## Chapter 6

## Exponential and Logarithmic Functions

## Section 6-4

## Transformations of Exponential and Logarithmic Functions

## Transforming Graphs of Exponential Functions

You can transform graphs of exponential and logarithmic functions in the same way you transformed graphs of functions in previous chapters. Examples of transformations of the graph of $f(x)=4^{x}$ are shown below.

## EXAMPLE 1 Translating an Exponential Function

Describe the transformation of $f(x)=\left(\frac{1}{2}\right)^{x}$ represented by $g(x)=\left(\frac{1}{2}\right)^{x}-4$.
Then graph each function.




## STUDY TIP

Notice in the graph that the vertical translation also shifted the asymptote 4 units down, so the range of $g$ is $y>-4$.

## EXAMPLE 2 Translating a Natural Base Exponential Function

Describe the transformation of $f(x)=e^{x}$ represented by $g(x)=e^{x+3}+2$. Then graph each function.


## STUDY TIP

Notice in the graph that the vertical translation also shifted the asymptote 2 units up, so the range of $g$ is $y>2$.


## EXAMPLE 3 Transforming Exponential Functions

Describe the transformation of $f$ represented by $g$. Then graph each function.
a. $f(x)=3^{x}, g(x)=3^{3 x-5}$
b. $f(x)=e^{-x}, g(x)=-\frac{1}{8} e^{-x}$





## Transforming Graphs of Logarithmic Functions

Examples of transformations of the graph of $f(x)=\log x$ are shown below.


| Transformation | $f(x)$ Notatio | Examples |  |
| :---: | :---: | :---: | :---: |
| Horizontal Translation Graph shifts left or right. | $f(x-h)$ | $\begin{aligned} & g(x)=\log (x-4) \\ & g(x)=\log (x+7) \end{aligned}$ | 4 units right <br> 7 units left |
| Vertical Translation <br> Graph shifts up or down. | $f(x)+k$ | $\begin{aligned} & g(x)=\log x+3 \\ & g(x)=\log x-1 \end{aligned}$ | 3 units up <br> 1 unit down |
| Reflection <br> Graph flips over $x$ - or $y$-axis. | $\begin{array}{r} f(-x) \\ -f(x) \end{array}$ | $\begin{aligned} & g(x)=\log (-x) \\ & g(x)=-\log x \end{aligned}$ | in the $y$-axis in the $x$-axis |
| Horizontal Stretch or Shrink <br> Graph stretches away from or shrinks toward $y$-axis. | $f(a x)$ | $\begin{aligned} & g(x)=\log (4 x) \\ & g(x)=\log \left(\frac{1}{3} x\right) \end{aligned}$ | shrink by a factor of $\frac{1}{4}$ <br> stretch by a factor of 3 |
| Vertical Stretch or Shrink <br> Graph stretches away from or shrinks toward $x$-axis. | $a \cdot f(x)$ | $\begin{aligned} & g(x)=5 \log x \\ & g(x)=\frac{2}{3} \log x \end{aligned}$ | stretch by <br> a factor of 5 <br> shrink by <br> a factor of $\frac{2}{3}$ |

## EXAMPLE 4 Transforming Logarithmic Functions

Describe the transformation of $f$ represented by $g$. Then graph each function.
a. $f(x)=\log x, g(x)=\log \left(-\frac{1}{2} x\right)$
b. $f(x)=\log _{1 / 2} x, g(x)=2 \log _{1 / 2}(x+4)$





## Writing Transformations of Graphs of Functions

## EXAMPLE 5 Writing a Transformed Exponential Function

Let the graph of $g$ be a reflection in the $x$-axis followed by a translation 4 units right of the graph of $f(x)=2^{x}$. Write a rule for $g$.

## EXAMPLE 6 Writing a Transformed Logarithmic Function

Let the graph of $g$ be a translation 2 units up followed by a vertical stretch by a factor of 2 of the graph of $f(x)=\log _{1 / 3} x$. Write a rule for $g$.

