Chapter 6
Exponential and Logarithmic Functions

Section 6-2
The Natural Base e

Core Concept

The Natural Base e

The natural base e is irrational. It is defined as follows:

As x approaches $+\infty$, $\left(1 + \frac{1}{x}\right)^x$ approaches $e \approx 2.71828182846$.

EXAMPLE 1 Simplifying Natural Base Expressions

Simplify each expression.

a. $e^3 \cdot e^6$ **b.** $\frac{16e^5}{4e^4}$ **c.** $(3e^{-4x})^2$

SOLUTION

a. $e^3 \cdot e^6 = e^{3+6}$

$= e^9$

b. $\frac{16e^5}{4e^4} = 4e^{5-4}$

$= 4e$

c. $(3e^{-4x})^2 = 3^2(e^{-4x})^2$

$= 9e^{-8x}$

$= \frac{9}{e^{8x}}$

Simplify the expression.

1. $e^7 \cdot e^4$ 2. $\frac{24e^8}{8e^5}$ 3. $(10e^{-3x})^3$

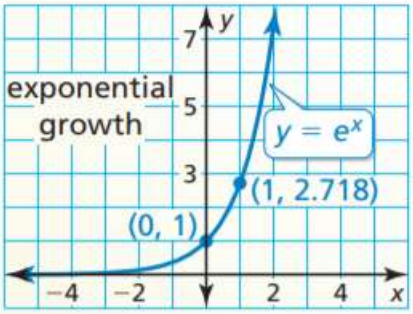
Core Concept

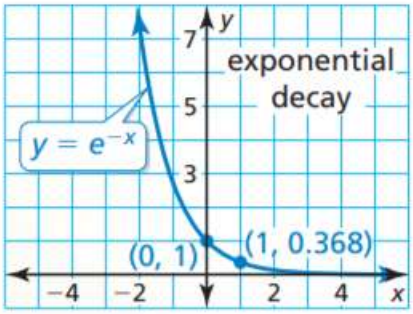
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- When $a > 0$ and $r > 0$, the function is an exponential growth function.
- When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.





EXAMPLE 2 Graphing Natural Base Functions

Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

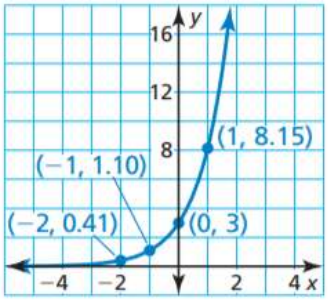
a. $y = 3e^x$

b. $f(x) = e^{-0.5x}$

SOLUTION

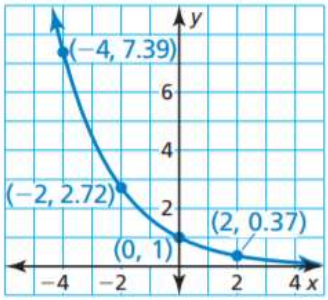
a. Because $a = 3$ is positive and $r = 1$ is positive, the function is an exponential growth function. Use a table to graph the function.

x	−2	−1	0	1
y	0.41	1.10	3	8.15



b. Because $a = 1$ is positive and $r = -0.5$ is negative, the function is an exponential decay function. Use a table to graph the function.

x	−4	−2	0	2
y	7.39	2.72	1	0.37



LOOKING FOR STRUCTURE

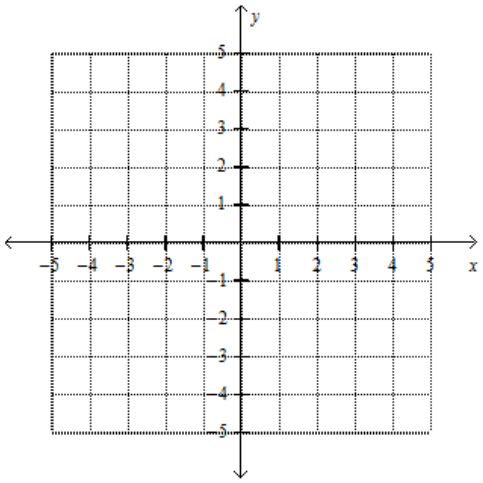
You can rewrite natural base exponential functions to find percent rates of change. In Example 2(b),

$$\begin{aligned} f(x) &= e^{-0.5x} \\ &= (e^{-0.5})^x \\ &\approx (0.6065)^x \\ &= (1 - 0.3935)^x. \end{aligned}$$

