-40 of the 1945 volume)

Evaluation of Electric Distribution Losses M. Mortara . . . 1

Two-Phase Co-ordinates of a Four-Phase Network Edward W. Kimbark . . . 7

Subway Supervisory Control W. A. Derr, C. J. Buck, J. A. Stoos . . . 10

Calculation of Distribution-System Fault Currents F. W. Linder . . . 16

Simulated-High-Altitude Testing H. H. Race, A. M. Ross, Jr. . . . 20

Aircraft Illumination W. W. Davies . . . 26

Damping Effect of D-C Marine Propulsion Motors Bernard Litman . . . 31

Speed-Control System F. W. Godsey, Jr., J. D. Miner, Jr., O. C. Walley . . . 37

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Grounding Principles and Practice I—Fundamental Considerations on Ground Currents

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IN THIS ARTICLE are considered the main scientific problems related to the conduction of electric currents to the earth and through the ground. While the overall performance of an electric system in which one or more points are grounded depends on the behavior of the wire network and the ground, this article is confined to the underground flow of the electric currents, which is not so easily measurable and therefore not so well understood as the behavior of the currents in the overhead network.

The earth is a body of three dimensions and therefore the beautiful simplicity of the linear wires by which electric currents usually are directed is lost. In the earth the currents spread out in the entire space, and it is necessary to follow their paths in order to analyze their performance in the underground.

The earth has been used as a conductor for electric currents since the beginning of engineering. However, after a brief period of preference for sending return currents through the ground, great difficulties and hazards were found from this in all branches of electrical engineering. Nowadays the earth is used mainly for fixing the neutral point of the electric system, and in many instances the inclusion of the earth as part of the circuit cannot be avoided.

RESISTANCE OF THE SOIL

Geological Strata. The electric conductivity of the materials constituting the earth's surface is very low compared with the high conductivity of the metals. Two main constituents of the earth, silicon oxide and aluminum oxide, actually are excellent insulators, and the conductivity of the ground is due in large measure to salts and moisture embedded between these insulators. On the other hand, even a semiconductor may carry a considerable amount of

This article is the first of five based on a series of lectures which was sponsored by the power and industrial group of the AIEE New York Section during the 1943-44 season. The behavior of currents flowing in the earth, as influenced by the resistivity of the soil, frequency of the current, and size and shape of electrodes, is analyzed here.

current if only the cross section is large enough, and in this respect the earth, by its great depth, presents no limitations.

Because of the high resistivity, all currents flowing through the ground suffer a considerable voltage drop. Therefore we must break with the popular concept that the potential of the ground is always zero. A substantial electric-field strength, or potential gradient, can develop and this may affect extended regions of the earth's surface. We have to distinguish between two different zones which will be considered in detail, namely, the spaces in proximity to the ground electrodes and the long path between such electrodes.

The ground under the surface of the earth is by no means homogeneous, and this makes a rigorous analysis of the distribution of currents very difficult if not impossible. Figure 1 represents a typical geological cross section of the underground indicating the great heterogeneity of the conducting body through which electric currents may pass. Random changes caused by weather and seasons, by rain, frost, and by other temperature variations, with their influence on the conductivity of the soil, increase the difficulty of analysis. It would not be wise, therefore, to develop here the laborious considerations for a very high degree of accuracy. However, a quantitative analysis of the electric phenom-

ena in the ground is necessary in order to make numerical calculations and to draw definite conclusions if the electric system from time to time shall not experience technical trouble or cause serious accidents.

Variation of Earth Resistivity. The resistivity of the soil depends on many parameters. It depends on the type of soil and therefore varies with distance as well as with depth, as indicated by Figure 1. Further, the resistivity is much smaller below the subsoil water level than above it. In frozen soil, as in the surface layer in winter, it is particularly high. It is important, therefore, for good ground electrodes to avoid the frost limit and to reach the subsoil water plane. Figure 2 shows the variation of resistivity ρ of a certain soil as measured (a) versus moisture content, (b) versus temperature change, and (c) versus added salt in per cent of weight. It is seen by the logarithmic scales used that these three parameters affect the earth resistivity by several powers of ten, thus very large variations may occur.

Attempts have been made to correlate the resistivity with the geological age of the strata. Although older material in general has higher resistivity, many exceptions were found. In the actual strata under the surface, loam, clay, and limestone usually have lower resistivity; sandy and rocky materials have higher resistivity. While for power networks and for radio waves the resistivity near the surface is more important, for telephone interference by action at a distance the resistivity of the deeper layers has greater significance.

SPHERICAL GROUND ELECTRODES

Current Paths and Grounding Resistances. There will be considered first the simpler electrode elements and later some of the more complex combinations used in practice. For quantitative calculations the meter-kilogram-second system will be used which leads directly to the practical electric units.

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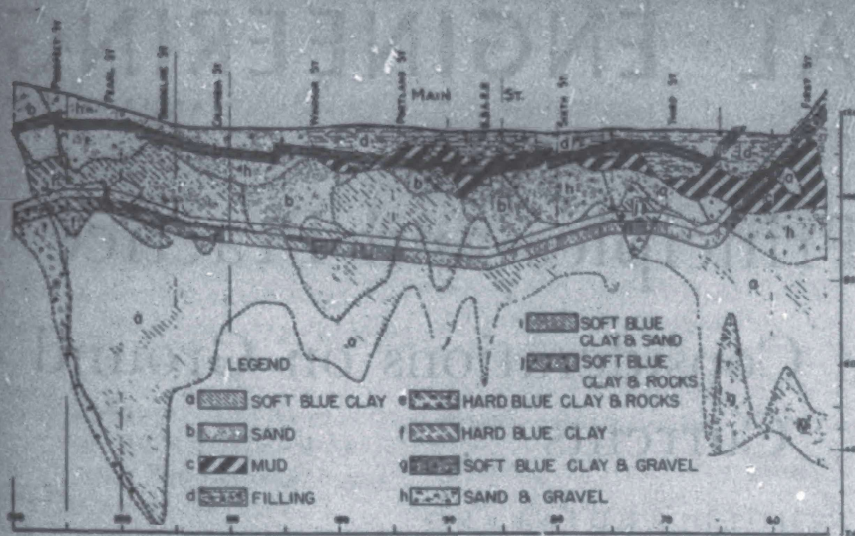


Figure 1. Subsoil strata under Main Street, Cambridge, Mass.

The simplest possible electrode is shown in Figure 3, namely, a sphere in the ground which is symmetrical in all directions. It may be entirely embedded in the ground, or only the lower hemisphere embedded in the half space of ground under the surface plane of the earth, a case which will be considered. If a current I flows through this electrode, spreading out radially in the ground, the current density at distance x from the center of the hemisphere is

$$i = \frac{I}{2\pi x^2} \quad (1)$$

According to Ohm's law such a current produces in the resistivity ρ of the soil an electric-field strength

$$e = \rho i = \frac{\rho I}{2\pi x^2} \quad (2)$$

The voltage, as line integral of the field strength from the surface of the conducting sphere of radius B to the distance x , is therefore

$$E = \int_B^x e dx = \frac{\rho I}{2\pi} \int_B^x \frac{dx}{x^2} = \frac{\rho I}{2\pi} \left[\frac{1}{B} - \frac{1}{x} \right] \quad (3)$$

Current density, field strength, and voltage, in their dependence on distance, are represented graphically near the top of Figure 3.

The total voltage between the spherical electrode and a far distant point with $x = \infty$ is, according to equation 3,

$$E = \frac{\rho I}{2\pi B} \quad (4)$$

and therefore the resistance experienced by the streamlines of current diverging from the hemisphere is

$$R = \frac{E}{I} = \frac{\rho}{2\pi B} \quad (5)$$

Any additional resistance due to incomplete contact of the electrode and the

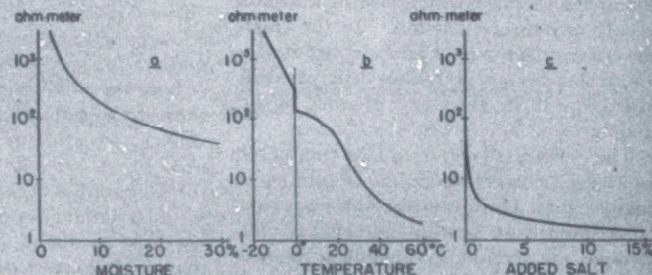
ground occurs only rarely in practice.

If we consider, for a numerical example, a hemispherical electrode of radius $B=1$ meter embedded at the surface of the earth of resistivity $\rho=10^3$ ohm-meter, the ground resistance of this electrode will be

$$R = \frac{10^3 \text{ ohm-m}}{2\pi \cdot 1\text{m}} = 16 \text{ ohms}$$

As the curves in Figure 3 show, this resistance is distributed over the entire half space; however, the major part of it is concentrated in proximity to the electrode.

Figure 2. Resistivity of the earth depending on moisture, temperature, and salt contents



If two such electrodes are used through which the current enters and leaves the ground, the streamlines of the current develop as superposition of two hemispherical current distributions. This is shown in Figure 4a under the assumption that the resistivity of the soil is uniform in space. If a voltage of 110 volts is impressed between the electrodes, one having a diameter of 10 meters and the other of 1 meter, and separated by a distance of 1,000 meters, the streamlines and equipotential lines develop as shown in Figure 4a, and concentrate mainly around the smaller electrode.

If, on the other hand, a more highly conducting layer should exist at the surface of the earth, for example after a heavy rain, then the streamlines do not develop in depth but remain near the surface. Their shape within this layer and the dis-

tribution of potential are shown in Figure 4b for electrodes of the same dimensions and distance apart as before. Here the voltage does not concentrate so heavily near the electrodes but spreads out over larger areas of the surface.

