Chapter 3
Quadratic Equations and Complex Numbers

Section 3-5
Solving Nonlinear Systems

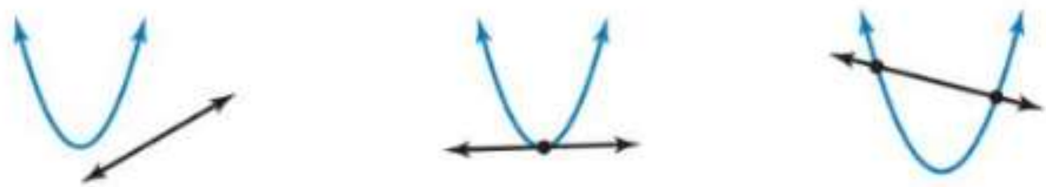
Systems of Nonlinear Equations

Previously, you solved systems of *linear* equations by graphing, substitution, and elimination. You can also use these methods to solve a system of *nonlinear* equations. In a **system of nonlinear equations**, at least one of the equations is nonlinear. For instance, the nonlinear system shown has a quadratic equation and a linear equation.

$y = x^2 + 2x - 4$ Equation 1 is nonlinear.

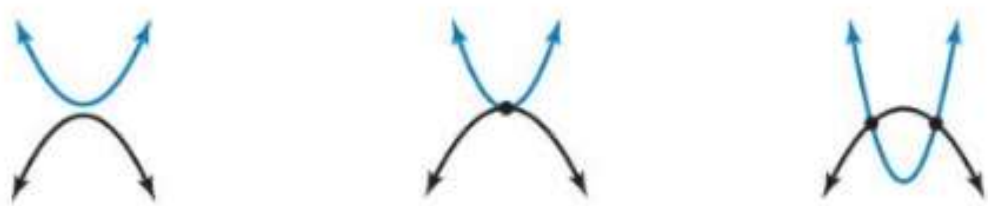
$y = 2x + 5$ Equation 2 is linear.

When the graphs of the equations in a system are a line and a parabola, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



No solution One solution Two solutions

When the graphs of the equations in a system are a parabola that opens up and a parabola that opens down, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



No solution One solution Two solutions

EXAMPLE 1 Solving a Nonlinear System by Graphing

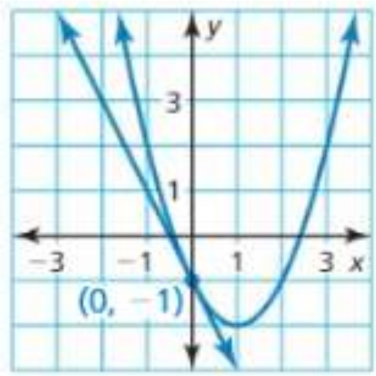
Solve the system by graphing.

$y = x^2 - 2x - 1$ Equation 1

$y = -2x - 1$ Equation 2

SOLUTION

Graph each equation. Then estimate the point of intersection. The parabola and the line appear to intersect at the point $(0, -1)$. Check the point by substituting the coordinates into each of the original equations.



Equation 1

$y = x^2 - 2x - 1$

$-1 \stackrel{?}{=} (0)^2 - 2(0) - 1$

$-1 = -1$ ✓

Equation 2

$y = -2x - 1$

$-1 \stackrel{?}{=} -2(0) - 1$

$-1 = -1$ ✓

▶ The solution is $(0, -1)$.

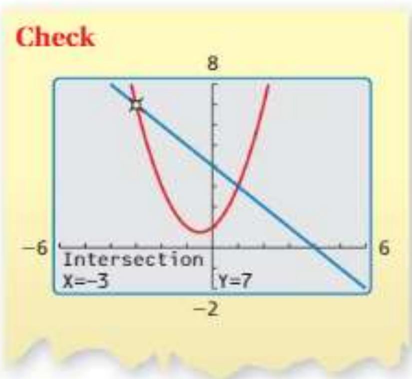
Remember that graphing is not the best way to solve equations. Sometimes it can be hard to look to see where two graphs intersect.

