Chapter 4 **Polynomial Functions**

Section 4-4 Factoring Polynomials

Factoring Polynomials

Previously, you factored quadratic polynomials. You can also factor polynomials with degree greater than 2. Some of these polynomials can be factored completely using techniques you have previously learned. A factorable polynomial with integer coefficients is factored completely when it is written as a product of unfactorable polynomials with integer coefficients.

EXAMPLE 1 Finding a Common Monomial Factor

Factor each polynomial completely.

a.
$$x^3 - 4x^2 - 5x$$

b.
$$3y^5 - 48y^3$$

c.
$$5z^4 + 30z^3 + 45z^2$$

Factor the polynomial completely.



1.
$$x^3 - 7x^2 + 10x$$



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 2. $3n^7 - 75n^5$



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Special Factoring Patterns

Sum of Two Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
 $64x^3 + 1 = (4x)^3 + 1^3$
= $(4x+1)(16x^2 - 4x + 1)$

Difference of Two Cubes

$$a^{3} - b^{3} = (a - b)(a^{2} + ab + b^{2})$$

$$27x^{3} - 8 = (3x)^{3} - 2^{3}$$

$$= (3x - 2)(9x^{2} + 6x + 4)$$

EXAMPLE 2 Factoring the Sum or Difference of Two Cubes

Factor (a) $x^3 - 125$ and (b) $16s^5 + 54s^2$ completely.

For some polynomials, you can **factor by grouping** pairs of terms that have a common monomial factor. The pattern for factoring by grouping is shown below.

$$ra + rb + sa + sb = r(a+b) + s(a+b)$$
$$= (r+s)(a+b)$$

EXAMPLE 3 Factoring by Grouping

Factor $z^3 + 5z^2 - 4z - 20$ completely.

An expression of the form $au^2 + bu + c$, where u is an algebraic expression, is said to be in quadratic form. The factoring techniques you have studied can sometimes be used to factor such expressions.

LOOKING FOR STRUCTURE



The expression $16x^4 - 81$ is in quadratic form because it can be written as $u^2 - 81$ where $u = 4x^2$.

EXAMPLE 4 Factoring Polynomials in Quadratic Form

Factor (a) $16x^4 - 81$ and (b) $3p^8 + 15p^5 + 18p^2$ completely.

Factor the polynomial completely.

$$\bigcirc$$
 4. $a^3 + 27$

6.
$$x^3 + 4x^2 - x - 4$$

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 5. $6z^5 - 750z^2$

READING

In other words, x - k is a factor of f(x) if and only if k is a zero of f.



The Factor Theorem

A polynomial f(x) has a factor x - k if and only if f(k) = 0.

EXAMPLE 5 Determining Whether a Linear Binomial Is a Factor

Determine whether (a) x - 2 is a factor of $f(x) = x^2 + 2x - 4$ and (b) x + 5 is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

EXAMPLE 6 Factoring a Polynomial

Show that x + 3 is a factor of $f(x) = x^4 + 3x^3 - x - 3$. Then factor f(x) completely.

ANOTHER WAY

Notice that you can factor f(x) by grouping.

$$f(x) = x^{3}(x + 3) - 1(x + 3)$$

$$= (x^{3} - 1)(x + 3)$$

$$= (x + 3)(x - 1) \cdot (x^{2} + x + 1)$$

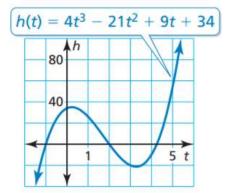
- 10. Determine whether x 4 is a factor of $f(x) = 2x^2 + 5x 12$.
- 11. Show that x 6 is a factor of $f(x) = x^3 5x^2 6x$. Then factor f(x) completely.



Because the x-intercepts of the graph of a function are the zeros of the function, you can use the graph to approximate the zeros. You can check the approximations using the Factor Theorem.

EXAMPLE 7 Real-Life Application

During the first 5 seconds of a roller coaster ride, the function $h(t) = 4t^3 - 21t^2 + 9t + 34$ represents the height h (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 5 seconds?



SOLUTION