
PHY 204
Midterm
WITH
EVERBODIES
FAVORITE CLOWN
STUDENT
NAME
REDACTED
Cirqure De
Physics

Step right up to find out if these are solutions to the differential equation!

$$x = x_0 cos\left[\left(\sqrt{\frac{KQ}{R^3}}\right)t + \frac{\pi^2}{9}\right]$$

$$x = x_0 e^{\left[i\left(\sqrt{\frac{KQ}{R^3}}\right)t + \frac{\pi^2}{9}\right]}$$

$$x = \frac{1}{2}Kt^2 + QRt + \frac{\pi^2}{9}$$

II) Math

assume diff. equation $\frac{d^2x}{dt^2} = -\left(\frac{KQ}{R^3}\right)x$ i) is $x = x_0 \cos\left[\sqrt{\frac{KQ}{R^3}}t + \frac{\pi^2}{9}\right]$ a solution?

To find out we must take the 1st & 2nd derivature:

$$\frac{dx}{dt} = -x_0\sqrt{\frac{KQ}{R^3}} \sin\left[\sqrt{\frac{KQ}{R^3}}t + \frac{\pi^2}{9}\right] \Rightarrow -x_0\left(\frac{KQ}{R^3}\right)\cos\left(\frac{KQ}{R^3} + \frac{\pi^2}{9}\right)$$

$$\frac{d^2x}{dt^2} = -\frac{KQ}{R^3} \cdot \underbrace{x_0 \cos\left(\frac{KQ}{R^3}t + \frac{\pi^2}{9}\right)}_{x}$$

$$\therefore \frac{d^2x}{dt^2} = -\left(\frac{KQ}{R^3}\right)x$$

So, yes! the above is a solution that satisfys the diff. equation bc when you take the 2nd derivature you get:

$\frac{d^2x}{dt} = -\left(\frac{KQ}{R^3}\right)x_0 \cos\left(\frac{KQ}{R^3} + \frac{\pi^2}{9}\right)$ and this can be replaced with the original function to make it $-\left(\frac{KQ}{R^3}\right)x$ which is the diff. equation. (the two match

II Math Continued:

ii) Is $x = x_0 e^{\left[i\left(\sqrt{\frac{KQ}{R^3}}t + \frac{\pi^2}{9}\right)\right]}$ a solution to the diff. equation?

What we know:

$y = e^x$ | i = a constant

$\frac{dy}{dx} = e^x$ | $i \equiv \sqrt{-1}$

To find out we must take the 1st & 2nd derivative:

$$\frac{dx}{dt} = i\sqrt{\frac{KQ}{R^3}}\, x_0 e^{\left[i\sqrt{\frac{KQ}{R^3}}t + \frac{\pi^2}{9}\right]}$$

$$\frac{d^2x}{dt^2} = i^2\left(\frac{KQ}{R^3}\right)\underbrace{x_0 e^{\left[i\sqrt{\frac{KQ}{R^3}} + \frac{\pi^2}{9}\right]}}_{x}$$

$i \equiv \sqrt{-1}$

$i^2 = -1$

$$\therefore \quad = -\left(\frac{KQ}{R^3}\right)x$$

So, yes! The above satisfys the diff. equation bc when the 2nd derivative is taken, $\frac{d^2x}{dt^2} = i^2\left(\frac{KQ}{R^3}\right)x_0 e^{\left[i\sqrt{\frac{KQ}{R^3}} + \frac{\pi^2}{9}\right]}$. $i = \sqrt{-1}$ which makes the function negative. Substituting the equation with x, it gives $-\left(\frac{KQ}{R^3}\right)x$ which is equal to the diff. equation

II Math Continued:

B) Is $x = \frac{1}{2}K\tau^2 + QR\tau + \frac{\pi^2}{9}a$ harmonic oscillator? (aka does x satisfy $\frac{d^2x}{dt^2} = -\omega^2 x$)

To find out we must take the 1st & 2nd derivative:

$\frac{dx}{dt} = K\tau + QR$

hear me out:

