Chapter 8
Sequences and Series

Section 8-1
Defining and Using Sequences and Series

Writing Terms of Sequences

Core Concept

Sequences

A **sequence** is an ordered list of numbers. A *finite sequence* is a function that has a limited number of terms and whose domain is the finite set $\{1, 2, 3, \ldots, n\}$. The values in the range are called the **terms of a sequence.**

Domain:	1	2	3	4	. . .	n	Relative position of each term
	↓	↓	↓	↓		↓	
Range:	a_1	a_2	a_3	a_4	. . .	a_n	Terms of the sequence

An *infinite sequence* is a function that continues without stopping and whose domain is the set of positive integers. Here are examples of a finite sequence and an infinite sequence.

Finite sequence: 2, 4, 6, 8 **Infinite sequence:** 2, 4, 6, 8, . . .

A sequence can be specified by an equation, or *rule*. For example, both sequences above can be described by the rule $a_n = 2n$ or $f(n) = 2n$.

The domain of a sequence may begin with 0 instead of 1. When this is the case, the domain of a finite sequence is the set $\{0, 1, 2, 3, \ldots, n\}$ and the domain of an infinite sequence becomes the set of nonnegative integers. Unless otherwise indicated, assume the domain of a sequence begins with 1.

EXAMPLE 1 Writing the Terms of Sequences

Write the first six terms of (a) $a_n = 2n + 5$ and (b) $f(n) = (-3)^{n-1}$.

Writing Rules for Sequences

When the terms of a sequence have a recognizable pattern, you may be able to write a rule for the nth term of the sequence.

STUDY TIP

When you are given only the first several terms of a sequence, there may be more than one rule for the nth term. For instance, the sequence 2, 4, 8, . . . can be given by $a_n = 2^n$ or $a_n = n^2 - n + 2$.

EXAMPLE 2 Writing Rules for Sequences

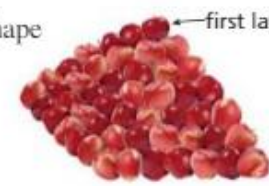
Tutorial

Describe the pattern, write the next term, and write a rule for the nth term of the sequences (a) $-1, -8, -27, -64, \ldots$ and (b) $0, 2, 6, 12, \ldots$.

EXAMPLE 3 Solving a Real-Life Problem

Tutorial

You work in a grocery store and are stacking apples in the shape of a square pyramid with seven layers. Write a rule for the number of apples in each layer. Then graph the sequence.



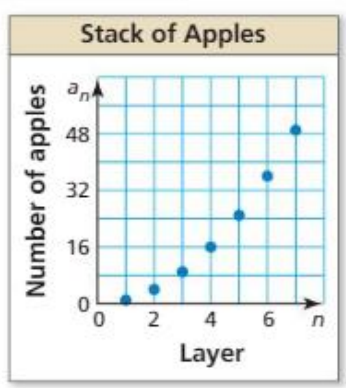
SOLUTION

Step 1 Make a table showing the number of fruit in the first three layers. Let a_n represent the number of apples in layer n.

Layer, n	1	2	3
Number of apples, a_n	 $1 = 1^2$	 $4 = 2^2$	 $9 = 3^2$

Step 2 Write a rule for the number of apples in each layer. From the table, you can see that $a_n = n^2$.

Step 3 Plot the points (1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36), and (7, 49). The graph is shown at the right.



COMMON ERROR

Although the plotted points in Example 3 follow a curve, do not draw the curve because the sequence is defined only for integer values of n, specifically $n = 1, 2, 3, 4, 5, 6$, and 7.

Writing Rules for Series

Core Concept

Series and Summation Notation

When the terms of a sequence are added together, the resulting expression is a **series**. A series can be finite or infinite.

Finite series: $2 + 4 + 6 + 8$

Infinite series: $2 + 4 + 6 + 8 + \cdots$

You can use **summation notation** to write a series. For example, the two series above can be written in summation notation as follows:

Finite series: $2 + 4 + 6 + 8 = \sum_{i=1}^{4} 2i$

Infinite series: $2 + 4 + 6 + 8 + \cdots = \sum_{i=1}^{\infty} 2i$

For both series, the *index of summation* is i and the *lower limit of summation* is 1. The *upper limit of summation* is 4 for the finite series and ∞ (infinity) for the infinite series. Summation notation is also called **sigma notation** because it uses the uppercase Greek letter *sigma*, written Σ.

READING

When written in summation notation, this series is read as "the sum of $2i$ for values of i from 1 to 4."

EXAMPLE 4 Writing Series Using Summation Notation

Write each series using summation notation.

a. $25 + 50 + 75 + \cdots + 250$

b. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$

The index of summation for a series does not have to be i—any letter can be used. Also, the index does not have to begin at 1. For instance, the index begins at 4 in the next example.

COMMON ERROR

Be sure to use the correct lower and upper limits of summation when finding the sum of a series.

Tutorial

EXAMPLE 5 Finding the Sum of a Series

Find the sum $\sum_{k=4}^{8}(3 + k^2)$.

For series with many terms, finding the sum by adding the terms can be tedious. Below are formulas you can use to find the sums of three special types of series.

Core Concept

Formulas for Special Series

Sum of n terms of 1: $\sum_{i=1}^{n} 1 = n$

Sum of first n positive integers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Sum of squares of first n positive integers: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

EXAMPLE 6 Using a Formula for a Sum

How many apples are in the stack in Example 3?

Section 8-1 Homework #5-59 Every other odd