Field Strength on Surface. If a man or an animal is walking through such a surface field of electric potential, for example in the neighborhood of a faulty transmission tower, as shown in Figure 5, the body diverts some current from the earth and may suffer damage from the potential gradient. The tower footing carrying a current I to the ground can be represented for the present purposes by an equivalent hemisphere of radius B , as shown by the dotted line in Figure 5. The field strength on the surface of the ground is then given by equation 2 and increases toward the tower to a maximum value at distance B

$$e_B = \frac{\rho I}{2\pi B^2} \quad (6)$$

With the figures of the former example and with a tower current to ground $I=100$ amperes, the maximum field strength at the surface is

$$e = \frac{10^3 \text{ ohm-m} \cdot 100 \text{ amp}}{2\pi \cdot (1\text{m})^2} = 1,600 \text{ volts per meter}$$

a remarkably high value.

This field strength produces a current in any living creature stepping over the surface. The maximum possible current develops if the internal body resistance is

small compared with the foot resistance on the ground, which is

$$r = \frac{\rho}{2\pi b} \quad (7)$$

if b signifies the equivalent radius of the creature's foot. For a man's foot this has been measured in an electrolytic trough as $b=7$ centimeters on the average. Since the voltage drop by the body current through the resistance $2r$ due to both feet is given by the voltage taken over the step length s

$$2ri_s = es \quad (8)$$

and therefore the maximum body current becomes

$$i_s = \frac{es}{2r} = \frac{\pi bes}{\rho} = \frac{sb}{2\pi B^2} I \quad (9)$$

if expressed in terms of the tower current I .

With the numerical values already used, there is obtained for a man taking a step one meter long at a distance of three meters from the center of the tower, a body current

$$i_s = \frac{1\text{m} \cdot 0.07\text{m} \cdot 100 \text{ amp}}{2 \cdot (3\text{m})^2} = 0.39 \text{ ampere}$$

and this maximum current is certainly deadly.

Measurements by Probes. Measurements of ground resistance require some considera-

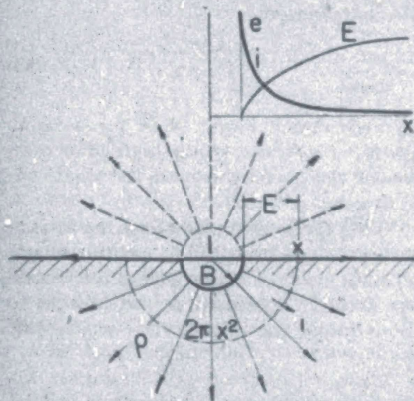


Figure 3. Radial flow of current from spherical electrode to ground

tion. Care should be taken that probes or auxiliary electrodes, as in Figure 6, are sufficiently far removed from the spreading-out zone of the main current, if the probes are to measure the entire resistance experienced by the current. The voltage E will be independent of the distance x between the main and auxiliary electrodes only if x is much greater than both b and B .

By application of equation 3 to both systems of current distribution of Figure 6 around the hemispheres of radii B and b , superposition gives the voltage between the two electrodes. Thus the total resistance is

$$\frac{E}{I} = \frac{\rho}{2\pi} \left[\frac{1}{B} + \frac{1}{b} - \frac{2}{x} \right] \quad (10)$$

which is smaller than the sum of the resistances R and r proper of the two electrodes. By equation 10, ρ can be determined from the measured quotient of voltage and current if the dimensions are known and if ρ is constant over the ground. If this latter cannot be assumed, the resistivity around the smaller electrode nevertheless can be measured if its radius b is made so small that it is much less than both B and x , so that not only the last term but also the first term in the bracket of equation 10 becomes negligible. This gives the resistivity

$$\rho = 2\pi b \frac{E}{I} \quad (11)$$

as determined only by the quotient of voltage and current and the linear dimension b of the probe. In this way numerous measurements have been performed showing that there are very great differences in the resistivity of the natural soil.

If no specific measurements for a definite spot of ground are made, the figures of Table I may be used as average resistivity over the underground.

Two Parallel Ground Electrodes. If two electrodes are arranged mutually within their spaces of high field strength, as in Figure 7, they experience mutual influence. For each of the spheres in Figure 7 the voltage from the sphere of radius b through distance y is, by equation 3,

$$E = \frac{\rho I}{4\pi} \left[\frac{1}{b} - \frac{1}{y} \right] = V_b - V \quad (12)$$

Since the voltage is determined by the difference in the bracket, which depends only on distances, the voltage can be expressed by the difference of two potentials, as on the right-hand side of equation 12. The potential V of a spherical source of current in space is therefore

$$V = \frac{\rho I}{4\pi y} \quad (13)$$

and such potentials always superpose if more than one source is present.

In the center plane between the two spheres of Figure 7, the potential is therefore

$$V_x = 2 \frac{\rho I}{4\pi y} \quad (14)$$

if both spheres carry equal currents I to the ground. On the other hand, at the surface of either sphere the self-potential is determined by the radius b , the mutual potential by the average distance $2z$ between the centers of the spheres. The total potential is therefore

$$V = E = \frac{\rho}{4\pi} \left(\frac{I}{b} + \frac{I}{2z} \right) = \frac{\rho I}{4\pi b} \left(1 + \frac{b}{2z} \right) \quad (15)$$

Hence the resistance of two spheres in parallel in the ground, carrying together the current $2I$, is

$$R = \frac{E}{2I} = \frac{\rho}{4\pi b} \frac{1}{2} \left(1 + \frac{b}{2z} \right) \quad (16)$$

The first quotient, by comparison with equation 5, gives the ground resistance of one sphere. The last factor for far-distant electrodes with $z = \infty$ becomes $1/2$; but for close electrodes, for example, with $z = b$

Table I. Average Resistivity of the Ground

Type of Ground	Resistivity ρ , Ohm-Meters
Wet organic soil.....	10
Moist soil.....	10^1
Dry soil.....	10^2
Bed rock.....	10^4

becomes $1/2(1+1/2) = 3/4$. Thus distant spheres are mutually independent in their resistances and their parallel resistance can be computed according to ordinary rules. Close electrodes, however, experience an increase of their ground resistance by mutual interference.

Depth Electrode. Figure 8 shows a spherical electrode buried in depth. Considering the arrangement of Figure 8 as half that of Figure 7, we obtain the resistance of such a depth electrode as twice that of equation 16, namely,

$$R = \frac{E}{I} = \frac{\rho}{4\pi b} \left(1 + \frac{b}{2z} \right) \quad (17)$$

Near the surface, for example at $z = b/2$, we

have $\left(1 + \frac{b}{2b/2} \right) = 2$, and the resistance becomes the same as with equation 5. Under the surface the resistance decreases and in greater depth the correction term $b/2z$ vanishes, thus cutting the resistance in half.

With a depth electrode the space of high current concentration is not accessible to creatures, thus the danger of stepping near the electrode decreases. At any point of the surface, as seen by Figure 8,

$$\frac{x}{y} = \sin \alpha = \frac{dy}{dx} \quad (18)$$

where x is the distance from the axis of the depth electrode and α the corresponding central angle. Thus the field strength at the surface is

$$e = \frac{dV_x}{dx} = \frac{dV_x}{dy} \sin \alpha \quad (19)$$

Substituting the potential V_x from equation 14, we obtain the field strength at the surface

$$e = \frac{\rho I}{2\pi} \frac{d(1/y)}{dy} \sin \alpha = \frac{\rho I}{2\pi} \frac{\sin \alpha}{y^2} \quad (20)$$

Now, for α near 90 degrees, $\sin \alpha / y^2 = 1/x^2$; while for small values of α , $\sin \alpha / y^2 = x/z^2$. Therefore the field strength at large distances decreases exactly like that around a hemisphere, while vertically over the depth electrode the field strength becomes zero and rises linearly for small values of x . At an angle determined by $\tan \alpha = 1/\sqrt{2}$, the field strength is a maximum, namely,

$$e = \frac{\rho I}{3\sqrt{3}\pi z^2} \quad (21)$$

The ratio of this value to the maximum value of equation 6 for a surface electrode is

$$\frac{2}{3\sqrt{3}} \left(\frac{b}{z} \right)^2 = 0.39 \left(\frac{b}{z} \right)^2 \quad (22)$$

which shows that a depth electrode, for sufficiently large depth z , minimizes the danger for stepping creatures.

ROD AND WIRE ELECTRODES

Driven Rod at Earth Surface. Spherical or hemispherical electrodes are not con-

venient for actual use. In practice rod or wire electrodes, having a relatively small cross section compared with the length, are preferred. We may subdivide with good approximation such a rod electrode driven into the ground, as in Figure 9, into a large number n of nearly spherical elements, which over the length l of the rod in the ground have a mutual distance

$$dl = \frac{l}{n} \quad (23)$$

each feeding a current I/n into the ground. If y is the distance from any element to a point at the surface of the earth and α is the angle from y to the axis of the rod, the small diagram in Figure 9 shows that

$$\sin \alpha = \frac{y d\alpha}{dl} \quad (24)$$

The potential dV of every element is given by equation 13 with current I/n . By substitution of the distance y from equation 24, and use of equation 23, the incremental potential at the surface of the earth becomes

$$dV = \frac{\rho I/n}{4\pi y} = \frac{\rho I}{4\pi l} \frac{d\alpha}{\sin \alpha} \quad (25)$$

If the limiting value of angle α is denoted by β , as shown in Figure 9, the potential in the center plane of a rod of length $2l$, containing $2n$ spheres and thus including the fictitious image above the ground, is given by the integral of dV from $+\beta$ to $-\beta$,

$$V = \frac{\rho I}{4\pi l} \int_{+\beta}^{-\beta} \frac{d\alpha}{\sin \alpha} = \frac{\rho I}{2\pi l} \log_e \left(\cot \frac{\beta}{2} \right) \quad (26)$$

This result would be rigorous if the density of the current emerging from the rod into the earth were uniform over the length, which actually is only a fair approximation.

