Chapter 8
Sequences and Series

Section 8-4
Finding Sums of Infinite Geometric Series

Partial Sums of Infinite Geometric Series

The sum S_n of the first n terms of an infinite series is called a **partial sum**. The partial sums of an infinite geometric series may approach a limiting value.

EXAMPLE 1 Finding Partial Sums

Consider the infinite geometric series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots.$$

Find and graph the partial sums S_n for $n = 1, 2, 3, 4,$ and 5. Then describe what happens to S_n as n increases.

SOLUTION

Step 1 Find the partial sums.

$$S_1 = \frac{1}{2} = 0.5$$

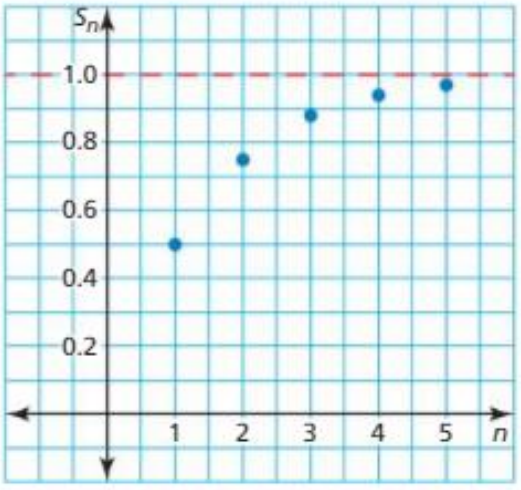
$$S_2 = \frac{1}{2} + \frac{1}{4} = 0.75$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \approx 0.88$$

$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \approx 0.94$$

$$S_5 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \approx 0.97$$

Step 2 Plot the points (1, 0.5), (2, 0.75), (3, 0.88), (4, 0.94), and (5, 0.97). The graph is shown at the right.



▶ From the graph, S_n appears to approach 1 as n increases.

Core Concept

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

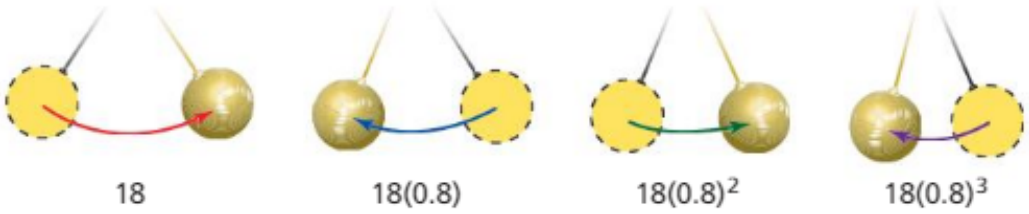
EXAMPLE 2 Finding Sums of Infinite Geometric Series

Find the sum of each infinite geometric series.

a. $\sum_{i=1}^{\infty} 3(0.7)^{i-1}$ **b.** $1 + 3 + 9 + 27 + \cdots$ **c.** $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \cdots$

EXAMPLE 3 Solving a Real-Life Problem

A pendulum that is released to swing freely travels 18 inches on the first swing. On each successive swing, the pendulum travels 80% of the distance of the previous swing. What is the total distance the pendulum swings?



18 $\quad$ $18(0.8)$ $\quad$ $18(0.8)^2$ $\quad$ $18(0.8)^3$

EXAMPLE 4 Writing a Repeating Decimal as a Fraction

Write $0.242424\ldots$ as a fraction in simplest form.

5. WHAT IF? In Example 3, suppose the pendulum travels 10 inches on its first swing. What is the total distance the pendulum swings?

Write the repeating decimal as a fraction in simplest form.

6. 0.555 . . .

7. 0.727272 . . .

8. 0.131313 . . .

Section 8-4 Homework #3-27 odd and 35