$\frac{1}{2}K\tau^2$ take the coefficient out

$\frac{1}{2}K \frac{dx}{dt}(\tau^2)$ bring down the exponent then -1

$2 \cdot \tau^{2-1} \to 2 \cdot \tau$ make them get married

$\cancel{\frac{1}{2}} \cdot K \cdot \cancel{2} \cdot \tau \to K\tau$

$\frac{dx}{dt} \frac{\pi^2}{9} \to 0$

$K\tau + QR + 0$

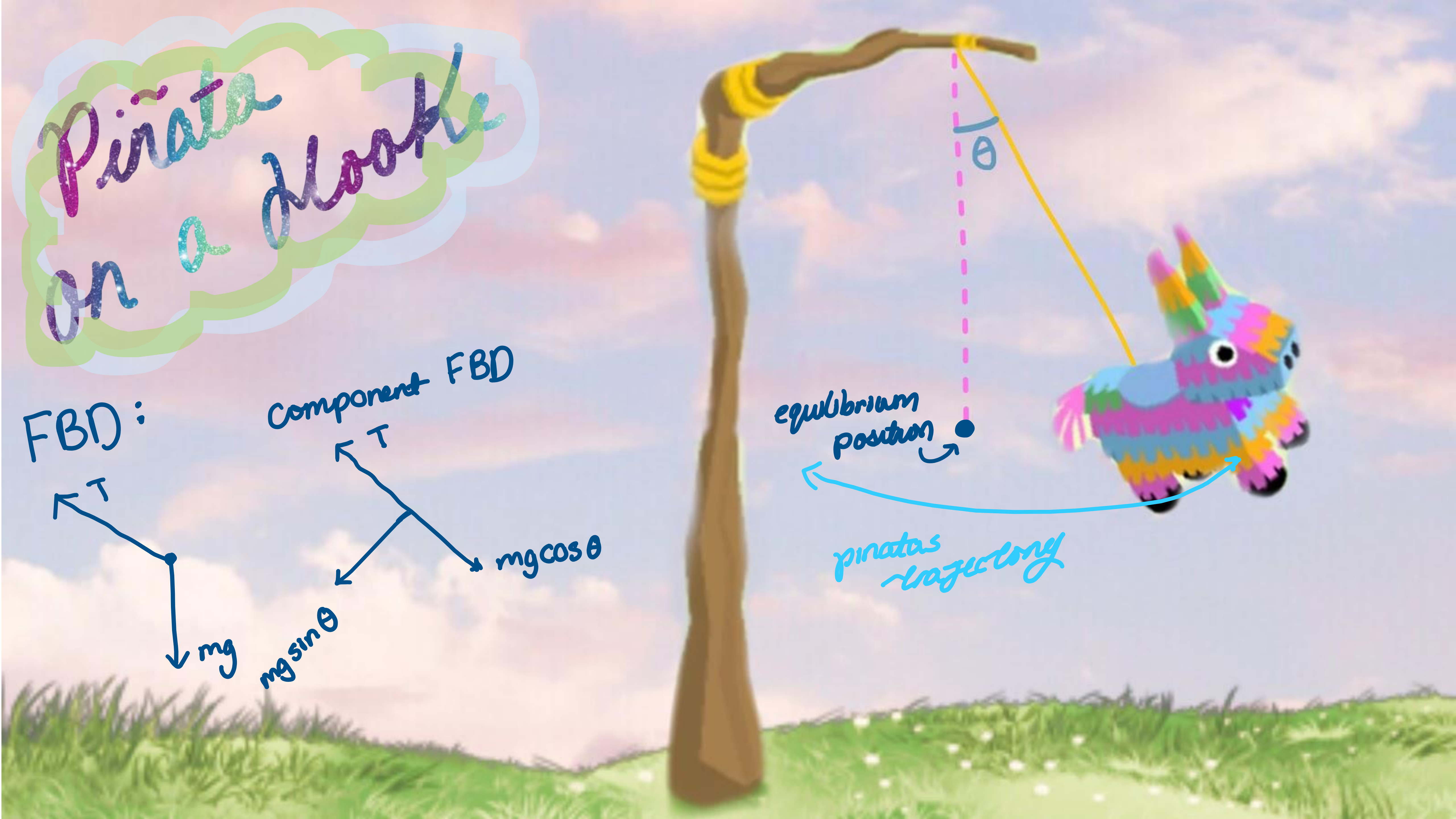
$\frac{d^2x}{dt^2} = \underline{K}$

$\underbrace{K\tau}_{\downarrow K} + \underbrace{QR}_{\hookrightarrow 0}$ if it represents a random #

holding τ as our constant

The above is not a SHO because when the 2nd derivative is taken, it is simply K. In order for it to be a SHO, the 2nd derivative must satisfy $\frac{d^2x}{dt^2} = -\left(\frac{KQ}{R^3}\right)x$ aka the general form $\frac{d^2x}{dt^2} = -\omega^2 x$.

Therefore, this is not a Simple Harmonic Oscillator.



Piñata on a Hooke
FBD:
T
mg
Component FBD
T
mgcosθ
mgsinθ
θ
equilibrium position
pinatas trajectory

Hooke on a Hooke cont:

$$\frac{d^2\theta}{dt^2} = -\left(\frac{\pi^2}{36}\right)\theta$$