The electric potential in the symmetry plane of the driven rod thus is dependent on only four parameters, namely: resistivity ρ of the ground, current I flowing into the rod, its length l within the ground, three data which are always given for a definite ground electrode, and the angle of vision β between the axis of the rod and the distance from the bottom of the rod to the point under consideration at the surface. For any point in this center plane β is the only variable parameter.

For large distance x of the point considered from the axis of the rod, the logarithm in equation 26 simplifies to

$$\log_e \left(\frac{1 + \cos \beta}{\sin \beta} \right) \approx \cos \beta \approx \frac{l}{x} \quad (27)$$

and the potential is

$$V_\infty = \frac{\rho I}{2\pi x} \quad (28)$$

which is identical with that for a hemisphere.

On the other hand, for the surface of the rod, where the potential V is identical with

the voltage E of the electrode, Figure 10 shows that for small radius a as compared to length l

$$\cot \frac{\beta}{2} \approx \frac{2l}{a} \quad (29)$$

Hence the resistance as quotient of voltage and current is

$$R = \frac{E}{I} = \frac{\rho}{2\pi l} \log_e \left(\frac{2l}{a} \right) \quad (30)$$

While the shape of the rod, determining the ratio l/a , is of minor significance since it forms only the argument of a logarithm, the length l of the rod is of major importance, for the ground resistance is nearly inversely proportional to the length.

A rod of radius $a = 2.5$ centimeters and of length $l = 6$ meters, dimensions often used in practice, driven into moist soil will present a resistance

$$R = \frac{10^8 \text{ ohm-m}}{2\pi \cdot 6\text{m}} \log_e \left(\frac{2 \cdot 6}{2.5 \cdot 10^{-2}} \right) \\ = 2.65 \cdot \log_e 480 = 16 \text{ ohms}$$

This value equals that for a hemisphere two meters in diameter.

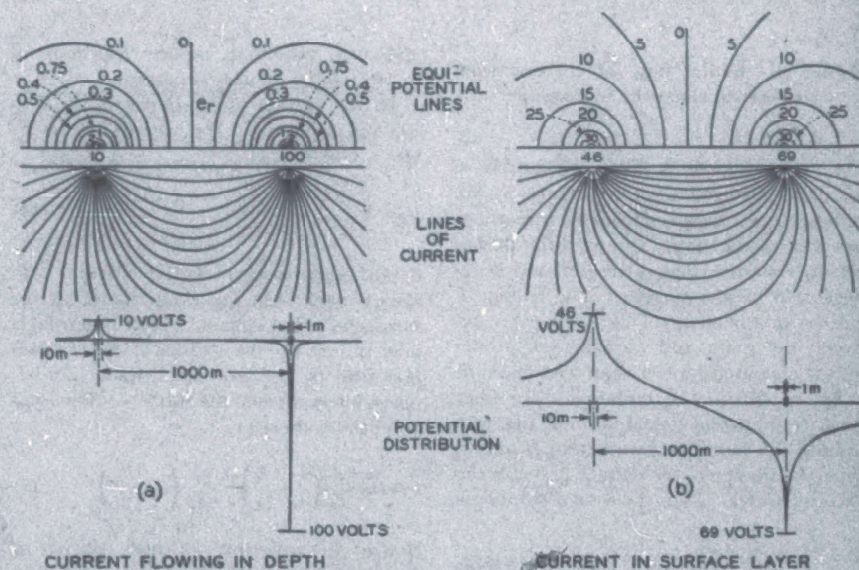


Figure 4. Flow of current between two ground electrodes

Field Strength Around Rod Electrode. On the surface of the ground there develops, according to Figure 11, a field strength

$$e = \frac{dV}{dx} = \frac{dV}{d\beta} \frac{d\beta}{dx} \quad (31)$$

where the angle β is introduced as the variable. It is correlated to x by

$$\tan \beta = \frac{x}{l} \quad (32)$$

$$\frac{dx}{d\beta} = \frac{l}{\cos^2 \beta}$$

where the last expression is merely the derivative of the first. Differentiation of

equation 26 with respect to β and substitution of equation 32 into equation 31 gives the field strength

$$e = \frac{\rho I}{4\pi l} \frac{2 \cos^2 \beta}{\sin \beta} = \frac{\rho I \cos \beta}{2\pi l x} \quad (33)$$

which is decreasing with both increased x and β .

For large distance x , $\cos \beta = l/x$, approximately, and the field strength

$$e_\infty = \frac{\rho I}{2\pi x^2} \quad (34)$$

becomes identical with that for a hemisphere. For small x , however, $\cos \beta \approx 1$, and the field strength

$$e_0 = \frac{\rho I}{2\pi l x} \quad (35)$$

becomes much larger than for a hemisphere. This is due to the high concentration of the current around the small circumference of a rod.

Along the length of the rod the density of the current from rod to earth is nearly constant over the major part of the length, but actually increases at the bottom to about double the value at the center plane,

due to the point effect of the end of the rod.

Simple Forms of Ground Electrodes. In a similar way the potential and the ground resistance of other forms of electrodes can be derived by superposition of spherical or cylindrical elements. Table II gives a survey of some simple forms of ground electrode. We see that a flat strip follows a relation quite like that of a circular rod and a strip thus is equivalent to a rod or a wire of a diameter equal to half the width of the strip.

The resistance of a ring of wire, having a periphery $2\pi b$, is only slightly greater than that of a straight wire of length $2l = 2\pi b$,

the increase being due to the absence of the ends of the rod with their higher current density.

The resistance of a ground electrode near the surface of the earth is always twice the resistance of that electrode in greater depth, because the distribution of the cur-

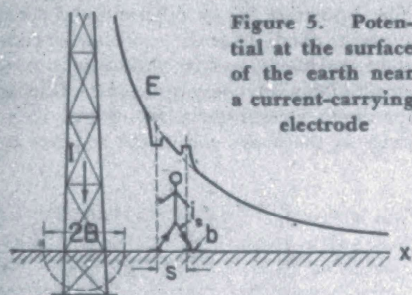


Figure 5. Potential at the surface of the earth near a current-carrying electrode

rent through the upper semispace is cut off.

As all the equations show, the resistance is determined primarily by the largest dimension of the electrode and only to a minor extent by the smaller dimensions, like cross section or thickness. Thus the surface of the electrode is unimportant and only the linear extension matters.

The simplest method for determining the resistance of a complicated shape of ground electrode, as for example the tower footing of a high-voltage line, or of a radio antenna, often consists in measuring the resistance of a scale model of the structure in an electrolytic trough. Then the resistance of the actual device is lower in the ratio of the linear dimensions and higher in the ratio of the resistivities of the surrounding medium. The radius of the equivalent hemisphere is always a convenient means of comparison.

Four-Point-Star Electrode. For some structures frequently used which are composed of several rods or wires, it is convenient to derive analytical formulas. In a four-point-star electrode, as in Figure 12, the self-potential of every straight wire of length $2l$ is, according to equations 26 and 29,

$$V_0 = \frac{\rho l}{2\pi l} \log_e \left(\frac{2l}{a} \right) \quad (36)$$

This is to be increased by the mutual potential between the transverse wires of the cross. Equation 26 gives the potential of a wire of length $2l$ in its center plane. For angles $\beta \leq 45$ degrees, as in Figure 12,

$$\cot \frac{\beta}{2} \approx \frac{2l}{x} \quad (37)$$

The mean value of the logarithm of $2l/x$ over the length l of the wire is

$$\begin{aligned} \frac{1}{l} \int_0^l \log_e \left(\frac{2l}{x} \right) dx &= -2 \int_0^l \log_e \xi \cdot d\xi \cdot \frac{1}{\xi} \\ &= -2 \int_0^1 \log_e \xi \cdot d\xi = 2 \left[\log_e \xi + \frac{1}{\xi} \right]_0^1 \\ &= \log_e 2 + 1 \end{aligned} \quad (38)$$

Thus the mean mutual potential between the crossed wires is

$$V_1 \approx \frac{\rho l}{2\pi l} (\log_e 2 + 1) \quad (39)$$

The voltage E of every conductor is given by the sum of the potentials or

$$E = V_0 + V_1 \approx \frac{\rho l}{2\pi l} \left[\log_e \left(\frac{2l}{a} \right) + \log_e 2 + 1 \right] \quad (40)$$

and therefore the resistance to ground of the entire four-point-star electrode is

$$R = \frac{E}{4I} \approx \frac{\rho}{8\pi l} \left[\log_e \left(\frac{4l}{a} \right) + 1 \right] \quad (41)$$

If such a star of wires is embedded near the surface of the earth at a depth small compared with the length dimension of the electrode, the ground resistance is twice that of equation 41 as indicated in the last example of Table II.

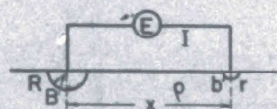


Figure 6. Measurement of the resistivity of the soil

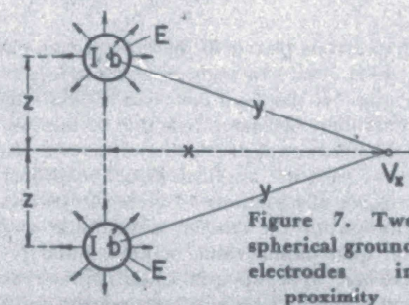


Figure 7. Two spherical ground electrodes in proximity

The value of the resistance of equation 41, by the addition of unity to the logarithm in the bracket, is slightly greater than for a straight wire of length $4l$. This increase is due to the mutual influence of the current distributions in the ground caused by neighboring parts of the entire electrode. With more star wires than four, such influence increases, preventing the resistance from being reduced in proportion to the increase of the number of the wires. For many wires, finally forming a circular disk, the resistance approaches

$$R = \frac{\rho}{8l} \quad (42)$$

For average values of the ratio radius to length of the wire, a comparison of equation 42 with equation 41 shows that by increasing the number of radial wires to infinity the resistance reduces to only

$$\frac{R^0}{R^+} = \frac{\pi}{\log_e \left(\frac{4l}{a} \right) + 1} \approx \frac{\pi}{9.3} \approx \frac{1}{3} \quad (43)$$

of that of the simple four-wire star. The gain with increasing number of wires is not very great.

