

## Chapter 7 Rational Functions

### Section 7-3 Multiplying and Dividing Rational Functions

## Simplifying Rational Expressions

A **rational expression** is a fraction whose numerator and denominator are nonzero polynomials. The *domain* of a rational expression excludes values that make the denominator zero. A rational expression is in **simplified form** when its numerator and denominator have no common factors (other than  $\pm 1$ ).

### Core Concept

#### Simplifying Rational Expressions

Let  $a$ ,  $b$ , and  $c$  be expressions with  $b \neq 0$  and  $c \neq 0$ .

**Property**  $\frac{ac}{bc} = \frac{a}{b}$  Divide out common factor  $c$ .

**Examples**  $\frac{15}{65} = \frac{3 \cdot \cancel{5}}{13 \cdot \cancel{5}} = \frac{3}{13}$  Divide out common factor 5.

$\frac{4(x+3)}{(x+3)(x+3)} = \frac{4}{x+3}$  Divide out common factor  $x+3$ .

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator. Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

#### **EXAMPLE 1** Simplifying a Rational Expression

Simplify  $\frac{x^2 - 4x - 12}{x^2 - 4}$ .

#### COMMON ERROR

Do not divide out variable terms that are not factors.

$$\frac{x-6}{x-2} \neq \frac{-6}{-2}$$



# Multiplying Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form. Similar to rational numbers, rational expressions are closed under multiplication.

## Core Concept

### Multiplying Rational Expressions

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $b \neq 0$  and  $d \neq 0$ .

**Property**  $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$  Simplify  $\frac{ac}{bd}$  if possible.

**Example**  $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \cdot 3 \cdot \cancel{x} \cdot x^2 \cdot \cancel{y^3}}{\cancel{10} \cdot 2 \cdot \cancel{x} \cdot \cancel{y^3}} = \frac{3x^2}{2}, x \neq 0, y \neq 0$

### **EXAMPLE 2** Multiplying Rational Expressions

Find the product  $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$ .

### **EXAMPLE 3** Multiplying Rational Expressions

Find the product  $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$ .

## EXAMPLE 4 Multiplying a Rational Expression by a Polynomial

Find the product  $\frac{x+2}{x^3-27} \cdot (x^2+3x+9)$ .

### STUDY TIP

Notice that  $x^2 + 3x + 9$  does not equal zero for any real value of  $x$ . So, no values must be excluded from the domain to make the simplified form equivalent to the original.



## Dividing Rational Expressions

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression. Rational expressions are closed under nonzero division.

### Core Concept

#### Dividing Rational Expressions

Let  $a$ ,  $b$ ,  $c$ , and  $d$  be expressions with  $b \neq 0$ ,  $c \neq 0$ , and  $d \neq 0$ .

**Property**  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$  Simplify  $\frac{ad}{bc}$  if possible.

**Example**  $\frac{7}{x+1} \div \frac{x+2}{2x-3} = \frac{7}{x+1} \cdot \frac{2x-3}{x+2} = \frac{7(2x-3)}{(x+1)(x+2)}$   $x \neq \frac{3}{2}$

**EXAMPLE 5** Dividing Rational Expressions

Find the quotient  $\frac{7x}{2x - 10} \div \frac{x^2 - 6x}{x^2 - 11x + 30}$ .

**EXAMPLE 6** Dividing a Rational Expression by a Polynomial

Find the quotient  $\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$ .

**EXAMPLE 7****Solving a Real-Life Problem**

The total annual amount  $I$  (in millions of dollars) of personal income earned in Alabama and its annual population  $P$  (in millions) can be modeled by

$$I = \frac{6922t + 106,947}{0.0063t + 1}$$

and

$$P = 0.0343t + 4.432$$

where  $t$  represents the year, with  $t = 1$  corresponding to 2001. Find a model  $M$  for the annual per capita income. (Per capita means per person.) Estimate the per capita income in 2010. (Assume  $t > 0$ .)