### Chapter 7 **Rational Functions**

### Section 7-3 Multiplying and Dividing Rational Functions

## Simplifying Rational Expressions

A rational expression is a fraction whose numerator and denominator are nonzero polynomials. The domain of a rational expression excludes values that make the denominator zero. A rational expression is in simplified form when its numerator and denominator have no common factors (other than  $\pm 1$ ).

# Core Concept

### Simplifying Rational Expressions

Let a, b, and c be expressions with  $b \neq 0$  and  $c \neq 0$ .

Property	$\frac{ae'}{be'} = \frac{a}{b}$	Divide out common factor c.
Examples	$\frac{15}{65} = \frac{3 \cdot \cancel{5}}{13 \cdot \cancel{5}} = \frac{3}{13}$	Divide out common factor 5.
	$\frac{4(x+3)}{(x+3)(x+3)} = \frac{4}{x+3}$	Divide out common factor $x + 3$ .

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator. Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

## EXAMPLE 1 Simplifying a Rational Expression

Simplify  $\frac{x^2 - 4x - 12}{x^2 - 4}.$ 

COMMON ERROR

Do not divide out variable terms that are not factors.

 $\frac{x-6}{x-2} \neq \frac{-6}{-2}$ 

## Multiplying Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form. Similar to rational numbers, rational expressions are closed under multiplication.

# 🔄 Core Concept

### **Multiplying Rational Expressions**

Let a, b, c, and d be expressions with  $b \neq 0$  and  $d \neq 0$ .

**Property**  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$  Simplify  $\frac{ac}{bd}$  if possible. Example  $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{10 \cdot 3 \cdot x \cdot x^2 \cdot y^3}{10 \cdot 2 \cdot x \cdot y^3} = \frac{3x^2}{2}, \quad x \neq 0, y \neq 0$ 

### EXAMPLE 2 Multiplying Rational Expressions

Find the product  $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$ .

### EXAMPLE 3 Multiplying Rational Expressions

Find the product  $\frac{3x-3x^2}{x^2+4x-5} \cdot \frac{x^2+x-20}{3x}$ .

Find the product  $\frac{x+2}{x^3-27} \cdot (x^2+3x+9)$ .

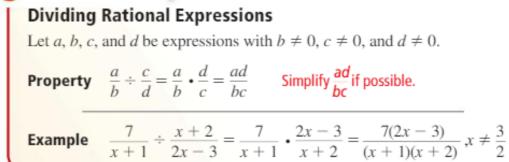
## STUDY TIP

Notice that  $x^2 + 3x + 9$ does not equal zero for any real value of x. So, no values must be excluded from the domain to make the simplified form equivalent to the original.

## Dividing Rational Expressions

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression. Rational expressions are closed under nonzero division.

# 🔄 Core Concept



## EXAMPLE 5 Dividing Rational Expressions

Find the quotient  $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$ .

## EXAMPLE 7 Solving a Real-Life Problem



The total annual amount 
$$I$$
 (in millions of dollars) of personal income earned in Alabama and its annual population  $P$  (in millions) can be modeled by

$$I = \frac{6922t + 106,947}{0.0063t + 1}$$
 and

$$P = 0.0343t + 4.432$$

where t represents the year, with t = 1 corresponding to 2001. Find a model M for the annual per capita income. (Per capita means per person.) Estimate the per capita income in 2010. (Assume t > 0.)