Multiple Rod Electrodes. Another combined electrode frequently used, namely, multiple driven rods connected in parallel, is shown in Figure 13. The potential at the earth's surface of every rod again is given by the general equation 26. The voltage at each of the electrodes is given by the sum of all the potentials produced by the rod considered and all the other rods. For rod number one, for example, the voltage is

$$E_1 = \frac{\rho}{2\pi l} \left[I_1 \log_e \left(\cot \frac{\beta_1}{2} \right) + I_2 \log_e \left(\cot \frac{\beta_2}{2} \right) + \dots \right] \quad (44)$$

The currents of the rods are not yet determined, but for the sake of simplicity the resistivity of the soil and the length of all the rods have been taken as uniform. As many equations of this type can be developed as there are rods in the electrode, the angles β signifying always the angles of vision from the bottom of each rod toward the top of the rod considered as seen by Figure 13. Therefore, for any arrangement of driven rods a sufficient number of equations is obtained to determine the distribution of the currents among the individual rods of the electrode.

The first cotangent of equation 44 refers to the rod considered and can be evaluated by equation 29. The further cotangents, which refer to the other rods, can be expressed, as shown in Figure 13, by the ratio of the length l of the rod to a distance s_n cut off by the angle $\beta/2$ on the distance S_n between the rods. Thus

$$\left. \begin{aligned} \cot \frac{\beta_1}{2} &= \frac{l}{a/2} \\ \cot \frac{\beta_n}{2} &= \frac{l}{s_n} \end{aligned} \right\} \quad (45)$$

If the rods are connected by zero resistance and are located symmetrically with

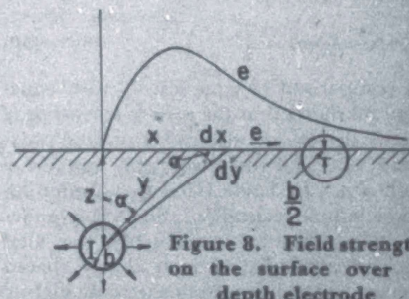


Figure 8. Field strength on the surface over a depth electrode

respect to one another, the voltages and currents are

$$E_1 = E_2 = E_3 \dots = E; \quad I_1 = I_2 = I_3 \dots = \frac{I}{n} \quad (46)$$

Therefore, with use of equation 45, all the equations 44 take the form

$$E = \frac{\rho}{2\pi l n} \left[\log_e \left(\frac{l}{a/2} \cdot \frac{l}{s_1} \cdot \frac{l}{s_2} \dots \right) \right] \quad (47)$$

the resistance strata of the earth in which the ground electrodes have to function.

HEATING OF THE GROUND

If a ground electrode is to be loaded continuously, or even over a short time only, the temperature rise θ of the soil must be considered in order to avoid an overloading which might evaporate the moisture. The current density about a spherical electrode with radius B embedded in the ground varies over the distance x as

$$i = \frac{I}{4\pi x^2} \quad (56)$$

as is indicated in Figure 17. Thereby an amount of heat ρi^2 is produced in every element of volume, which may be considerable because of the high value of the resistivity ρ . This heat is partly stored in the volume elements of the ground, which have an average specific heat $\gamma \approx 1.75 \cdot 10^4$ $\text{ws/m}^3\text{C}$ (watt-seconds/meter³·degrees centigrade); partly conducted from higher to lower temperature within the ground, the average heat conductivity being $\lambda \approx 1.2$ w/mC (watts/meter·degrees centigrade). Although these two thermal constants of the soil are of major significance, their actual values seldom have been measured accurately.

The differential equation of the radial heat conduction about a sphere is

$$\gamma \frac{d\theta}{dt} - \frac{\lambda}{x} \frac{d^2(x\theta)}{dx^2} = \rho i^2 \quad (57)$$

It is difficult to obtain a general solution of this equation since i varies with x . However, we can discuss two interesting phases of the heating, giving particular integrals, namely the steady state and the short-time state.

Steady-State Heating. For continuous ground currents the time derivative in

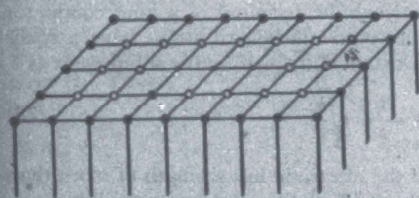


Figure 14. Extended bed of rod electrodes

equation 57 vanishes, and the differential equation becomes

$$\lambda \frac{d^2(x\theta)}{dx^2} + \frac{\rho}{x^3} \left(\frac{I}{4\pi} \right)^2 = 0 \quad (58)$$

With two simple integrations the solution for the temperature distribution over distance x becomes

$$\theta = \frac{\rho}{\lambda} \left(\frac{I}{4\pi} \right)^2 \frac{1}{x} \left(\frac{1}{B} - \frac{1}{2x} \right) \quad (59)$$

which is plotted against x in Figure 17. The maximum earth temperature at the electrode with $x=B$ is

$$\theta_B = \frac{1}{2\lambda} \left(\frac{I}{4\pi B} \right)^2 \quad (60)$$

and depends only on the two constants ρ and λ of the ground and on the linear cur-

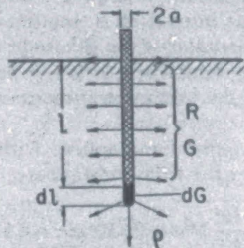


Figure 15. Increment of ground conductance with length of a rod

rent density I/B . The admissible current to ground from the spherical electrode is therefore

$$I = 4\pi B \sqrt{2 \frac{\lambda}{\rho} \theta} = \frac{1}{R} \sqrt{2\rho\lambda\theta} \quad (61)$$

where the right-hand term is obtained by substituting the resistance $R = \rho/4\pi B$. The current-carrying capacity of the electrode is thus determined, in addition to the ground constants, by resistance and temperature rise θ only.

Since the heat flow and the current distribution around electrodes of any shape follow the same mathematical law, namely Laplace's differential equation, the right-hand expression of equation 61 is valid not only for spherical electrodes but for any

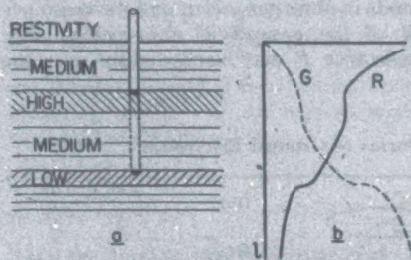


Figure 16. Measurement of rod resistance against depth

shape of electrode whether it be a rod, a ring, a disk, or a more complicated form. Therefore, if the resistance is known, which differs with the shape, the continuous current-carrying capacity can be determined easily.

The voltage at the ground electrode, measured from the metal to a far distant point is, from the last term of equation 61

$$E = IR = \sqrt{2\rho\lambda\theta} \quad (62)$$

Besides the ground constants E depends only on the temperature rise in the steady

state. Conversely, the voltage applied to a ground electrode determines its steady-state temperature. These conclusions are true for any shape of electrode.

As an example, for $\theta = 60$ degrees centigrade in moist soil,

$$E = \sqrt{2 \cdot 10^4 \cdot 1.2 \cdot 60} = 120 \text{ volts}$$

This voltage must not be surpassed if that temperature rise is not to be exceeded.

The admissible current density at the surface of the ground electrode, where the highest temperature exists, determined for a spherical electrode from equation 61 is

$$i = \frac{I}{4\pi B^2} = \frac{1}{B} \sqrt{2 \frac{\lambda}{\rho} \theta} \quad (63)$$

For the former example

$$i = \frac{1}{1\text{m}} \sqrt{2 \frac{1.2}{10^4} 60} = 1.2 \text{ amperes per square meter}$$

Thus the current density permitted for continuous loading of the electrode is fairly small. The soil surrounding the electrode must not heat up to 100 degrees centigrade, since the moisture would then evaporate completely, and the current would be interrupted by the enormously increased resistance.

Short-Time Heating. For short-time loading of the ground electrode the second member of equation 57, the heat-conduction term, plays only a minor role and thus may be neglected. The remaining differential equation then becomes

$$\frac{d\theta}{dt} = \frac{\rho}{\gamma} i^2 \quad (64)$$

The temperature rise thus follows the same law as it does with ordinary linear conduc-

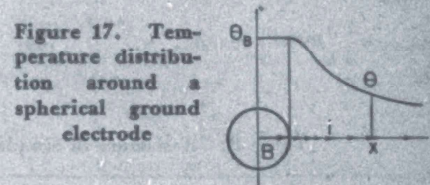


Figure 17. Temperature distribution around a spherical ground electrode

tors. If resistivity ρ and specific heat γ are constant, the temperature rise is linear with time, and thus the admissible current density is

$$i = \sqrt{\frac{\gamma}{\rho} \frac{\theta}{t}} \quad (65)$$

Because the length dimension x does not appear in equation 64, this differential equation is true for every volume element of the soil, independent of the pattern of flow of the current. Hence equation 65 is valid for any shape of the ground electrode.

For example, an electrode in moist soil and for a temperature rise not exceeding 60 degrees centigrade is permitted to be loaded over a period of 100 seconds by a current density

$$i = \sqrt{\frac{1.75 \times 10^4}{10^3} \cdot \frac{60}{100}} = 100 \text{ amperes per square meter}$$

which is a fairly high value. For sandy soil with ten times higher resistivity a short-time loading at the same current density is permitted for ten seconds only. Higher current density or longer loading period would evaporate the moisture of the soil in a very short time, and an explosion of the space surrounding the electrode might follow, and is sometimes experienced.

