Chapter 6
Exponential and Logarithmic Functions

Section 6-5
Properties of Logarithms

Properties of Logarithms

You know that the logarithmic function with base b is the inverse function of the exponential function with base b. Because of this relationship, it makes sense that logarithms have properties similar to properties of exponents.

Core Concept

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

EXAMPLE 1 Using Properties of Logarithms

Use $\log_2 3 \approx 1.585$ and $\log_2 7 \approx 2.807$ to evaluate each logarithm.

a. $\log_2 \frac{3}{7}$ **b.** $\log_2 21$ **c.** $\log_2 49$

SOLUTION

a. $\log_2 \frac{3}{7} = \log_2 3 - \log_2 7$ Quotient Property

$\approx 1.585 - 2.807$ Use the given values of $\log_2 3$ and $\log_2 7$.

$= -1.222$ Subtract.

b. $\log_2 21 = \log_2(3 \cdot 7)$ Write 21 as $3 \cdot 7$.

$= \log_2 3 + \log_2 7$ Product Property

$\approx 1.585 + 2.807$ Use the given values of $\log_2 3$ and $\log_2 7$.

$= 4.392$ Add.

c. $\log_2 49 = \log_2 7^2$ Write 49 as 7^2.

$= 2 \log_2 7$ Power Property

$\approx 2(2.807)$ Use the given value $\log_2 7$.

$= 5.614$ Multiply.

COMMON ERROR

Note that in general $\log_b \frac{m}{n} \neq \frac{\log_b m}{\log_b n}$ and $\log_b mn \neq (\log_b m)(\log_b n)$.

Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm.

1. $\log_6 \frac{5}{8}$ **2.** $\log_6 40$ **3.** $\log_6 64$ **4.** $\log_6 125$

Rewriting Logarithmic Expressions

You can use the properties of logarithms to expand and condense logarithmic expressions.

EXAMPLE 2 Expanding a Logarithmic Expression

Expand $\ln \frac{5x^7}{y}$.

SOLUTION

$$\ln \frac{5x^7}{y} = \ln 5x^7 - \ln y \qquad \text{Quotient Property}$$

$$= \ln 5 + \ln x^7 - \ln y \qquad \text{Product Property}$$

$$= \ln 5 + 7 \ln x - \ln y \qquad \text{Power Property}$$

EXAMPLE 3 Condensing a Logarithmic Expression

Condense $\log 9 + 3 \log 2 - \log 3$.

SOLUTION

$$\log 9 + 3 \log 2 - \log 3 = \log 9 + \log 2^3 - \log 3 \qquad \text{Power Property}$$

$$= \log(9 \cdot 2^3) - \log 3 \qquad \text{Product Property}$$

$$= \log \frac{9 \cdot 2^3}{3} \qquad \text{Quotient Property}$$

$$= \log 24 \qquad \text{Simplify.}$$

Expand the logarithmic expression.

5. $\log_6 3x^4$

6. $\ln \frac{5}{12x}$

Condense the logarithmic expression.

7. $\log x - \log 9$

8. $\ln 4 + 3 \ln 3 - \ln 12$

Change-of-Base Formula

Logarithms with any base other than 10 or e can be written in terms of common or natural logarithms using the *change-of-base formula*. This allows you to evaluate any logarithm using a calculator.

Core Concept

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}.$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

EXAMPLE 4 **Changing a Base Using Common Logarithms**

Evaluate $\log_3 8$ using common logarithms.

SOLUTION

$$\log_3 8 = \frac{\log 8}{\log 3} \qquad \log_c a = \frac{\log a}{\log c}$$

$$\approx \frac{0.9031}{0.4771} \approx 1.893 \qquad \text{Use a calculator. Then divide.}$$

ANOTHER WAY

In Example 4, $\log_3 8$ can be evaluated using natural logarithms.

$\log_3 8 = \frac{\ln 8}{\ln 3} \approx 1.893$

Notice that you get the same answer whether you use natural logarithms or common logarithms in the change-of-base formula.

EXAMPLE 5 **Changing a Base Using Natural Logarithms**

Evaluate $\log_6 24$ using natural logarithms.

SOLUTION

$$\log_6 24 = \frac{\ln 24}{\ln 6} \qquad \log_c a = \frac{\ln a}{\ln c}$$

$$\approx \frac{3.1781}{1.7918} \approx 1.774 \qquad \text{Use a calculator. Then divide.}$$

Use the change-of-base formula to evaluate the logarithm.

9. $\log_5 8$ **10.** $\log_8 14$ **11.** $\log_{26} 9$ **12.** $\log_{12} 30$

EXAMPLE 5 Changing a Base Using Natural Logarithms

Evaluate $\log_6 24$ using natural logarithms.

SOLUTION

$$\log_6 24 = \frac{\ln 24}{\ln 6} \qquad \log_c a = \frac{\ln a}{\ln c}$$

$$\approx \frac{3.1781}{1.7918} \approx 1.774 \qquad \text{Use a calculator. Then divide.}$$

EXAMPLE 6 Solving a Real-Life Problem

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the intensity of the sound doubles. By how many decibels does the loudness increase?

SOLUTION

Section 6-5 Homework #3-39 odd and 43, 45