ii) What is the angular frequency?

$$\frac{d^2\theta}{dt^2} = -\underbrace{\frac{\pi^2}{36}}_{\omega^2}\theta \quad \text{so} \quad \boxed{\omega \simeq \frac{\pi}{6} \approx 0.524 \text{ rad/sec}}$$

iii) How many cycles/sec do we expect?

$$T = \frac{2\pi}{\omega}, \quad \omega = \frac{\pi}{6} \rightarrow \frac{1}{\omega} = \frac{6}{\pi} \rightarrow \boxed{T = \frac{2\cancel{\pi}\cdot 6}{\cancel{\pi}} = 12 \text{ cycles/sec}}$$

iv) What is the particles speed @ $t = \frac{T}{4}$ sec

$$\frac{12}{4} = 3 \text{ sec.}$$

$$\frac{d^2\theta}{dt^2} = -(\omega^2)\theta, \text{ then } \theta = \theta_0 \cos(\omega t)$$

$$\theta = \theta_0 \cos(\omega t)$$

$$= \frac{\pi}{12} \cos\left(\frac{\pi}{6} \cdot 3\right)$$

$$\theta = \frac{\pi}{12} \cdot 0 \leftarrow \text{equilibrium position}$$

using phy 203 Knowledge:

$$mgh + \cancel{\tfrac{1}{2}mv^2} = mgh + \cancel{\tfrac{1}{2}mv^2}$$

$$gh = \sqrt{2gh}$$

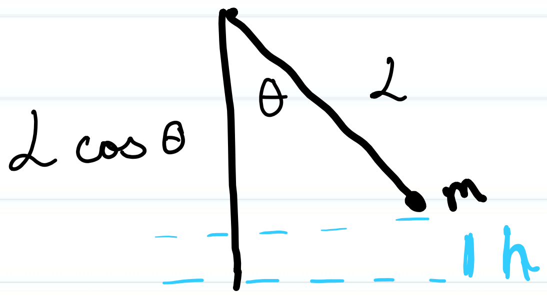
Hooke on a Hook cont.