The voltage at the electrode with short-time loading may attain a very high value. In the example for moist soil, the voltage is higher than in the steady-state example by the ratio of the current densities, and therefore it reaches here the value

$$E = 120 \cdot \frac{100}{1.2} = 10,000 \text{ volts}$$

The initial rise of temperature over a short time as given by equation 64 is shown in Figure 18. The final temperature rise, with fixed constants of the soil, would be given by equation 60, and also is indicated in Figure 18. The intermediate curve follows the complete differential equation 57 but is difficult to derive analytically. However, we can easily determine the time constant τ , defined as the time in which the linear initial rise would reach the steady-state temperature, see Figure 18. For this intersection we have, with use of equations 60 and 64 and substitution in the former the current density for the current

$$\theta_{\text{steady}} = \frac{1}{2} \frac{\rho}{\lambda} B^2 i^2 = \frac{\rho}{\gamma} i^2 \tau = \theta_{\text{rise}} \quad (66)$$

Hence the time constant of heating for a spherical ground electrode and its surrounding soil is

$$\tau = \frac{1}{2} \frac{\gamma}{\lambda} B^2 \quad (67)$$

In addition to the thermal constants of the soil, τ is dependent only on the square of the radius B . For other forms of electrodes the time-constant follows a similar form except that instead of B another characteristic dimension of the electrode must be used, being somewhere between the maximum and the minimum dimension of the electrode.

For example, for a sphere with radius $B = 1$ meter the time constant is

$$\tau = \frac{1}{2} \frac{1.75 \cdot 10^4}{1.2} \text{ sec} = 0.73 \cdot 10^4 \text{ sec} = 8.5 \text{ days}$$

This is a very long period, due to the very low heat conductivity of the soil, but it is completely in accord with test results.

LINEARLY EXTENDED FIELDS

Buried Long Electrodes. In the foregoing discussions there have been considered only electrodes with constant voltage over their length. However, with the use of long electrodes such as wires or strips, cable sheaths, railroad or tramway tracks, pipe lines, or similar ground electrodes of considerable length, the current may suffer an ohmic drop due to the internal resistance r of the electrode before it spreads out in the resistance R of the ground. Figure 19 shows how the total current I_0 enters a ground wire, follows its length, gradually being attenuated, and leaving the wire as spatial current i . There are measured now the wire current I in amperes, the ground current i in amperes per meter, the resistance r of the electrode in ohms per meter, and the resistance R of the ground in ohm-meters. The resistance R may vary slightly over the

length x of the wire due to the concentration of the ground-current density i near the end points. However, taking a mean value over the length is sufficiently accurate for this problem.

According to Kirchhoff's laws, at any point x of the long wire the sum of the inflowing and outflowing current within the wire and the current flowing to ground is zero. On the other hand, the voltage over a narrow rectangle formed by a wire resistance element $r \cdot dx$ and the two adjacent ground resistances R is zero. Therefore

$$\left. \begin{aligned} i + \frac{dI}{dx} &= 0 \\ R \frac{di}{dx} + rI &= 0 \end{aligned} \right\} \quad (68)$$

from which can be obtained the differential equation, for the wire current I , for example,

$$\frac{d^2 I}{dx^2} - \frac{r}{R} I = 0 \quad (69)$$

If we reckon x from the far end of the wire of length l , and write for abbreviation

$$\delta = \sqrt{\frac{r}{R}} \quad (70)$$

the solution of equation 69 gives the current distribution in the wire as

$$I = \frac{\sinh \delta x}{\sinh \delta l} I_0 \quad (71)$$

Correspondingly, the ground-current distribution results from the first equation 68 as

$$i = \delta \cdot \frac{\cosh \delta x}{\sinh \delta l} I_0 \quad (72)$$

The variation of both currents along the length x of the electrode is shown in Figure 19b.

Near the terminal of a long conductor the hyperbolic distributions have a space constant X defined by the subtangent as

$$X = \frac{1}{\delta} = \sqrt{\frac{R}{r}} \quad (73)$$

If the electrode has a length of more than about $3X$, the ground current will have attenuated to an insignificant remainder over that length. Thus the effect of buried long electrodes is limited by gradual attenuation, determined by the ratio of wire to ground resistance, as seen by equation 73.

For low values of wire resistance r or length l the product δl is small and there is

$$\left. \begin{aligned} \sinh \delta l &\approx \delta l \\ \cosh \delta l &\approx 1 \end{aligned} \right\} \quad (74)$$

Therefore

$$\left. \begin{aligned} I &= \frac{x}{l} I_0 \\ i &= I_0 / l \end{aligned} \right\} \quad (75)$$

Table II. Resistance of Simple Forms of Ground Electrodes

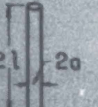
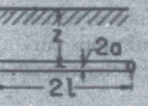
	Sphere: $R = \frac{\rho}{4\pi B}$		Disk: $R = \frac{\rho}{8b}$
	Rod: $R = \frac{\rho}{4\pi l} \log_e \left(\frac{2l}{a} \right)$		Ring: $R = \frac{\rho}{4\pi b} \log_e \left(\frac{8b}{a} \right)$
	Strip: $R = \frac{\rho}{4\pi l} \log_e \left(\frac{4l}{w} \right)$		Deep wire: $R = R_0 \left[1 + \frac{\log_e \left(\frac{l}{z} \right)}{\log_e \left(\frac{2l}{a} \right)} \right]$
	Equivalent rod and strip: $A = W/2$		Surface electrode: $R_0 = 2R$

Table III. Geometric Mean Distances for Parallel-Rod Electrodes

n	Arrangement	Geometric Mean Distance
2		$A = \sqrt{aS}$
3		$A = \sqrt[3]{aS^2}$
4		$A = \sqrt[4]{2aS^3}$

Table IV. Resistance and Dimensions of Equivalent Ground-Return Conductor

$l =$	50	150	500	5,000	c
$R =$	0.05	0.15	0.5	5	Ω/km
$d =$	2.1	1.3	0.7	0.2	cm
$D =$	800	460	250	80	m

Thus the wire current decreases linearly with x and the ground current is uniformly distributed over l , as is the case with constant-voltage electrodes.

For large values of wire resistance r or length l , excluding, however, the point $x=0$ and its neighborhood, there is

$$\sinh \delta x \approx \cosh \delta x \approx e^{\delta x}/2 \quad (76)$$

Therefore, according to equations 71 and 72, both currents I and i attenuate exponentially from their original values, and the ground current is related to the wire current by

$$i = \sqrt{\frac{r}{R}} I \quad (77)$$

The resistance R_0 of the entire electrode to ground is given by the ratio of voltage E to terminal current I_0 , the voltage being equal to the ohmic drop of the initial ground current i_0 across its ground resistance R . Thus

$$R_0 = \frac{E}{I_0} = \frac{Ri_0}{I_0} \quad (78)$$

Using equation 72 at the terminal $x=l$, and substituting equation 70, the ground resistance of an extended buried electrode is generally

$$R_0 = \sqrt{rR} \cdot \coth \delta l \quad (79)$$

and for large δl , where $\coth \delta l$ becomes 1, is simply

$$R_0 = \sqrt{rR} \quad (80)$$

Hence the ground resistance of a long electrode depends equally on the electrode's metallic resistance electrode and the soil's resistance soil, the geometric mean value being effective.

For example, for the lead sheath of a cable of radius $a=2$ centimeters and length $l=1$ kilometer buried in moist soil close to the surface of the earth, the ground resistance of each meter of cable, using equation 30, is

$$R = \frac{\rho}{2\pi} \log_e \left(\frac{l}{a} \right) = \frac{10^9}{2\pi} \log_e \left(\frac{10^3}{2 \cdot 10^{-2}} \right) = 170 \text{ ohm-meters}$$

This would give a total resistance of the lead sheath to ground of $R/l=0.17$ ohm, if the voltage of the electrode were kept constant over the entire length. The resistance of the lead sheath is about $r=1.5 \cdot 10^{-3}$ ohms per meter, giving a space constant

$$X = \sqrt{\frac{170}{1.5 \cdot 10^{-3}}} = 337 \text{ meters}$$

and since $\coth 1,000/337=1$, a resultant resistance of lead sheath and ground

$$R_0 = \sqrt{1.5 \cdot 10^{-3} \cdot 170} = 0.5 \text{ ohm}$$

Hence the voltage on the sheath will decrease over the cable length to nearly zero and the effective resistance R_0 is three times as great as were this voltage constant.

Separate Conductors Buried in the Earth. If cross section and conductance of the electrode are considerable, the currents may be led to much greater distances. Such conductors sometimes play a tricky role, even if they are not directly connected to any electric system. They collect the streamlines of current in one region and carry them into a distant region of the ground.

For a long pipe in an extended ground field, as in Figure 20, an equilibrium is established between the parallel current density i_1 in the ground and i_2 in the pipe extending in the direction of the streamlines. The field strengths e in soil and conductor are the same, and since the current density is field strength over resistivity, the ratio of the current densities in electrode and soil is

$$\frac{i_2}{i_1} = \frac{e_2/\rho_2}{e_1/\rho_1} = \frac{\rho_1}{\rho_2} \quad (81)$$

Thus the current densities are inversely as the resistivities.

For a steel conductor buried in moist ground, there must be expected

$$\frac{i_{\text{steel}}}{i_{\text{soil}}} = \frac{10^9}{10^{-1}} = 10^{10}$$

and a still greater ratio if the ground is dry. Therefore a steel pipe of ten square centimeters cross section can collect current from a ground area of

$$10^6 \frac{10}{10^4} = 10^4 \text{ square meters} = 1 \text{ square kilometer}$$

of cross section. Hence such a pipe is able to suck ground current from wide areas and carry it to distant points.

