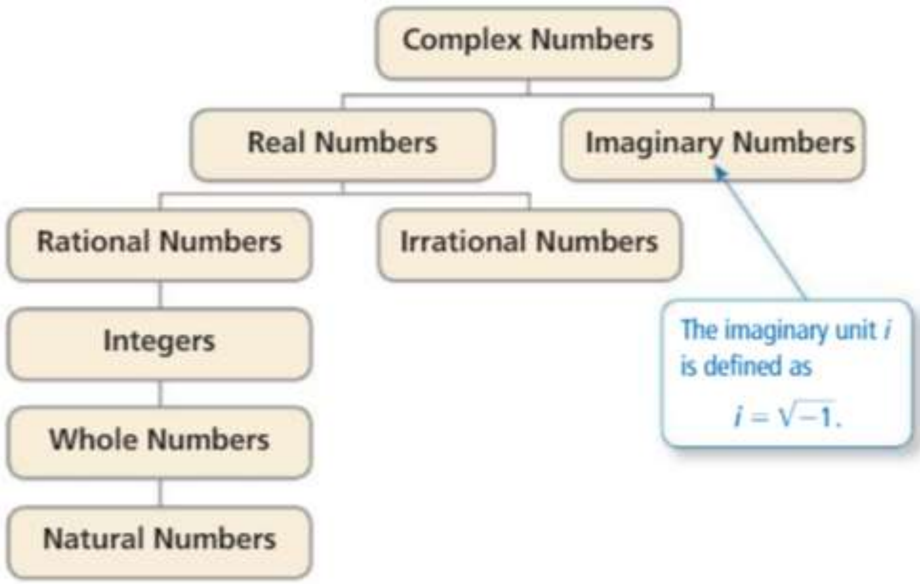
Chapter 3
Quadratic Equations and Complex Numbers

Section 3-2
Complex Numbers

Essential Question What are the subsets of the set of complex numbers?

In your study of mathematics, you have probably worked with only *real numbers*, which can be represented graphically on the real number line. In this lesson, the system of numbers is expanded to include *imaginary numbers*. The real numbers and imaginary numbers compose the set of *complex numbers*.



EXPLORATION 1 Classifying Numbers

Determine which subsets of the set of complex numbers contain each number.

a. $\sqrt{9}$ **b.** $\sqrt{0}$ **c.** $-\sqrt{4}$

d. $\sqrt{\frac{4}{9}}$ **e.** $\sqrt{2}$ **f.** $\sqrt{-1}$

EXPLORATION 2 Complex Solutions of Quadratic Equations

Use the definition of the imaginary unit i to match each quadratic equation with its complex solution. Justify your answers.

a. $x^2 - 4 = 0$ **b.** $x^2 + 1 = 0$ **c.** $x^2 - 1 = 0$

d. $x^2 + 4 = 0$ **e.** $x^2 - 9 = 0$ **f.** $x^2 + 9 = 0$

A. i **B.** $3i$ **C.** 3

D. $2i$ **E.** 1 **F.** 2

The Imaginary Unit i

Not all quadratic equations have real-number solutions. For example, $x^2 = -3$ has no real-number solutions because the square of any real number is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the **imaginary unit i**, defined as $i = \sqrt{-1}$. Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of *any* negative number.

Core Concept

The Square Root of a Negative Number

Property	**Example**
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

Find the square root of the number.

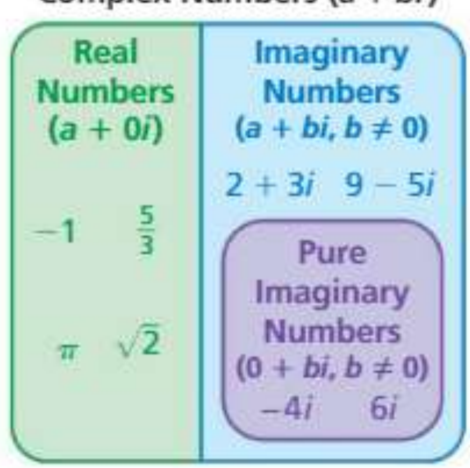
1. $\sqrt{-4}$ **2.** $\sqrt{-12}$ **3.** $-\sqrt{-36}$ **4.** $2\sqrt{-54}$

A **complex number** written in *standard form* is a number $a + bi$ where a and b are real numbers. The number a is the *real part*, and the number bi is the *imaginary part*.

$$a + bi$$

If $b \neq 0$, then $a + bi$ is an **imaginary number**. If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number**. The diagram shows how different types of complex numbers are related.

