Chapter 9
Trigonometric Ratios and Functions

Section 9-2
Angles and Radian Measure

Drawing Angles in Standard Position

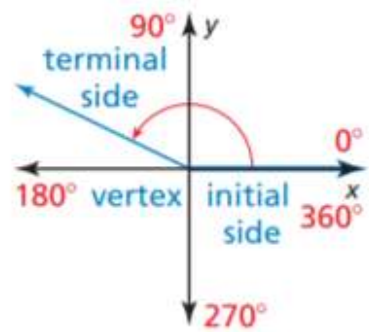
In this lesson, you will expand your study of angles to include angles with measures that can be any real numbers.

Core Concept

Angles in Standard Position

In a coordinate plane, an angle can be formed by fixing one ray, called the **initial side**, and rotating the other ray, called the **terminal side**, about the vertex.

An angle is in **standard position** when its vertex is at the origin and its initial side lies on the positive x-axis.



The measure of an angle is positive when the rotation of its terminal side is counterclockwise and negative when the rotation is clockwise. The terminal side of an angle can rotate more than 360°.

EXAMPLE 1 **Drawing Angles in Standard Position**

Draw an angle with the given measure in standard position.

a. 240° **b.** 500° **c.** −50°

Finding Coterminal Angles

In Example 1(b), the angles 500° and 140° are **coterminal** because their terminal sides coincide. An angle coterminal with a given angle can be found by adding or subtracting multiples of 360°.

STUDY TIP

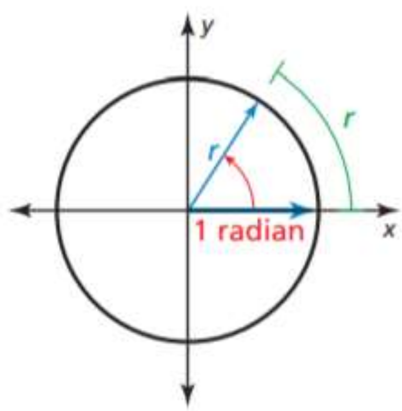
If two angles differ by a multiple of 360°, then the angles are coterminal.

EXAMPLE 2 **Finding Coterminal Angles**

Find one positive angle and one negative angle that are coterminal with (a) $-45°$ and (b) 395°.

Using Radian Measure

Angles can also be measured in *radians*. To define a radian, consider a circle with radius r centered at the origin, as shown. One **radian** is the measure of an angle in standard position whose terminal side intercepts an arc of length r.



Because the circumference of a circle is $2\pi r$, there are 2π radians in a full circle. So, degree measure and radian measure are related by the equation $360° = 2\pi$ radians, or $180° = \pi$ radians.

Core Concept

Converting Between Degrees and Radians

Degrees to radians

Multiply degree measure by

$$\frac{\pi \text{ radians}}{180°}.$$

Radians to degrees

Multiply radian measure by

$$\frac{180°}{\pi \text{ radians}}.$$

EXAMPLE 3 **Convert Between Degrees and Radians**

Convert the degree measure to radians or the radian measure to degrees.

a. 120°

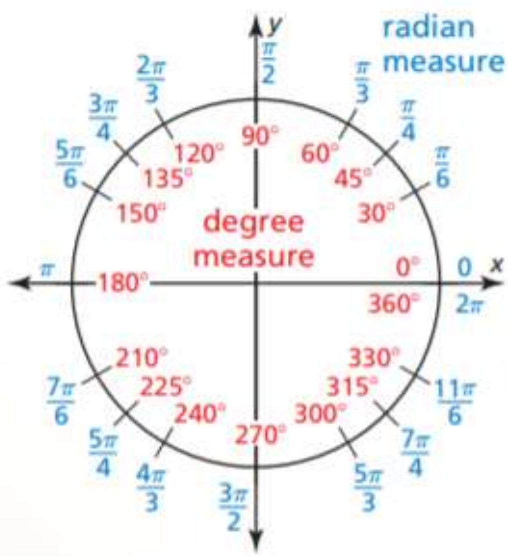
b. $-\frac{\pi}{12}$

Concept Summary

Degree and Radian Measures of Special Angles

The diagram shows equivalent degree and radian measures for special angles from 0° to 360° (0 radians to 2π radians).

You may find it helpful to memorize the equivalent degree and radian measures of special angles in the first quadrant and for $90^\circ = \frac{\pi}{2}$ radians. All other special angles shown are multiples of these angles.



A **sector** is a region of a circle that is bounded by two radii and an arc of the circle. The **central angle** θ of a sector is the angle formed by the two radii. There are simple formulas for the arc length and area of a sector when the central angle is measured in radians.

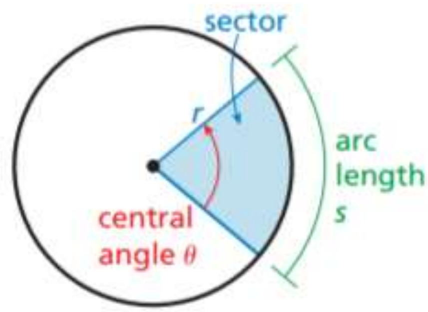
Core Concept

Arc Length and Area of a Sector

The arc length s and area A of a sector with radius r and central angle θ (measured in radians) are as follows.

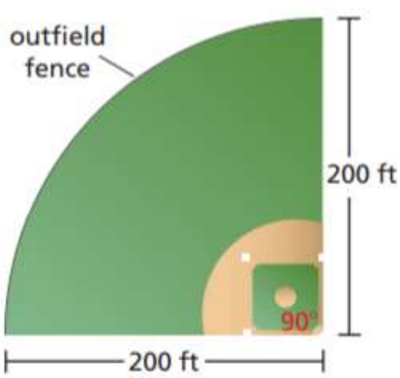
Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$



EXAMPLE 4 Modeling with Mathematics

A softball field forms a sector with the dimensions shown. Find the length of the outfield fence and the area of the field.



Section 9-2 Homework # 5-39 odd