For a short pipe in an extended ground field, as in Figure 21, streamlines of current will enter at one end and leave from the other end. The voltage over the length $2l$ of the pipe in the undisturbed ground field is

$$E = 2l\epsilon = 2l\rho i \quad (82)$$

This voltage works on the entrance and exit ground resistance of each end of the pipe and produces a current I in the center of the pipe, the metallic resistance being insignificant. Thus, with use of equation 30,

$$E = 2RI = \frac{2\rho l}{2\pi} \log_e \left(\frac{2l}{a} \right) \quad (83)$$

Equating equations 82 and 83 gives a pipe current

$$I = \frac{2\pi l^2}{\log_e \left(\frac{2l}{a} \right)} i \quad (84)$$

related to the current density i in the undisturbed underground field and independent of resistivity ρ . If groups of parallel pipes are present instead of a single pipe, their geometric mean distance A rather than the radius a of a single pipe is to be used, see equation 50.

For example, a pipe of half length $l=100$ meters and radius $a=10$ centimeters will collect from a field of current density $i=10^{-4}$ amperes per square meter a maximum current

$$I = \frac{2\pi 100^2}{\log_e \left(\frac{2 \cdot 100}{0.1} \right)} 10^{-4} = 0.83 \text{ ampere}$$

Figure 21 shows that with such a pipe the current lines over a circular area are collected, the diameter D of which can be found from

$$I = \frac{\pi}{4} D^2 i = \frac{2\pi l^2}{\log_e \left(\frac{2l}{a} \right)} i \quad (85)$$

Thus the diameter of collection is

$$D = 2l \sqrt{\frac{2}{\log_e \left(\frac{2l}{a} \right)}} = 2l \sqrt{\frac{2}{6 \rightarrow 10}} \approx l \quad (86)$$

if an average value of the logarithm of about 8 is used.

Such pipes may suffer electrolytic corrosion if the d-c density exceeds a value of the order of 0.1 ampere per square meter. The average entrance or exit current density over the surface of the pipe in Figure 21 is

$$\bar{i} = \frac{I}{2\pi al} = \frac{l/a}{\log_e \left(\frac{2l}{a} \right)} i \quad (87)$$

showing a high current concentration over the density i in the free field, and this may even be doubled at the pipe ends. For the

example the mean current density along the pipe is

$$i = \frac{100/0.1}{7.6} = 132 \text{ } i = 132 \cdot 10^{-4} = 1.32 \cdot 10^{-2} \text{ ampere per square meter}$$

which is under the corrosion limit.

For a long pipe passing through a limited ground field, as in Figure 22, the problem of the development of the current lines in space is highly complex. Ollendorf found an approximate solution for the maximum current I_p developed in a pipe of radius a which passes at distance S an electrode from which a current I enters the ground, namely,

$$I_p = \frac{\sinh^{-1} \sqrt{\frac{S}{a}}}{\log_4 \left(4 \frac{S}{a} \right)} I \quad (88)$$

The current collected in this way, as shown by Figure 22, may spread out over vast distances depending on the resistivities of ground and pipe. The maximum surface density of the current entering the pipe, according to Ollendorf's solution, is

$$i = \frac{I}{\pi a S \log_4 \left(4 \frac{S}{a} \right)} \quad (89)$$

With current $I=100$ amperes entering the ground through an electrode at distance $S=30$ meters from a pipe the current density entering the pipe of radius $a=10$ centimeters will be

$$i = \frac{100}{\pi \cdot 0.1 \cdot 30 \log_4 \left(4 \frac{30}{0.1} \right)} = 1.5 \text{ amperes per square meter}$$

a value which will produce severe electrolytic corrosion, if it is direct current.

Alternating Return Current Under Transmission Lines. This article has dealt up to now with the effects of the resistivity of the ground without considering any in-

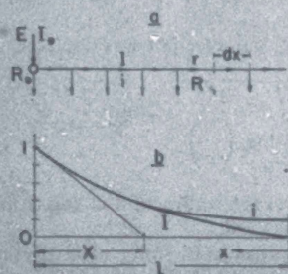


Figure 19. Current distribution along an extended electrode

ductive action of the currents. These derivations are true for direct current flowing through the ground. However, they are equally valid for alternating current as far as the proximity of the electrodes is concerned, for here the inductive action of the changing magnetic fields is negligible compared with the effect of the high resistivity

of the ground. Only for extremely rapid change with time this may be modified, as will be shown later.

If ground current flows over a long distance between entrance and exit electrode, the current lines in the d-c case spread out over such a broad transverse area in the earth, as seen by Figure 4a, that the resistance is negligibly small except for the proximity of the electrodes, as previously shown. For alternating current, however, with decreasing effect of the resistance the self-inductive action of the magnetic field becomes preponderant, and thus the distribution of the current lines in the ground under a-c conditions will be determined mainly by the inductive effects. It is well known that the currents in such a case are so distributed that the energy of the magnetic field and thereby the self-inductance tend to become a minimum.

Therefore an alternating return current in the ground under a long transmission line will not spread to a great distance, as direct current does, but will concentrate on paths in proximity to the transmission line, as shown in Figure 23. Hence the distribution of alternating current in the ground is governed by the laws of skin effect, laterally as well as in depth. Because

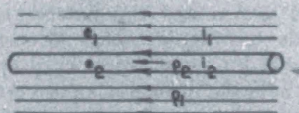


Figure 20. Long pipe embedded in an extended ground field

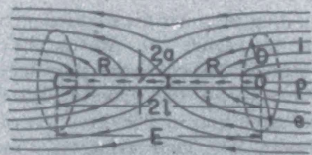


Figure 21. Short pipe buried in an extended ground field

of the high resistivity of the soil, however, much larger dimensions come into consideration than with metallic conductors.

A single transmission line, as in Figure 23, will develop concentric magnetic lines of force of nearly circular form, at least in the neighborhood of the line, where the magnetic effect of the return current in the ground is of lesser importance because of its low density. Therefore the phenomena will be dependent primarily on the distance y from the center of the wire, carrying the total current I , to the point considered at the earth. The error caused by this assumption is usually small as compared to that caused by subsoil heterogeneity. With angular frequency ω , the differential equation of the distribution over the ground of the current density i then is

$$\frac{1}{y} \frac{d}{dy} \left(y \frac{di}{dy} \right) - j \cdot 2\pi \mu \frac{\omega}{\rho} i = 0 \quad (90)$$

a complex relation similar to that for other skin-effect problems. The value $\mu=10^{-7}$ is the permeability of nonmagnetic material in the meter-kilogram-second system.

The solution of this equation is given by a Bessel function H_0 of zero order. With abbreviation

$$\kappa = \sqrt{2\pi \mu \frac{\omega}{\rho}} \quad (91)$$

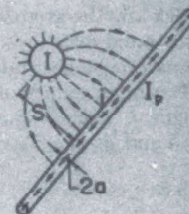
the current density, up to frequencies of about 5,000 cycles per second, is

$$i = \frac{I}{2} H_0(\sqrt{j} \kappa y) \quad (92)$$

where the argument of the Bessel function depending on distance y is complex, indicating that the distribution of the return current through the ground occurs in waves.

For two instants, 90 degrees displaced in time, the values of the Bessel function H_0 are shown in Figure 23, determining the current density against distance. From

Figure 22. Long pipe running through a limited ground field



numerical tables of this Bessel function it can be derived that directly under an overhead power line of height $y=h=10$ meters the current density at 60 cycles per second in moist soil of $\rho=10^8$ ohm-meters is nearly

$$i = 3 \cdot I \text{ in amperes per square kilometer}$$

Thus for every ampere of line current there develops a current density of three amperes per square kilometer in the ground under the line, and naturally a still smaller current density at greater distances.

The current density in the ground decreases to an insignificant magnitude at a distance of about $y=3/\kappa$, as seen by a discussion of the function H_0 . Hence for moist soil the zone of extinction is

For current of 60 cycles per second about 2,000 meters,

For current of 500 cycles per second about 700 meters,

and a still smaller distance for higher frequencies. Thus a return current under a transmission line flows only in a moderately broad zone through the ground under and to both sides of the line. This zone follows any trace of the line, it may be straight or curved, since the distribution of the return current always seeks to reduce the self-inductance to a minimum and therefore to avoid any open loop between line current and return path.

The field strength in the ground, with use of equations 91 and 92, is

$$e = \rho i = \pi \mu \omega l \cdot H_0 (\sqrt{j} \kappa y) \quad (93)$$

Therefore the voltage over a distance l on the ground surface immediately under the transmission line, namely for $y=h$, is

$$E = \pi \mu \omega l \cdot H_0 (\sqrt{j} \kappa h) = \omega l \left[\frac{\pi}{2} + j 2 \log_e \left(\frac{1.12}{\kappa h} \right) \right] 10^{-7} \quad (94)$$

Herein an asymptotic approximation of the Bessel function is used for small values of the complex argument, resulting in a form which has distinct real and imaginary components. The voltage over the ground, therefore, is complex, consisting of two perpendicular vector components, where

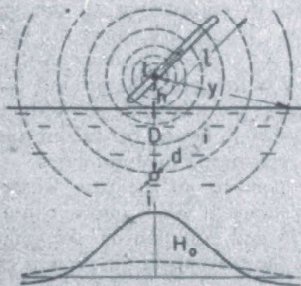


Figure 23. Distribution of ground-return currents under a-c transmission lines

the real part gives the effective resistance of the return current

$$R = \frac{\pi}{2} \mu \omega l = \pi^2 f l \cdot 10^{-7} \quad (95)$$

while the imaginary part determines the effective self-inductance

$$L = 2l \cdot \log_e \left(\frac{0.178}{h} \sqrt{\frac{\rho}{f}} \right) \cdot 10^{-7} \quad (96)$$

In the last two equations the frequency f per second is used instead of the angular frequency ω .