$V = \sqrt{2gh}$

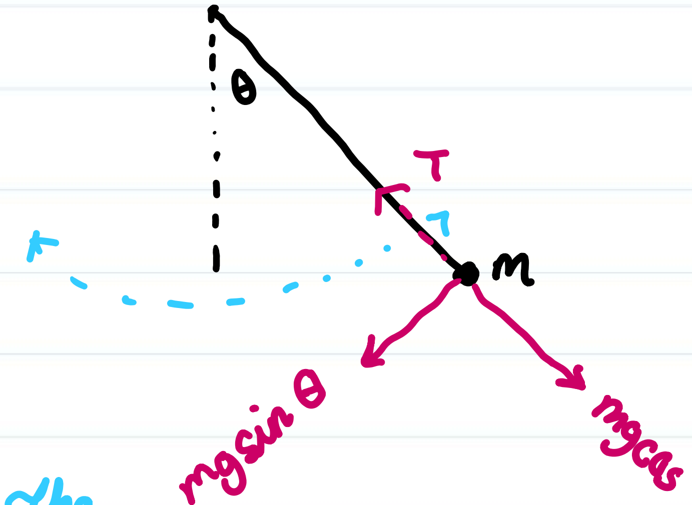
$V = \sqrt{2g(L \cdot 1 - \cos\theta)}$

$V = \sqrt{2\left(4\pi^2 \cdot \frac{5m}{12s}\right)\left(5m \cdot 1 - \cos\frac{\pi}{12}\right)}$

$\boxed{V = 2.37 \text{ m/s}}$



$h = L(1 - \cos\theta)$



g ≠ 10 m/s² because the axis is going horizontal rather than verticle to the earth ↓

$T = \frac{2\pi}{\omega}$ $\sqrt{\omega^2} = \sqrt{\frac{g}{L}}$

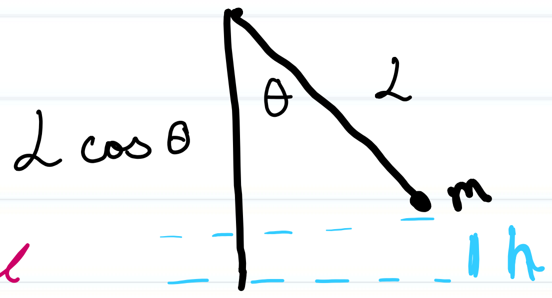
$\frac{1}{\omega} = \frac{L}{g}$

$T = 2\pi\sqrt{\frac{L}{g}}$

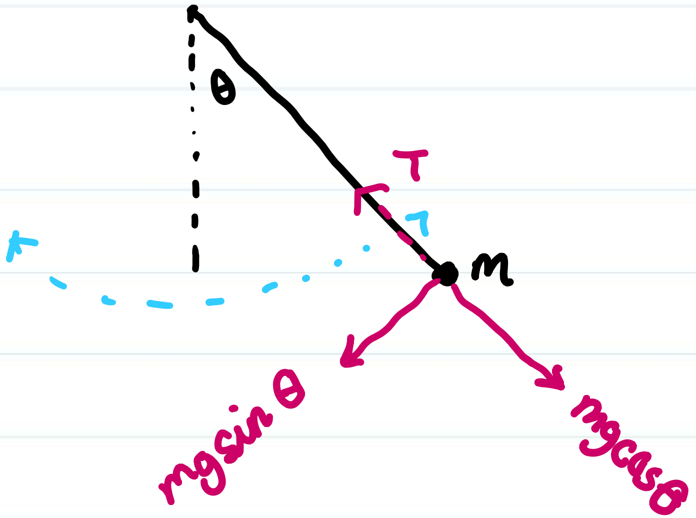
Solving for g we get

$g = 4\pi^2 \frac{L}{T}$

Hooke on a Hook cont.



$h = L(1-\cos\theta)$



Using the component FBD to solve for Normal force: $F_N + mg\sin\theta = mg\cos\theta - mg\cos\theta$

$F_N = -mg\sin\theta$

$F = ma$

$\therefore \; ma = -mg\sin\theta$

phy203 Knowledge I summon thee!

V) $\sum \vec{F} = m\vec{a}$

$\sum \vec{F}_{Tan} = m\vec{a}_{Tan}$ ↗ We want this (as opposed to centripetal) because it is the axis of motion & the axis at which the mass displaces from equilibrium.

