Chapter 5
Rational Exponents and Radical Functions

Section 5-1
***n*th Roots and Rational Exponents**

*n*th Roots

You can extend the concept of a square root to other types of roots. For example, 2 is a cube root of 8 because $2^3 = 8$. In general, for an integer n greater than 1, if $b^n = a$, then b is an ***n*th root of *a***. An nth root of a is written as $\sqrt[n]{a}$, where n is the **index** of the radical.

You can also write an nth root of a as a power of a. If you assume the Power of a Power Property applies to rational exponents, then the following is true.

$$(a^{1/2})^2 = a^{(1/2) \cdot 2} = a^1 = a$$

$$(a^{1/3})^3 = a^{(1/3) \cdot 3} = a^1 = a$$

$$(a^{1/4})^4 = a^{(1/4) \cdot 4} = a^1 = a$$

Because $a^{1/2}$ is a number whose square is a, you can write $\sqrt{a} = a^{1/2}$. Similarly, $\sqrt[3]{a} = a^{1/3}$ and $\sqrt[4]{a} = a^{1/4}$. In general, $\sqrt[n]{a} = a^{1/n}$ for any integer n greater than 1.

Rational Exponents

A rational exponent does not have to be of the form $\frac{1}{n}$. Other rational numbers, such as $\frac{3}{2}$ and $-\frac{1}{2}$, can also be used as exponents. Two properties of rational exponents are shown below.

Core Concept

Rational Exponents

Let $a^{1/n}$ be an nth root of a, and let m be a positive integer.

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{-m/n} = \frac{1}{a^{m/n}} = \frac{1}{(a^{1/n})^m} = \frac{1}{(\sqrt[n]{a})^m},\ a \neq 0$$

EXAMPLE 2 Evaluating Expressions with Rational Exponents

Evaluate each expression.

a. $16^{3/2}$ **b.** $32^{-3/5}$

SOLUTION

Rational Exponent Form	Radical Form

EXAMPLE 3 Approximating Expressions with Rational Exponents

Evaluate each expression using a calculator. Round your answer to two decimal places.

a. $9^{1/5}$ **b.** $12^{3/8}$ **c.** $\left(\sqrt[4]{7}\right)^3$

SOLUTION

a. $9^{1/5} \approx 1.55$

b. $12^{3/8} \approx 2.54$

c. Before evaluating $\left(\sqrt[4]{7}\right)^3$, rewrite the expression in rational exponent form.

$$\left(\sqrt[4]{7}\right)^3 = 7^{3/4} \approx 4.30$$

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9^(1/5)
            1.551845574
12^(3/8)
            2.539176951
7^(3/4)
            4.303517071
```

Evaluate the expression without using a calculator.

5. $4^{5/2}$ **6.** $9^{-1/2}$ **7.** $81^{3/4}$ **8.** $1^{7/8}$

Evaluate the expression using a calculator. Round your answer to two decimal places when appropriate.

9. $6^{2/5}$ **10.** $64^{-2/3}$ **11.** $\left(\sqrt[4]{16}\right)^5$ **12.** $\left(\sqrt[3]{-30}\right)^2$

Solving Equations Using *n*th Roots

To solve an equation of the form $u^n = d$, where u is an algebraic expression, take the nth root of each side.

EXAMPLE 4 **Solving Equations Using *n*th Roots**

Find the real solution(s) of (a) $4x^5 = 128$ and (b) $(x - 3)^4 = 21$.

COMMON ERROR

When n is even and $a > 0$, be sure to consider both the positive and negative nth roots of a.

13. $8x^3 = 64$ **14.** $\frac{1}{2}x^5 = 512$ **15.** $(x + 5)^4 = 16$ **16.** $(x - 2)^3 = -14$

EXAMPLE 5 **Real-Life Application**

A hospital purchases an ultrasound machine for $50,000. The hospital expects the useful life of the machine to be 10 years, at which time its value will have depreciated to $8000. The hospital uses the declining balances method for depreciation, so the annual depreciation rate r (in decimal form) is given by the formula

$$r = 1 - \left(\frac{S}{C}\right)^{1/n}.$$

In the formula, n is the useful life of the item (in years), S is the salvage value (in dollars), and C is the original cost (in dollars). What annual depreciation rate did the hospital use?

SOLUTION

Section 5-1 Homework #11-45 odd and 49