Chapter 8
Sequences and Series

Section 8-2
Analyzing Arithmetic Sequences and Series

Identifying Arithmetic Sequences

In an **arithmetic sequence**, the difference of consecutive terms is constant. This constant difference is called the **common difference** and is denoted by d.

EXAMPLE 1 Identifying Arithmetic Sequences

Tell whether each sequence is arithmetic.

a. $-9, -2, 5, 12, 19, \ldots$

b. $23, 15, 9, 5, 3, \ldots$

Writing Rules for Arithmetic Sequences

Core Concept

Rule for an Arithmetic Sequence

Algebra The nth term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d$$

Example The nth term of an arithmetic sequence with a first term of 3 and a common difference of 2 is given by:

$$a_n = 3 + (n - 1)2, \text{ or } a_n = 2n + 1$$

EXAMPLE 2 Writing a Rule for the nth Term

Write a rule for the nth term of each sequence. Then find a_{15}.

a. 3, 8, 13, 18, . . .

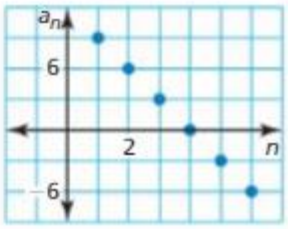
b. 55, 47, 39, 31, . . .

EXAMPLE 3 Writing a Rule Given a Term and Common Difference

One term of an arithmetic sequence is $a_{19} = -45$. The common difference is $d = -3$. Write a rule for the nth term. Then graph the first six terms of the sequence.

Use the rule to create a table of values for the sequence. Then plot the points.

n	1	2	3	4	5	6
a_n	9	6	3	0	−3	−6



ANALYZING RELATIONSHIPS

Notice that the points lie on a line. This is true for any arithmetic sequence. So, an arithmetic sequence is a linear function whose domain is a subset of the integers. You can also use function notation to write sequences:

$f(n) = -3n + 12$.

Tutorial

EXAMPLE 4 Writing a Rule Given Two Terms

Two terms of an arithmetic sequence are $a_7 = 17$ and $a_{26} = 93$. Write a rule for the nth term.

SOLUTION

Step 1 Write a system of equations using $a_n = a_1 + (n - 1)d$. Substitute 26 for n to write Equation 1. Substitute 7 for n to write Equation 2.

$a_{26} = a_1 + (26 - 1)d \Rightarrow 93 = a_1 + 25d$ Equation 1

$a_7 = a_1 + (7 - 1)d \Rightarrow 17 = a_1 + 6d$ Equation 2

Step 2 Solve the system.

$76 = 19d$ Subtract.

$4 = d$ Solve for d.

$93 = a_1 + 25(4)$ Substitute for d in Equation 1.

$-7 = a_1$ Solve for a_1.

Step 3 Write a rule for a_n.

$a_n = a_1 + (n - 1)d$ Write general rule.

$= -7 + (n - 1)4$ Substitute for a_1 and d.

$= 4n - 11$ Simplify.

Check

Use the rule to verify that the 7th term is 17 and the 26th term is 93.

$a_7 = 4(7) - 11 = 17$ ✓

$a_{26} = 4(26) - 11 = 93$ ✓

Write a rule for the nth term of the sequence. Then graph the first six terms of the sequence.

5. $a_{11} = 50, d = 7$

6. $a_7 = 71, a_{16} = 26$

Finding Sums of Finite Arithmetic Series

The expression formed by adding the terms of an arithmetic sequence is called an **arithmetic series**. The sum of the first n terms of an arithmetic series is denoted by S_n.

Core Concept

The Sum of a Finite Arithmetic Series

The sum of the first n terms of an arithmetic series is

$$S_n = n\left(\frac{a_1 + a_n}{2}\right).$$

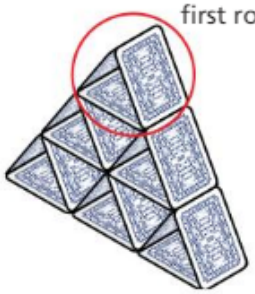
In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.

EXAMPLE 5 Finding the Sum of an Arithmetic Series

Find the sum $\sum_{i=1}^{20}(3i + 7)$.

EXAMPLE 6 Solving a Real-Life Problem

You are making a house of cards similar to the one shown.



a. Write a rule for the number of cards in the nth row when the top row is row 1.

b. How many cards do you need to make a house of cards with 12 rows?

SOLUTION

a. Starting with the top row, the number of cards in the rows are 3, 6, 9, 12, ... These numbers form an arithmetic sequence with a first term of 3 and a common difference of 3. So, a rule for the sequence is:

$$a_n = a_1 + (n - 1)d \qquad \text{Write general rule.}$$
$$= 3 + (n - 1)(3) \qquad \text{Substitute 3 for } a_1 \text{ and 3 for } d.$$
$$= 3n \qquad \text{Simplify.}$$

b. Find the sum of an arithmetic series with first term $a_1 = 3$ and last term $a_{12} = 3(12) = 36$.

$$S_{12} = 12\left(\frac{a_1 + a_{12}}{2}\right) = 12\left(\frac{3 + 36}{2}\right) = 234$$

So, you need 234 cards to make a house of cards with 12 rows.

Check

Use a graphing calculator to check the sum.

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sum(seq(3X,X,1,1
2))
                234
```

Find the sum.

7. $\sum_{i=1}^{10} 9i$

8. $\sum_{k=1}^{12} (7k + 2)$

9. $\sum_{n=1}^{20} (-4n + 6)$

10. WHAT IF? In Example 6, how many cards do you need to make a house of cards with eight rows?

Section 8-2 Homework #3-55 odd and 61