Hence alternating return current through the ground experiences a resistance which is proportional to length l and to frequency f but independent of resistivity ρ of the soil. Table IV gives the return resistance R in ohms per kilometer for various frequencies in moist soil of $\rho = 10^3$ ohm-meters. The values are not unimportant for heavier currents or for frequencies above the power range. The same return resistance would be attained by a fictitious copper wire of an equivalent diameter d as given in the third line of Table IV.

The self-inductance of the return currents as given by equation 96 is also proportional to length l , depends slightly on the height h of the transmission line and, to an even lesser degree, on resistivity and frequency, these being under the logarithm of a square root. Thus for a large range of

such data the self-inductance is very nearly

$$L = 0.9 \text{ millihenry per kilometer}$$

or

$$\omega L = 0.34 \text{ ohm per kilometer}$$

This self-inductance of the ground current adds to the self-inductance of the line current, which is also determined by a well-known logarithmic relation. So is also the self-inductance of a fictitious return conductor of diameter d and depth D under the surface, the ohmic and inductive effects of which may be taken equivalent to those of the ground return. By comparison of equation 96 with the self-inductance of such an ideal return system, the equivalent depth can be determined. We see from the last two lines in Table IV that the equivalent diameter d of a copper return wire would be of the order of two centimeters for 50 cycles per second but of only 0.2 centimeters for 5,000 cycles per second. On the other hand, the equivalent depth D would vary from 800 meters to 80 meters over the same frequency range.

Inductive Interference of Parallel Lines. If at a distance y a neighboring line runs parallel to the transmission line with cur-

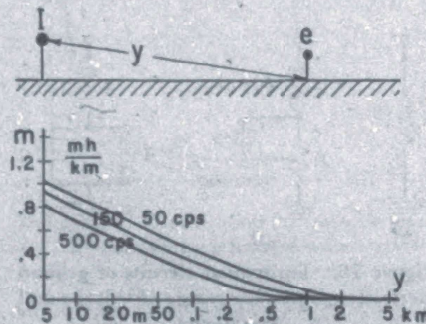


Figure 24. Mutual inductance between lines with ground return

rent I returning through the ground, as shown in Figure 24, and the two ends of the secondary line are directly or indirectly grounded, then a voltage develops over a length l of this line according to equation 93

$$E = \pi \mu \omega l \cdot H_0 (\sqrt{j} \kappa y) \quad (97)$$

This is a voltage of mutual effect between primary and secondary line caused mainly by action of the ground-return currents and their magnetic field.

Hence the coefficient of mutual induction for the unit length of both lines is

$$m = \pi \cdot H_0 (\sqrt{j} \kappa y) \cdot 10^{-7} \quad (98)$$

Since the value of the Bessel function H_0 is complex, this mutual inductance has amplitude and phase angle in contrast to the straight inductance of two linear circuits alone. This is effected by the alternating currents produced in a spatially extended third conductor, namely, the

ground. Figure 24 shows the amplitude of the mutual inductance in millihenry per kilometer dependent on distance y between primary and secondary circuit for various frequencies, if the resistivity of the intermediate ground is $\rho = 10^3$ ohm-meters, as for moist soil. In all practical cases the resistivity will not be uniform over surface and depth of the ground between and around such transmission lines, and thus an average resistivity of the entire field should be taken into account. High resistivity of the underground increases the action to a greater distance.

If, for example, a short-circuit current $I = 500$ amperes in a power line flows through an arcing ground to earth and returns through it to the grounded neutral of the power station, then, according to equation 97, a telephone line at a distance $y = 1$ kilometer, running parallel to the power line over a length $l = 10$ kilometers and using the ground for return, will suffer a voltage of

$$E = \pi \cdot 10^{-7} \cdot 377 \cdot 500 \cdot 10 \cdot 10^3 \cdot 0.20 = 118 \text{ volts}$$

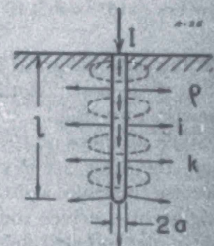
a value which would annihilate any telephone conversation.

IMPULSE CHARACTERISTICS

Performance of Driven Rods. It may be asked how closely the current in an electrode and the surrounding ground can follow a rapid variation of the voltage. The best criterion for this is given by the values of the electric time-constants and the natural periods of oscillation.

In Figure 25 the current I impinges on the rod electrode and enters the ground,

Figure 25. Impulse current spreading from driven rod



which in addition to its resistivity has a dielectric constant k . Thus in parallel to the conductive current in the ground there develops a capacitive or displacement current in case the electrode voltage changes with time. The displacement current follows exactly the lines of the conductive current, and thus the ground electrode will have a capacitance, as reciprocal to the resistance of equation 30,

$$C = \frac{k l}{2 \cdot \log_e \left(\frac{2l}{a} \right)} \cdot \frac{10^{-9}}{9} \text{ farad} \quad (99)$$

The currents in electrode and ground, furthermore, form a magnetic field indicated by the dashed lines in Figure 25.

This field is highest where the current is most concentrated; namely around the rod electrode. Therefore the inductance of the ground currents is mainly given by the distribution of the currents in the rod, having full value at the top and decreasing to zero at the bottom. The inductance of such a rod is

$$L = 2l \cdot \log_e \left(\frac{2l}{a} \right) \cdot 10^{-7} \text{ henry} \quad (100)$$

The capacitive time constant of any circuit element is given by the product of capacitance and resistance, and thus a rod electrode has a time constant

$$\tau_c = CR = \frac{\rho k}{4\pi \cdot 9 \cdot 10^9} \quad (101)$$

It depends only on resistivity and dielectric constant k , the value of which for ordinary soil may be of the order of 9, in view of the high value for water.

For example, for moist soil

$$\tau_c = \frac{10^4 \cdot 9}{4\pi \cdot 9 \cdot 10^9} = 8 \cdot 10^{-8} \text{ second}$$

which is an extremely small value. For rock as underground the value is about $8 \cdot 10^{-7}$ second.

The inductive time constant as quotient of self-inductance and resistance is

$$\tau_L = \frac{L}{R} = 4\pi \frac{\rho}{p} \cdot 10^{-7} \quad (102)$$

It is proportional to the square of the length l of the electrode, and therefore greatly dependent on this dimension. For example, for a rod of $l = 6$ meters in moist soil

$$\tau_L = 4\pi \frac{6^2}{10^4} \cdot 10^{-7} = 4.5 \cdot 10^{-7} \text{ second}$$

For the same electrode in rock the value is $4.5 \cdot 10^{-8}$ second. Both these values are likewise very small.

Since all these numerical values are less than the time usually assumed for the shortest possible lightning impulse impinging on a driven rod, current and voltage in the rod will follow such impressed impulse of the order of 10^{-8} seconds without any significant time delay and will behave as in the steady state. Hence capacitance and inductance of a driven rod of moderate length play no significant part even with rapid lightning phenomena.

The natural oscillations of which such a rod is capable have a period

$$T = 2\pi\sqrt{LC} = \frac{2\pi\sqrt{k}}{3 \cdot 10^9} \cdot l \quad (103)$$

depending only on dielectric constant and length. For the same example, the natural period is

$$T = \frac{2\pi\sqrt{9}}{3 \cdot 10^9} \cdot 6 = 3.8 \cdot 10^{-7} \text{ seconds}$$

This also is beyond the duration usually assumed for lightning strokes, and indi-

cates the frequency up to which the rod behaves as an ohmic resistance.

The damping of the free oscillations is determined by the value of the quotient of resistance and surge impedance in comparison to the value one half, namely,

$$\frac{R}{\sqrt{L/C}} = \frac{\rho\sqrt{k}}{120\pi l} \begin{cases} > 1/2 \text{ periodic} \\ < 1/2 \text{ aperiodic} \end{cases} \quad (104)$$

For the same rod in moist soil

$$\frac{R}{\sqrt{L/C}} = \frac{10^4\sqrt{9}}{120\pi \cdot 6} = 7.5$$

so that any natural oscillation would be aperiodically damped. For rock, however, the quotient assumes the value 13, indicating that here the driven rod can oscillate periodically.

An equivalent diagram is often convenient to represent the properties of a circuit element in a simplified form. In view of the foregoing numerical values there is no doubt that for low frequencies a driven rod and all its derived electrodes can be represented by a lumped resistance, as in Figure 26a. This remains true for a ground electrode of moderate length up to high and even very high frequencies. However, it should be noticed that a

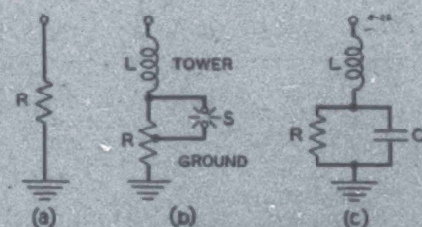


Figure 26. Equivalent circuits of ground electrodes over wide range of voltage and frequency

- (a). Low voltage, low frequency
- (b). High voltage, high frequency
- (c). Ultrahigh frequency

tower, or a lead, through which a lightning discharge reaches the electrode may have considerable self-inductance because of its height. This is indicated in Figure 26b. If the voltage at the rod electrode is very high, as is frequently the case with lightning discharges, the field strength at the rod becomes so great that the electric strength of the ground is broken down and sparks discharge from the rod into the neighboring ground thereby considerably decreasing the entire ground resistance. This performance is indicated in the equivalent diagram of Figure 26b by a spark by-pass to a part of the ground resistance.

If the discharge to the ground electrode is extremely rapid or the equivalent frequency is extremely high, of an order of more than 10^8 cycles per second, a considerable displacement current in the

capacitance of the ground will parallel the conductive current in the resistance, and together with the self-inductance of both of these currents through the rod, will effect an equivalent diagram as in Figure 26c. The time-constants, the natural oscillations, and the damping of the electrode, as in equations 101 to 104, now will be determinative.