Using the component FBD: $-\cancel{m}g\sin\theta = \cancel{m}a \rightarrow -g\sin\theta = a$

$\theta = \frac{x}{L} \rightarrow x = L\cdot\theta$ $\quad \frac{d^2x}{dt} = a$

Therefore, angular frequency depends on both the length of the string & free fall acc. of gravity!

$\theta(t) = \theta_{max}\cos(\omega t + \phi)$

$\therefore -g\sin\theta = \frac{d^2\theta}{dt^2}$

$-\frac{g}{L}\sin\theta = \frac{d^2\theta}{dt^2}$

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and for small $\measuredangle$, $\sin\theta \to \theta$. So,

$-\frac{g}{L}\theta = \frac{d^2\theta}{dt^2} \rightarrow \boxed{\frac{g}{L} = \omega^2}$

$T = \frac{2\pi}{\omega}$ $\quad \sqrt{\omega^2} = \sqrt{\frac{g}{L}}$

$\frac{1}{\omega} = \frac{L}{g}$

$T = 2\pi\sqrt{\frac{L}{g}}$

Continue:

$$\sqrt{\frac{g}{l}} = \sqrt{\omega^2}$$

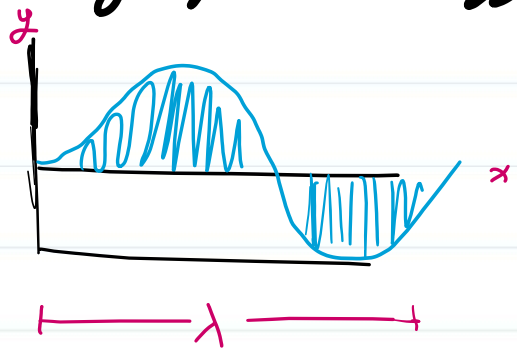
$$\sqrt{\frac{g}{l}} = \omega$$

$\therefore$ angular frequency (ω) depends on the length (l) of the string and free fall acceleration of gravity


TRY YOUR LUCK!
DUNK A SQUID
SS Yaverbaum

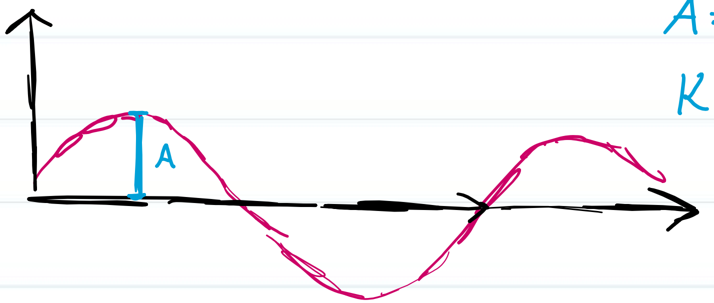
Wave Pulse - how it can be understood as a collection of phase staggered oscillators

a wave pulse is a non periodic wave formed by a single input of energy



from lab!

Wave equation : $y = A\cos(\omega t)$ or $y = A\cos(Kx)$



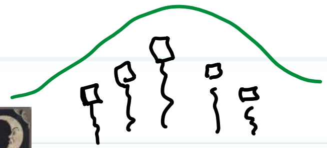
A = amplitude λ = wavelength V = speed of a wave

$K = \frac{2\pi}{\lambda}$

(Yaverbaum, 2022)

Propagation of a wave disturbance is produced and communicated via a medium

The particles started oscillation w each other in a different time period.



The medium

The different properties in a wave are due to disturbances of particles in a medium.

The Wave Equation

What we Know:

$$\frac{\partial^2 y}{\partial t^2} = V^2 \frac{\partial^2 y}{\partial x^2} \quad \text{aka} \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 y}{\partial t^2} \quad \Big\{ \quad V = \frac{\omega}{K} = \frac{\overset{\text{wavelength}}{\lambda}}{\underset{\text{period}}{T}} = \lambda \underset{\text{frequency}}{f}$$

