Chapter 9
Trigonometric Ratios and Functions

Section 9-7
Using Trigonometric Identities

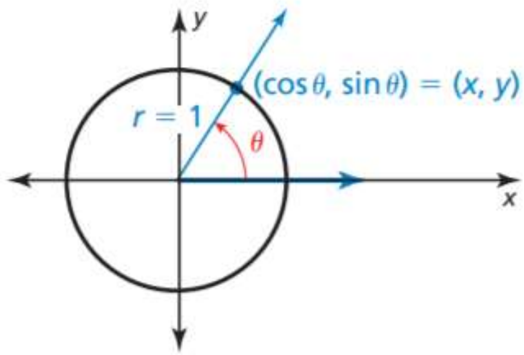
Using Trigonometric Identities

Recall that when an angle θ is in standard position with its terminal side intersecting the unit circle at (x, y), then $x = \cos\theta$ and $y = \sin\theta$. Because (x, y) is on a circle centered at the origin with radius 1, it follows that

$$x^2 + y^2 = 1$$

and

$$\cos^2\theta + \sin^2\theta = 1.$$



The equation $\cos^2\theta + \sin^2\theta = 1$ is true for any value of θ. A trigonometric equation that is true for all values of the variable for which both sides of the equation are defined is called a **trigonometric identity**. In Section 9.1, you used reciprocal identities to find the values of the cosecant, secant, and cotangent functions. These and other fundamental trigonometric identities are listed below.

STUDY TIP

Note that $\sin^2\theta$ represents $(\sin\theta)^2$ and $\cos^2\theta$ represents $(\cos\theta)^2$.

Core Concept

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

Negative Angle Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

In this section, you will use trigonometric identities to do the following.

- Evaluate trigonometric functions.
- Simplify trigonometric expressions.
- Verify other trigonometric identities.

EXAMPLE 1 Finding Trigonometric Values

Given that $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the values of the other five trigonometric functions of θ.

EXAMPLE 2 Simplifying Trigonometric Expressions

Simplify (a) $\tan\left(\frac{\pi}{2} - \theta\right)\sin\theta$ and (b) $\sec\theta\tan^2\theta + \sec\theta$.

Simplify the expression.

2. $\sin x \cot x \sec x$

3. $\cos\theta - \cos\theta\sin^2\theta$

Verifying Trigonometric Identities

You can use the fundamental identities from this chapter to verify new trigonometric identities. When verifying an identity, begin with the expression on one side. Use algebra and trigonometric properties to manipulate the expression until it is identical to the other side.

EXAMPLE 3 **Verifying a Trigonometric Identity**

Verify the identity $\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$.

Notice that verifying an identity is not the same as solving an equation. When verifying an identity, you cannot assume that the two sides of the equation are equal because you are trying to verify that they are equal. So, you cannot use any properties of equality, such as adding the same quantity to each side of the equation.

EXAMPLE 4 **Verifying a Trigonometric Identity**

Verify the identity $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$.

LOOKING FOR STRUCTURE

To verify the identity, you must introduce $1 - \sin x$ into the denominator. Multiply the numerator and the denominator by $1 - \sin x$ so you get an equivalent expression.

Verify the identity.

5. $\cot(-\theta) = -\cot \theta$

6. $\csc^2 x(1 - \sin^2 x) = \cot^2 x$

7. $\cos x \csc x \tan x = 1$

8. $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

Section 9-7 Homework #3-29odd