Performance of Buried Wires. In ground electrodes of considerable length, as with buried long wires, the self-inductance will play a larger part with respect to impulse currents, since the inductive time constant, see equation 102, increases as the square of the length. The capacitive effect of the ground, see equation 101, will not change, however, and may be neglected. Since buried wires usually have substantial cross section, the internal resistance of the electrode also may be neglected in comparison to the self-inductance. Figure 27a shows the remaining data of the problem, namely, the wire current I in amperes, the ground current i in amperes per meter, the self-inductance L of the wire in henrys per meter, and the ground resistance R in ohm-meters.

Corresponding to the case where the wire resistance is taken into account, see Figure 18 and equation 68, the relations between the currents in the present example are

$$\begin{aligned} i + \frac{dI}{dx} &= 0 \\ \frac{dI}{dx} + L \frac{di}{dt} &= 0 \end{aligned} \quad (105)$$

Their combination gives the differential equation for the current in the wire

$$\frac{d^2 I}{dx^2} - \frac{L}{R} \frac{dI}{dt} = 0 \quad (106)$$

and a corresponding equation for the ground current i . Equation 106 is the well-known differential equation of heat diffusion. A solution for a short impulse is

$$I = \frac{K}{\sqrt{t}} e^{-\frac{L}{R} \frac{x^2}{4t}} \quad (107)$$

where the amplitude is still free as determined by a constant K of integration, while the exponent depends, in addition to L and R , on the square of distance x and on time t . This function is plotted in Figure 27b against distance x and in Figure 28a against time t . Equation 107 represents an impulse curve over the time axis, which increases at first slowly and then steeply, reaches a maximum peak, and decreases gradually toward zero. Such a curve might well be used to approximate the time characteristic of a lightning discharge to ground which is known from many experimental investigations.

Figure 27b shows the manner in which the current in the wire, from a narrow zone in the beginning, spreads gradually

over the length of the wire. It is not necessary to identify the origins of x and of t in equation 107 with the origin of the lightning current. Figures 27 and 28 show that displacements in space x_0 and in time t_0 can be used for constants of the solution (107) and by proper selection of these every actual case can be approximated closely.

The points at which the spatial curves of Figure 27b are steepest determine the front zone of the attenuated propagating wave of current. Their location can be derived from equation 107 and is given by

$$\frac{L}{R} \frac{x^2}{t} = \frac{1}{2} \quad (108)$$

These points travel with a velocity

$$v = \frac{dx}{dt} = \frac{1}{4} \frac{R/L}{x} = \frac{10^8 \rho}{4\pi x} \quad (109)$$

where R/L is taken from equation 102, both data, however, being here related to the unit length. Hence the velocity of the front is not constant but decreases rapidly with distance x .

Buried long wires are often used as electric counterpoises additional to the tower footing conductance of high-voltage transmission lines, in case the resistivity of the soil is so high that it is difficult otherwise to obtain a low ground resistance. With a resistivity, therefore, of $\rho = 10^3$ ohm-meters, the front velocity of the current wave will be, for example, at distance $x = 10$ meters on the wire,

$$v = \frac{10^7 \cdot 10^3}{4\pi \cdot 10} = 0.8 \cdot 10^8 \text{ meters per second}$$

This is about one-fourth the velocity of light, a velocity reached in straight conductors in air. In ground wires of considerable length, therefore, the joint effect of self-inductance and ground resistance leads to a lower and ever decreasing wave velocity.

The voltage at every point of the wire is determined by the ohmic drop of the ground current i in the ground resistance R . By using the first equation 105 and differentiating equation 107, the voltage is

$$E = Ri = -R \frac{dI}{dx} = 2L \frac{x}{t} I \quad (110)$$

whereby E is reduced for every distance and time to equation 107 for the current I . Since E is proportional to I , a definite value of the impedance is obtained, which, however, is dependent on distance and time. The surge impedance is

$$Z = \frac{E}{I} = 2L \frac{x}{t} \quad (111)$$

and is proportional to the self-inductance L per unit length of the wire. For any point x of the wire, Z is inversely proportional to time.

There may be correlated to the beginning of the steep ascent of the current with time, as shown by t_0 in Figure 28a,

a location x_0 corresponding to about half the value of equation 108, so that

$$\frac{x_0^2}{t_0} = \frac{1}{4} \frac{R}{L} \quad (112)$$

By substitution of this value of x in equation 111 and use for R and L of values per unit length derived from equations 30 and 100 for a finite wire, the surge impedance for the terminal of the wire becomes

$$Z_0 = \frac{t_0}{t} \sqrt{\frac{RL}{t_0}} = \frac{t_0}{t} \sqrt{\frac{2\rho \cdot 10^{-7}}{\pi t_0}} \log_e \left(\frac{2l}{a} \right) \quad (113)$$

Since buried wires usually are embedded near the surface of the ground, extending to both sides from the terminal, the value of equation 30 is doubled for the resistance here in question.

We consider for an example a wire of length $l = 75$ meters and of radius $a = 0.5$ centimeter, buried in ground of high resistivity $\rho = 10^3$ ohm-meter, and we estimate the time until the steep ascent of the

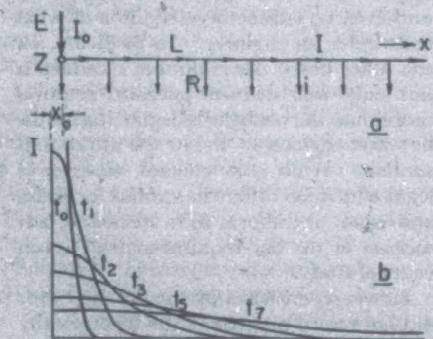


Figure 27. Distribution of impulse current along a buried wire

lightning current occurs as $t_0 = 0.2 \cdot 10^{-8}$ second. Then the surge impedance for the practical start of the current at $t = t_0$ will be

$$Z_0 = \sqrt{\frac{2 \cdot 10^3 \cdot 10^{-7}}{\pi \cdot 0.2 \cdot 10^{-8}}} \log_e \left(\frac{2 \cdot 75}{0.5 \cdot 10^{-2}} \right) = 184 \text{ ohms}$$

and this value decreases hyperbolically with increasing time as shown in Figure 28b.

The impedance would decrease gradually to zero, see equation 113, due to the omission of ohmic resistance of the infinite wire in this calculation. Actually the impedance cannot drop lower than the d-c ground resistance of the finite buried wire, which in this example is

$$R_0 = \frac{10^3}{\pi \cdot 75} \log_e (3 \cdot 10^4) = 44 \text{ ohms}$$

thus limiting the decrease of Z , as shown in Figure 28b by the dashed line.

CONCLUSION

By using for the development of the current distribution in the ground such mathematical analyses as are best suited to each special problem, rather than any

general method of rigorous calculation, the treatment does not depart far from the physical aspect, and the solutions become fairly simple and easy to survey, even in complicated examples. In view of the great heterogeneity of the ground under the surface of the earth, the effect of which can never be taken into account

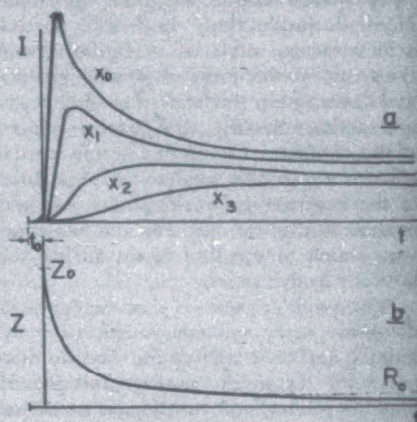


Figure 28. Change with time of current and surge impedance in buried wires

by any rigorous deduction, such a simplified foundation of the principles of ground currents appears appropriate from the viewpoint of engineering as well as from the viewpoint of the economic use of science.

REFERENCES

1. Ground Connections for Electrical Systems, O. S. Peters. National Bureau of Standards, technological paper 108, 1918.
2. Wave Propagation in Overhead Wires With Ground Return, J. R. Carson. Bell System Technical Journal, volume 5, 1926, page 539.
3. Grundlagen der Bemessung von Starkstromordern, K. Fohlhausen. VDE Fachberichte, 1927, page 39.
4. Erdströme (book), F. Ollendorf. Berlin, 1928.
5. Elektrische Schaltvorgänge (book), R. Rüdenberg. Berlin, Germany, 1933, third edition.
6. Theory and Tests of the Counterpoise, L. V. Bewley. Electrical Engineering, volume 53, August 1934, pages 1163-72.
7. Earth Resistivity and Geological Structure, R. H. Card. Electrical Engineering, volume 54, November 1935, pages 1153-61.
8. The Current-Loading Capacity of Earth Electrodes, H. G. Taylor. Journal of the Institution of Electrical Engineers, volume 77, 1935, page 542.
9. Calculation of Resistances to Ground, H. B. Dwight. Electrical Engineering, volume 55, 1936, pages 1319-28.
10. Practical Aspects of Earthing, K. Fawcett, H. W. Grinnitt, G. F. Shetter, H. G. Taylor. Journal of the Institution of Electrical Engineers, volume 37, 1940, page 357.
11. Grounding Electric Circuits Effectively, J. R. Eaton. General Electric Review, volume 44, 1941, pages 323, 397, 451.
12. Study of Driven Rods and Counterpoise Wires in High-Resistance Soil on 140-Kv System, J. G. Homstreet, W. W. Lewis, C. M. Frost. AIEE Transactions, volume 61, 1942, September section, pages 628-34.
13. Impulse and 60-Cycle Characteristics of Driven Grounds; part I, P. L. Bellaschi, AIEE Transactions, volume 60, 1941, March section, pages 123-8; part II, P. L. Bellaschi, R. E. Armington, A. E. Snowden, AIEE Transactions, volume 61, 1942, pages 349-63; part III, P. L. Bellaschi, R. E. Armington, AIEE Transactions, volume 62, 1943, pages 334-45.