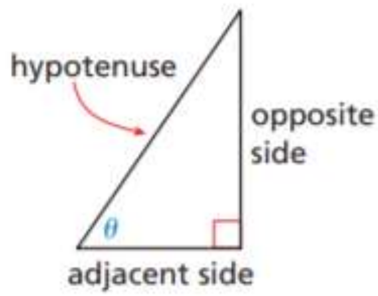
**Chapter 9**
**Trigonometric Ratios and Functions**

**Section 9-1**
**Right Triangles Trigonometry**

## The Six Trigonometric Functions

Consider a right triangle that has an acute angle $\theta$ (the Greek letter *theta*). The three sides of the triangle are the *hypotenuse*, the side *opposite* $\theta$, and the side *adjacent* to $\theta$.




Ratios of a right triangle's side lengths are used to define the six trigonometric functions: **sine**, **cosine**, **tangent**, **cosecant**, **secant**, and **cotangent**. These six functions are abbreviated sin, cos, tan, csc, sec, and cot, respectively.

## Core Concept

### Right Triangle Definitions of Trigonometric Functions

Let $\theta$ be an acute angle of a right triangle. The six trigonometric functions of $\theta$ are defined as shown.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

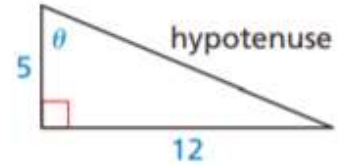
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

The abbreviations *opp.*, *adj.*, and *hyp.* are often used to represent the side lengths of the right triangle. Note that the ratios in the second row are reciprocals of the ratios in the first row.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

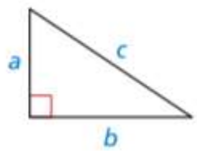
**EXAMPLE 1** **Evaluating Trigonometric Functions**

Evaluate the six trigonometric functions of the angle $\theta$.




**REMEMBER**

The Pythagorean Theorem states that $a^2 + b^2 = c^2$ for a right triangle with hypotenuse of length $c$ and legs of lengths $a$ and $b$.




**EXAMPLE 2** **Evaluating Trigonometric Functions**

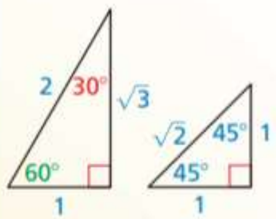
In a right triangle, $\theta$ is an acute angle and $\sin \theta = \frac{4}{7}$. Evaluate the other five trigonometric functions of $\theta$.

The angles 30°, 45°, and 60° occur frequently in trigonometry. You can use the trigonometric values for these angles to find unknown side lengths in special right triangles.

## Core Concept

### Trigonometric Values for Special Angles

The table gives the values of the six trigonometric functions for the angles 30°, 45°, and 60°. You can obtain these values from the triangles shown.

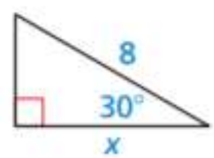



| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\csc\theta$ | $\sec\theta$ | $\cot\theta$ |
|---|---|---|---|---|---|---|
| 30° | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | 2 | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$ |
| 45° | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 |
| 60° | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{2\sqrt{3}}{3}$ | 2 | $\frac{\sqrt{3}}{3}$ |

## Finding Side Lengths and Angle Measures

### EXAMPLE 3 Finding an Unknown Side Length

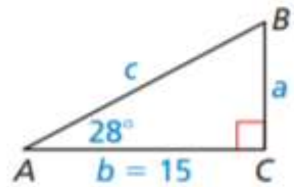
Find the value of $x$ for the right triangle.



Finding all unknown side lengths and angle measures of a triangle is called *solving the triangle*. Solving right triangles that have acute angles other than 30°, 45°, and 60° may require the use of a calculator. Be sure the calculator is set in *degree* mode.

## EXAMPLE 4 Using a Calculator to Solve a Right Triangle

Solve $\triangle ABC$.




### SOLUTION

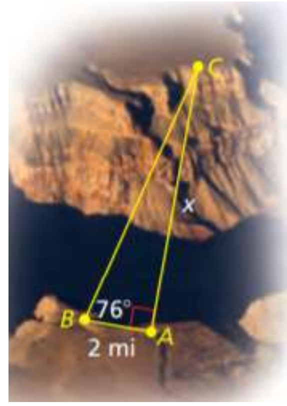
Because the triangle is a right triangle, $A$ and $B$ are complementary angles. So, $B = 90° - 28° = 62°$.

**READING**

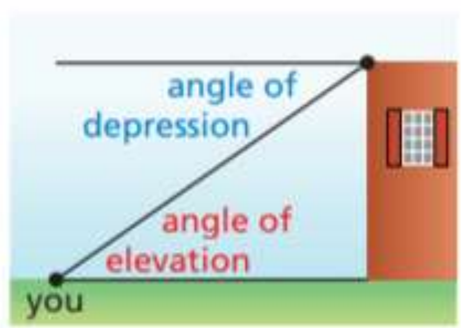
Throughout this chapter, a capital letter is used to denote both an angle of a triangle and its measure. The same letter in lowercase is used to denote the length of the side opposite that angle.

## EXAMPLE 5 Using Indirect Measurement

You are hiking near a canyon. While standing at *A*, you measure an angle of 90º between *B* and *C*, as shown. You then walk to *B* and measure an angle of 76° between *A* and *C*. The distance between *A* and *B* is about 2 miles. How wide is the canyon between *A* and *C*?




If you look at a point above you, such as the top of a building, the angle that your line of sight makes with a line parallel to the ground is called the *angle of elevation*. At the top of the building, the angle between a line parallel to the ground and your line of sight is called the *angle of depression*. These two angles have the same measure.



EXAMPLE 6 **Using an Angle of Elevation**

A parasailer is attached to a boat with a rope 72 feet long. The angle of elevation from the boat to the parasailer is 28°. Estimate the parasailer's height above the boat.

Section 9-1 Homework # 5-39 Every Other Odd and 41, 43, 45