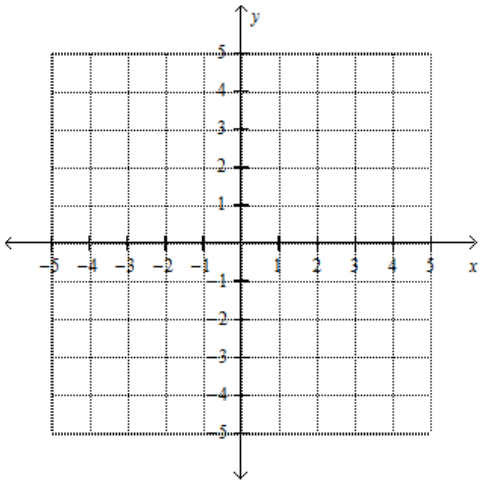
So, the percent decrease is about 39.35%.

Tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

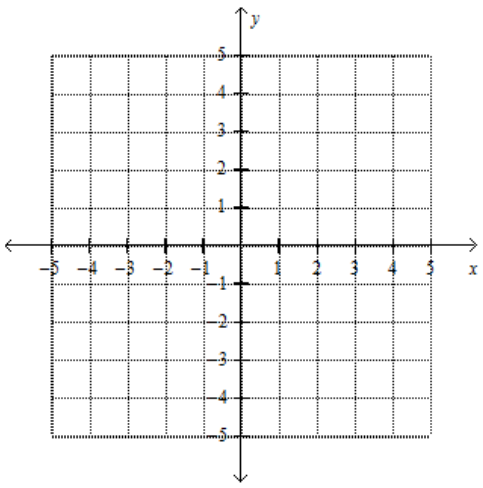
4. $y = \frac{1}{2}e^x$ **5.** $y = 4e^{-x}$ **6.** $f(x) = 2e^{2x}$



X	Y



X	Y



X	Y

Solving Real-Life Problems

You have learned that the balance of an account earning compound interest is given by $A = P\left(1 + \frac{r}{n}\right)^{nt}$. As the frequency n of compounding approaches positive infinity, the compound interest formula approximates the following formula.

Core Concept

Continuously Compounded Interest

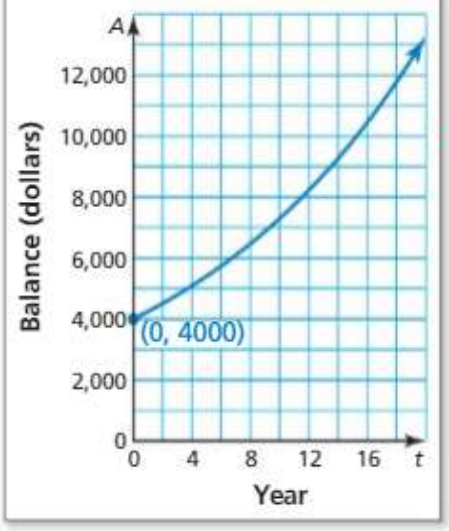
When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

EXAMPLE 3 Modeling with Mathematics

Your Friend's Account



You and your friend each have accounts that earn annual interest compounded continuously. The balance A (in dollars) of your account after t years can be modeled by $A = 4500e^{0.04t}$. The graph shows the balance of your friend's account over time. Which account has a greater principal? Which has a greater balance after 10 years?

MAKING CONJECTURES

You can also use this reasoning to conclude that your friend's account has a greater annual interest rate than your account.

Section 6-2 Homework #3-35 odd