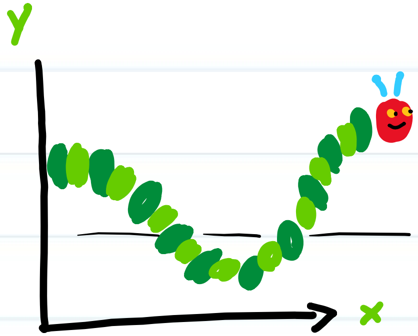
$$\frac{\partial^2 y}{\partial t^2} = (\omega^2)\frac{\partial^2 y}{\partial x}, \quad y = A\cos(\omega t + Kx)$$

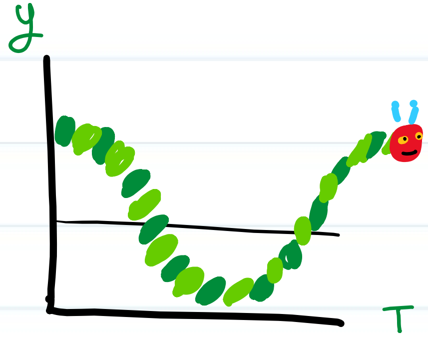
Given:

$$\frac{\partial^2 y}{\partial x^2} = \left(0.0025\,\frac{s^2}{m^2}\right)\frac{\partial^2 y}{\partial t^2}$$

$A = 8\ m$

$\omega = 0.1\ rad/sec$





A) find λ in meters:

$$V = \frac{\omega}{K} \quad \omega = \frac{2\pi}{T} \quad K = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{K}$$

$$= \frac{2\pi}{5\times10^{-3}\ rad/sec}$$

$$\boxed{\lambda \approx 1257\ m}$$

Since $\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 y}{\partial t^2}$

$$\sim = \left(\frac{1}{0.0025}\right)\sim$$

$$\sqrt{V^2} = \sqrt{\frac{1}{0.0025}}$$

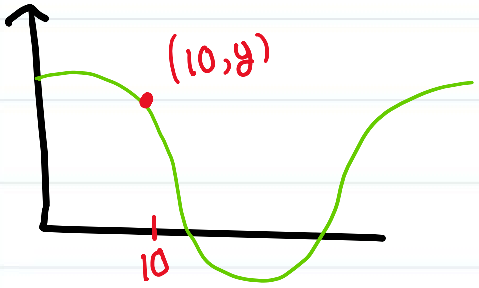
$$V = 20$$

$$V = \frac{\omega}{K} \rightarrow K = \frac{\omega}{V}$$

$$K = \frac{0.1}{20} = 5\times10^{-3}$$

Wave equation cont:

B) @ $\tau = 3\,sec$, $x = 10\,m$. How high or low on the oscillator is the particle?



$$\frac{d^2y}{dt^2} = \left(\frac{1}{V^2}\right)\frac{\partial^2 y}{\partial \tau^2}, \quad V^2 = \left(\frac{\omega}{K}\right)^2 \text{ so,}$$

$$\frac{d^2y}{dx^2} = \left(\frac{K}{\omega}\right)^2 \frac{dy^2}{dx^2}$$

then, $y = A\cos(\omega t + Kx)$ is a soln. to the diff equation!

Here: $y = 8\cos\left[\omega(3\,sec) + K\,\frac{rad}{s} \cdot 10\,m\right]$

$V = \frac{\omega}{K} \rightarrow K = \frac{\omega}{V}$

$K = \frac{0.1}{20} = 5\times10^{-3}$

$$y = 8\cos\left(\left(0.1\,\frac{rad}{\cancel{s}} \cdot 3\,\cancel{s}\right) + \left(5\times10^{-3}\,\frac{rad}{s} \cdot 10\,m\right)\right)$$

$$\boxed{y = 7.51\,m}$$

Wave Equation cont:

c) The new concavity is twice as concave as before. How does a compare

$$\frac{d^2y}{dt^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$$

acceleration is directly proportional to concavity

$\therefore$ The acceleration is twice as large as before

$\underbrace{\text{acceleration} \quad \text{concavity}}_{\text{direct relationship}}$


Don't forget your prize :)
Your favorite crustacean