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MATRICES

Class 12 - Mathematics

Section A

1. If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, then find (x - y). **[1]**

   a) 7 b) 10
   c) 12 d) 15

2. Suppose 2×2 matrix A = [a_{ij}], whose elements are given by $a_{ij} = \frac{i}{j}$, then matrix A is **[1]**

   a) $\begin{bmatrix}1 & \frac{1}{2}\\2 & 1\end{bmatrix}$ b) $\begin{bmatrix}\frac{1}{2} & 1\\2 & 1\end{bmatrix}$
   c) $\begin{bmatrix}1 & 2\\\frac{1}{2} & 1\end{bmatrix}$ d) $\begin{bmatrix}\frac{1}{2} & 1\\1 & 2\end{bmatrix}$

3. The matrix $\begin{bmatrix}0 & -5 & 8\\5 & 0 & 12\\-8 & -12 & 0\end{bmatrix}$ is a **[1]**

   a) scalar matrix b) skew-symmetric matrix
   c) diagonal matrix d) symmetric matrix

4. If A is any square matrix then which of the following is not symmetric? **[1]**

   a) A^tA b) $A - A^t$
   c) AA^t d) $A+A^t$

5. A and B are square matrices of same order. If $(A + B)^2 = A^2 + B^2$, then: **[1]**

   a) AB = -BA b) AB = BA
   c) BA = O d) AB = O

[2023]

6. If $A + B = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and $A - 2B = \begin{bmatrix}-1 & 1\\1 & -1\end{bmatrix}$, then A = ? **[1]**

   a) $\frac{1}{3}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$ b) $\frac{1}{3}\begin{bmatrix}1 & 1\\3 & 1\end{bmatrix}$
   c) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ d) $\frac{1}{2}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$

7. If $\begin{bmatrix}1 & 2 & 1\\2 & 3 & 1\\3 & a & 1\end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is **[1]**

a) {0}

b) ℝ - {4}

c) ℝ

d) {4}

[2023]

8. If A = $[a_{ij}]_{3\times4}$ is matrix given by $\boldsymbol{A} = \begin{bmatrix} 4 & -2 & 1 & 3 \\ 5 & 7 & 9 & 6 \\ 21 & 15 & 18 & -25 \end{bmatrix}$. Then, $a_{23} + a_{24}$ will be equal to the element **[1]**

a) a_{14}

b) a_{32}

c) a_{44}

d) a_{13}

9. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to **[1]**

a) AB

b) 2 AB

c) A + B

d) 2 BA

10. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^4 = **[1]**

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

11. Compute the indicated product $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ **[1]**

12. If A is a square matrix such that A^2 = A. then write the value of 7A - $(I + A)^3$. where I is an identity matrix. **[1]**

[2014]

13. If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$ then write the order of matrix A. **[1]**

[2016]

14. Let A = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and b = –2. **[1]**

Show that $(bA)^T = b\,A^T$

15. Construct a 2 × 2 matrix, A = [aij], whose element $a_{ij} = \frac{i}{j}$ **[1]**

16. If [x 2] $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ = 2, find the value of x **[1]**

17. For A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}. **[1]**

[2020]

18. If A = $\begin{bmatrix} 2 & -3 & 5 \\ -1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -2 \\ 4 & -3 & 1 \end{bmatrix}$ verify that (A + B) = (B + A). **[1]**

19. Let A be a matrix of order 3×4. If R_1 denotes the first row of A and C_2 denotes its second column, then determine the orders of matrices R_1 and C_2. **[1]**

20. If $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, find a matrix X such that 3A - 2B + X = 0 **[1]**

Section B

21. If A_α = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then prove that $A_\alpha A_\beta = A_{\alpha+\beta}$ **[2]**

[2004]

22. Construct $A_{2\times2}$ matrix, where $a_{ij} = |-2i+3j|$ **[2]**

23. If A =$[a_{ij}]$ = $\begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and B = $[b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find $a_{11}\, b_{11} + a_{22}\, b_{22}$ **[2]**

24. If A = $\begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that (A + A ') is symmetric. **[2]**

25. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements? **[2]**

26. If $A = \begin{bmatrix} 5 & 4 \\ 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 5 & 1 \\ 6 & 8 & 4 \end{bmatrix}$, find AB and BA whichever exists. **[2]**

27. If $\begin{bmatrix} a+b & 2 \\ 5 & b \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$, then find a. **[2]**

28. Show that AB $\neq$ BA in the case: $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ **[2]**

29. If A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then find AB, BA. Show that AB $\neq$ BA **[2]**

30. Let A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that $(AB)^T = B^T A^T$ **[2]**

Section C

31. Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$. **[3]**

32. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ **[3]**

33. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. **[3]**

[2016]

34. If I is the identity matrix and A is a square matrix such that $A^2 = A$, then what is the value of $(I + A)^2 - 3A$? **[3]**

[2012]

35. If A = $[a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric. **[3]**

36. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11I = O$, Hence find A^{-1}. **[3]**

[2020, 2018]

37. A trust fund has Rs. 35000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an NGO (Cancer Ad Society). **[3]**
Using matrix multiplication, determine how to divide Rs. 35000 among two types of bonds, if the trust fund obtains an annual total interest of Rs. 3200. What are the values reflected in this question?

