VERTEX AND INTERCEPT FORMS OF A QUADRATIC FUNCTION

FORM OF QUADRATIC FUNCTION	CHARACTERISTICS OF GRAPH
Vertex form $y=a(x-h)^2+k$	The vertex is (h,k). The axis of symmetry is $x=h$.

Writing Transformations of Quadratic Functions

The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex**. The **vertex form** of a quadratic function is $f(x)=a(x-h)^2+k$, where $a\neq 0$ and the vertex is (h, k).

$$f(x)=a(x-h)^2+k$$

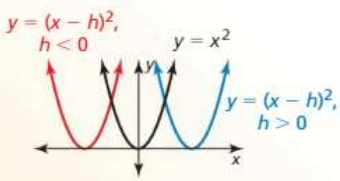
- a indicates a reflection in the x-axis and/or a vertical stretch or shrink.
- h indicates a horizontal translation.
- k indicates a vertical translation.

Core Concept

Horizontal Translations

$f(x)=x^2$

$f(x-h)=(x-h)^2$

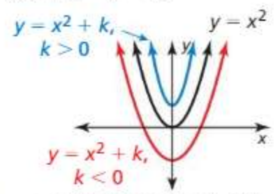


- shifts left when $h<0$
- shifts right when $h>0$

Vertical Translations

$f(x)=x^2$

$f(x)+k=x^2+k$



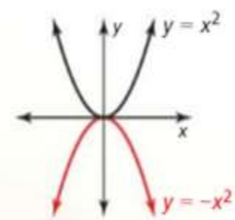
- shifts down when $k<0$
- shifts up when $k>0$

Core Concept

Reflections in the x-Axis

$f(x)=x^2$

$-f(x)=-(x^2)=-x^2$

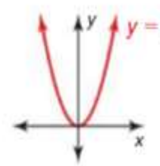


flips over the x-axis

Reflections in the y-Axis

$f(x)=x^2$

$f(-x)=(-x)^2=x^2$

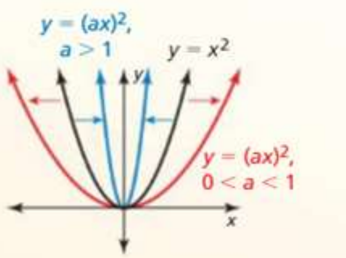


$y=x^2$ is its own reflection in the y-axis.

Horizontal Stretches and Shrinks

$f(x)=x^2$

$f(ax)=(ax)^2$

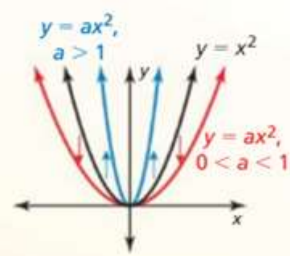


- horizontal stretch (away from y-axis) when $0<a<1$
- horizontal shrink (toward y-axis) when $a>1$

Vertical Stretches and Shrinks

$f(x)=x^2$

$a\cdot f(x)=ax^2$



- vertical stretch (away from x-axis) when $a>1$
- vertical shrink (toward x-axis) when $0<a<1$

Core Concept

THE GRAPH OF A QUADRATIC FUNCTION

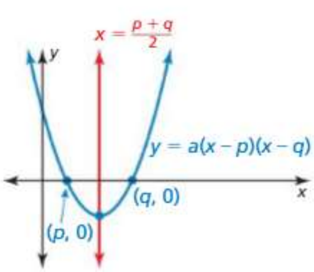
Standard Form

The graph $y=ax^2+bx+c$ is a parabola with these characteristics.

- The parabola opens up if $a>0$ and opens down if $a<0$. The parabola is wider than the graph of $y=x^2$ if $|a|<1$ and narrower than the graph of $y=x^2$ if $|a|>1$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is the vertical line $x=-\frac{b}{2a}$.

Intercept form $y=a(x-p)(x-q)$

The x intercepts are p and q. The axis of symmetry is halfway between $(p,0)$ and $(q,0)$.



For both forms, the graph opens up if $a>0$ and opens down if $a<0$.

Standard Form Vertex

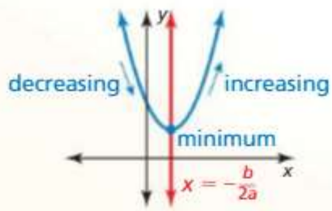
$$\text{Vertex}=\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

Core Concept

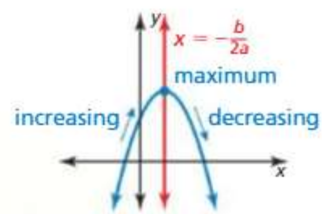
Minimum and Maximum Values

For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum value** of the function when $a > 0$ and the **maximum value** when $a < 0$.

$a > 0$



$a < 0$



Core Concept

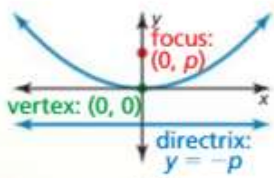
Standard Equations of a Parabola with Vertex at the Origin

Vertical axis of symmetry ($x = 0$)

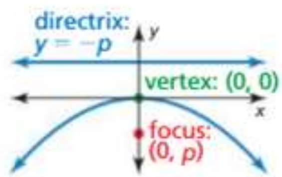
Equation: $y = \frac{1}{4p}x^2$

Focus: $(0, p)$

Directrix: $y = -p$



$p > 0$



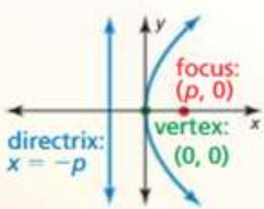
$p < 0$

Horizontal axis of symmetry ($y = 0$)

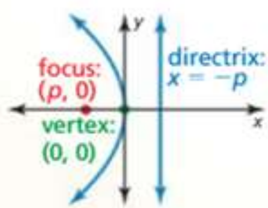
Equation: $x = \frac{1}{4p}y^2$

Focus: $(p, 0)$

Directrix: $x = -p$



$p > 0$



$p < 0$

Core Concept

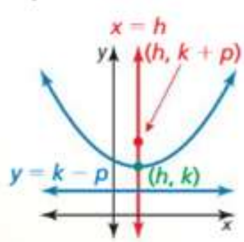
Standard Equations of a Parabola with Vertex at (*h*, *k*)

Vertical axis of symmetry ($x = h$)

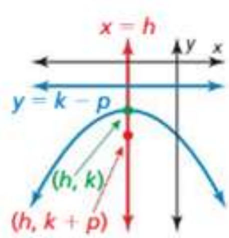
Equation: $y = \frac{1}{4p}(x - h)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$



$p > 0$



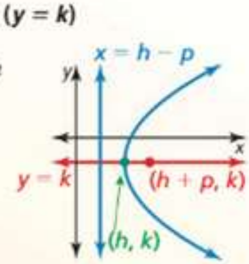
$p < 0$

Horizontal axis of symmetry ($y = k$)

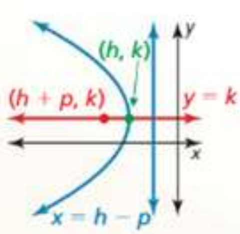
Equation: $x = \frac{1}{4p}(y - k)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



$p > 0$



$p < 0$