EXAMPLE 2 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$x^2 + x - y = -1$ Equation 1

$x + y = 4$ Equation 2

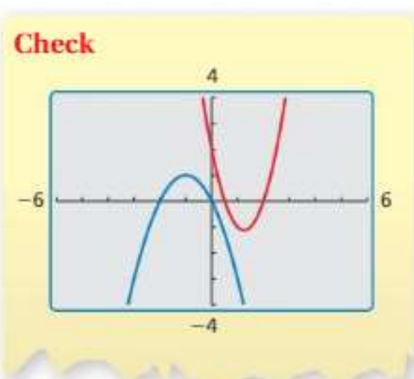


EXAMPLE 3 Solving a Nonlinear System by Elimination

Solve the system by elimination.

$x^2 - 5x - y = -2$ Equation 1

$x^2 + 2x + y = 0$ Equation 2

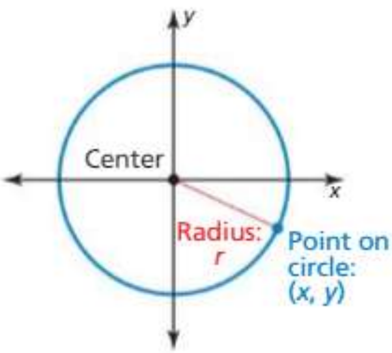


Solve the system using any method. Explain your choice of method.

1. $y = -x^2 + 4$
   $y = -4x + 8$

2. $x^2 + 3x + y = 0$
   $2x + y = 5$

3. $2x^2 + 4x - y = -2$
   $x^2 + y = 2$

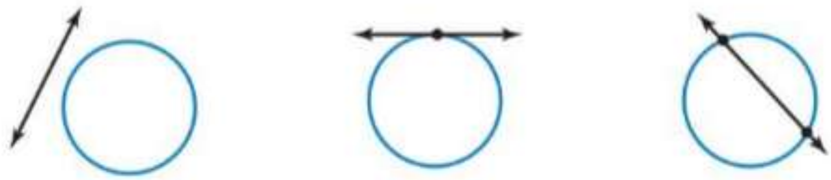


Some nonlinear systems have equations of the form

$$x^2 + y^2 = r^2.$$

This equation is the standard form of a circle with center (0, 0) and radius r.

When the graphs of the equations in a system are a line and a circle, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



Tutorial

EXAMPLE 4 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$x^2 + y^2 = 10$ Equation 1

$y = -3x + 10$ Equation 2

SOLUTION

Substitute $-3x + 10$ for y in Equation 1 and solve for x.

$x^2 + y^2 = 10$	Write Equation 1.
$x^2 + (-3x + 10)^2 = 10$	Substitute $-3x + 10$ for y.
$x^2 + 9x^2 - 60x + 100 = 10$	Expand the power.
$10x^2 - 60x + 90 = 0$	Write in standard form.
$x^2 - 6x + 9 = 0$	Divide each side by 10.
$(x - 3)^2 = 0$	Perfect Square Trinomial Pattern
$x = 3$	Zero-Product Property

COMMON ERROR

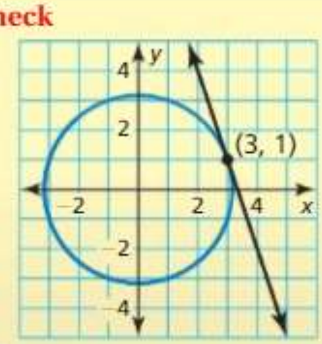
You can also substitute $x = 3$ in Equation 1 to find y. This yields two *apparent* solutions, (3, 1) and (3, −1). However, (3, −1) is *not* a solution because it does not satisfy Equation 2. You can also see (3, −1) is not a solution from the graph.

To find the y-coordinate of the solution, substitute $x = 3$ in Equation 2.

$$y = -3(3) + 10 = 1$$

► The solution is (3, 1). Check the solution by graphing the system. You can see that the line and the circle intersect only at the point (3, 1).

Check



Solve the system.

4. $x^2 + y^2 = 16$
$y = -x + 4$

5. $x^2 + y^2 = 4$
$y = x + 4$

6. $x^2 + y^2 = 1$
$y = \frac{1}{2}x + \frac{1}{2}$

Section 3-5 Homework #11-31 odd