[2015]

38. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of a and b. **[3]**

[2015]

39. For the matrices, A and B, verify that (AB)′ = B′A′, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ **[3]**

40. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$, obtain the values of a, b, c, x, y and z. **[3]**

Section D

41. **Read the following text carefully and answer the questions that follow:** **[4]**

In a city, there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys, the number of units of types I, II, and III respectively are 80, 70, and 65 in factory A and 85, 65, and 72 are in factory B. For girls the number of units of types I, II, and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

i. Represent the number of units of each type produced by factory A for both boys and girls and number of units of each type produced by factory B for both boys and girls in matrix form. (1)
ii. Find the total production of sports clothes of each type for boys. (1)
iii. Find the total production of sports clothes of each type for girls. (2)

OR

Let R be a 3×2 matrix that represent the total production of sports clothes of each type for boys and girls, then find the transpose of R. (2)

42. **Read the following text carefully and answer the questions that follow:** **[4]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

Based on the above information, answer the following questions.

i. If ₹ 15000 is invested in bond X, then what is the matrix representation of A and B? (1)
ii. If ₹ 15,000 is invested in bond X, how can we determine the total amount of interest received on both bonds? (1)
iii. How much is the investment in two bonds if the trust fund obtains an annual total interest of ₹3200? (2)

OR

What is the amount of investment in bond Y if the interest given to the old age home is ₹500? (2)

43. **Read the following text carefully and answer the questions that follow:** **[4]**

In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories type I, type II and type III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

i. Write the matrix P, if P represents the matrix of number of units of each type produced by factory A for both boys and girls. (1)

ii. Write the matrix Q, if Q represents the matrix of number of units of each type produced by factory B for both boys are girls. (1)

iii. Find the total production of sports clothes of each type for boys. (2)

OR

Find the total production of sports clothes of each type for girls. (2)

44. **Read the following text carefully and answer the questions that follow:** **[4]**

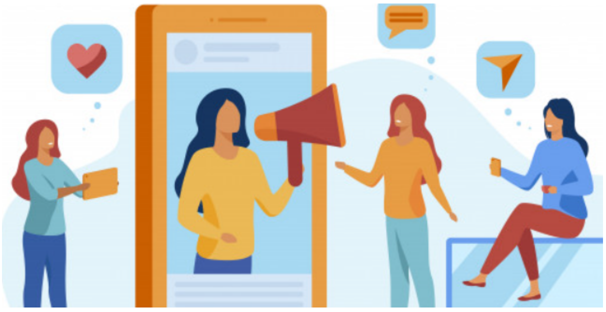
To promote the making of toilets for women, an organization tried to generate awareness through

i. house calls

ii. emails and

iii. announcements.

The cost for each mode per attempt is given below:



1. ₹ 50
2. ₹ 20
3. ₹ 40

The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given modes is

1. 2%
2. 4%
3. 20%

i. Find total number of toilets that can be expected after the promotion in village X. (1)

ii. Find the percentage of toilets that can be expected after the promotion in all the three-villages? (1)

iii. Find the cost incurred by the organization on village X. (2)

OR

Find the total cost incurred by the organization on for all the three villages? (2)

45. **Read the following text carefully and answer the questions that follow:** **[4]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association). Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.


i. Represent the given information in matrix algebra. (1)

ii. If ₹15000 is invested in bond X, then find total amount of interest received on both bonds? (1)

iii. If the trust fund obtains an annual total interest of ₹ 3200, then find the investment in two bonds. (2)

OR

If the amount of interest given to old age home is ₹500, then find the amount of investment in bond Y. (2)

Section E

46. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A + I)$, where I is 3×3 unit matrix. **[5]**

47. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then use the principle of mathematical induction to show that A^n = $\begin{bmatrix} 1 & n & n(n+1)/2 \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$ for every positive integer n **[5]**

48. Let A = $\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrices X and Y such that X + Y = A, where X is a symmetric and Y is a skew-symmetric matrix. **[5]**

49. For a matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that: **[5]**

i. (A + A') is a symmetric matrix.

ii. (A – A') is a skew symmetric matrix.

50. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, Prove I + A = (I - A)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **[5]**

51. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ then show that P(x).P(y) = P(x + y) = P(y).P(x). **[5]**

52. If $A = \begin{bmatrix} 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \end{bmatrix}$ then find a non-zero matrix C such that AC = BC. **[5]**

53. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ **[5]**

54. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **[5]**

55. Given the matrices **[5]**

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Verify that (A + B) + C = A + (B + C).