Complex Numbers ($a + bi$)



Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$.

Tutorial

EXAMPLE 2 Equality of Two Complex Numbers

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

SOLUTION

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x = 10$	Equate the real parts.	$-7i = yi$	Equate the imaginary parts.
$x = 5$	Solve for x.	$-7 = y$	Solve for y.

▶ So, $x = 5$ and $y = -7$.

Find the values of x and y that satisfy the equation.

5. $x + 3i = 9 - yi$

6. $9 + 4yi = -2x + 3i$

EXAMPLE 3 Adding and Subtracting Complex Numbers

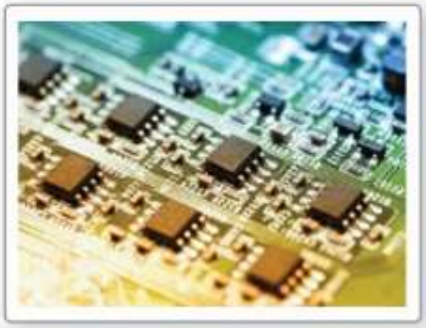
Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i)$

b. $(7 - 6i) - (3 - 6i)$

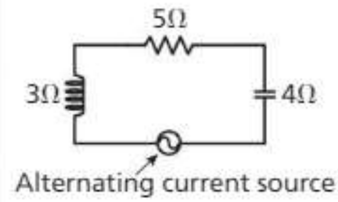
c. $13 - (2 + 7i) + 5i$

EXAMPLE 4 Solving a Real-Life Problem



Electrical circuit components, such as resistors, inductors, and capacitors, all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. Each of these quantities is measured in ohms. The symbol used for ohms is Ω, the uppercase Greek letter omega.

Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance (in ohms)	R	L	C
Impedance (in ohms)	R	Li	$-Ci$



The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled. The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit.

To multiply two complex num
just as you do when multiplying real numbers or algebraic expressions.

EXAMPLE 5 Multiplying Complex Numbers

Multiply. Write the answer in standard form.

a. $4i(-6 + i)$ **b.** $(9 - 2i)(-4 + 7i)$

STUDY TIP

When simplifying an expression that involves complex numbers, be sure to simplify i^2 as -1.

Complex Solutions and Zeros

EXAMPLE 6 **Solving Quadratic Equations**

Solve (a) $x^2 + 4 = 0$ and (b) $2x^2 - 11 = -47$.

LOOKING FOR STRUCTURE

Notice that you can use the solutions in Example 6(a) to factor $x^2 + 4$ as $(x + 2i)(x - 2i)$.

Solve the equation.

17. $x^2 - 8 = -36$ **18.** $3x^2 - 7 = -31$ **19.** $5x^2 + 33 = 3$

Find the zeros of the function.

20. $f(x) = x^2 + 7$ **21.** $f(x) = -x^2 - 4$ **22.** $f(x) = 9x^2 + 1$

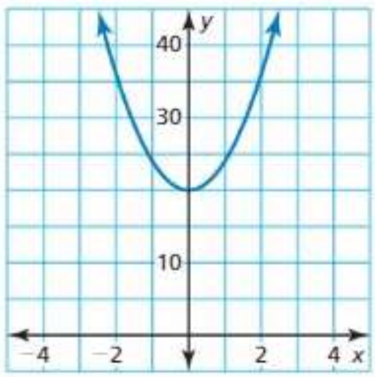
Tutorial

EXAMPLE 7 Finding Zeros of a Quadratic Function

Find the zeros of $f(x) = 4x^2 + 20$.

FINDING AN ENTRY POINT

The graph of f does not intersect the x-axis, which means f has no real zeros. So, f must have complex zeros, which you can find algebraically.



Section 3-2 Homework

#5,7,9,11,13,15,23,25,27,29,33,37,41,43,45,49,51,53,55,59,63,69,83