Solution

ARITHMETIC PROGRESSION

Class 10 - Mathematics

Section A

1.

(b) 7th

Explanation:

Let $T_n < 0$. Then, $25 + (n - 1) \times (-5) < 0$.

$30 < 5n \Rightarrow 5n > 30 \Rightarrow n > 6$.

$\Rightarrow$ So, the required term is 7th.

2.

(d) 2500

Explanation:

$Sn = (\frac{50}{2})(2(1) + (50-1)2)$

$= 25(2 + 49 \times 2)$

$= 25(2 + 98)$

$= 25(100)$

$= 2500$

Therefore, the sum of the first 50 odd natural numbers is 2500.

3. **(a)** 5, 10, 15, 20

Explanation:

Let the 4 numbers are a , a + d , a + 2d , a + 3d.

Sum of 4 numbers AP = 50

a + a + d + a + 2d + a + 3d = 50

$\Rightarrow$ 4a + 6d = 50

$\Rightarrow$ 2a + 3d = 25 ....(1)

Also given the greatest number is 4 times the least.

4(a) = a + 3d

4a - a = 3d

a = d

putting a = d in (1) , we obtain

5d = 25

d = 5

a = 5 but d = a.

$\therefore$ First four terms are 5 , 10 ,15 , 20

4.

(b) (n + 1) : n

Explanation:

Let a and d be the first term and common difference respectively of the given A.P.

Now, S_1 = Sum of odd terms

$\Rightarrow S_1 = a_1 + a_3 + a_5 + ... + a_{2n+1}$

$\Rightarrow S_1 = \frac{n+1}{2}\{a_1 + a_{2n+1}\}$

$\Rightarrow S_1 = \frac{n+1}{2}\{a + a + (2n + 1 - 1)d\}$

$\Rightarrow S_1 = (n + 1)(a + nd)$

and, S_2 = Sum of even terms

$\Rightarrow S_2 = a_2 + a_4 + a_6 + ... + a_{2n} \Rightarrow S_2 = \frac{n}{2}[a_2 + a_{n2}]$

$\Rightarrow S_2 = \frac{n}{2}[(a + d) + \{a + (2n - 1)d\}]$

$\Rightarrow S_2 = n(a + nd)$

$\therefore S_1 : S_2 = (n + 1)(a + nd) : n(a + nd) = (n + 1) : n$

5. **(a)** 38

Explanation:

Given

First term, a = 1,

Nth term, $a_n = 20$,

Sum of n terms, $S_n = 399$

Using the formula

$Sn = \frac{n}{2}(a + l)$

Where S_n = Sum of first n terms

n = no of terms

$l = a_n$ = last term

$2S_n = n(a + a_n)$

2(399) = n(1 + 20)

n = 798/21

n = 38

6.

(b) 24

Explanation:

Let the middle term be a,

then the first term is a - d and next term is a + d

$\therefore a - d + a + a + d = 72$

$\Rightarrow 3a = 72$

$\Rightarrow a = 24$

7.

(b) 5, 7, 9, 11

Explanation:

Given $a_n = 2n + 3$

$\therefore a_1 = 2 \times 1 + 3 = 2 + 3 = 5$

$a_2 = 2 \times 2 + 3 = 4 + 3 = 7$

$a_3 = 2 \times 3 + 3 = 6 + 3 = 9$

$a_4 = 2 \times 4 + 3 = 11$

Therefore, the first four terms are 5, 7, 9, 11.

8.

(b) 1

Explanation:

If a, b and c are in A.P.,

b - a = c - b

-(a - b) = -(b - c)

a - b = b - c

dividing both sides by b - c

$\frac{a-b}{b-c} = \frac{b-c}{b-c}$

$\frac{a-b}{b-c} = 1$

9.

(c) 13

Explanation:

For 1st A.P.; a = 63 and d = 2 , hence T_n = 63 + (n - 1)2

⇒ T_n = 2n + 61 ... (i)

for 2nd AP a = 3, d = 7, hence t_n = 3 + (n - 1)7

t_n = 7n - 4 ... (ii)

by condition, from (i) & (ii),

7n - 4 = 2n + 61

⇒ 5n = 65

∴ n = 13

10.

(d) 3.5

Explanation:

Given: a = 3.5, d = 0 and n = 101, then

$a_n = a + (n - 1)d$

= $3.5 + (101 - 1) \times 0$ =

=3.5 + 0 = 3.5

11. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both are correct. Reason is the correct reasoning for Assertion.

Assertion, S_{10} = $\frac{10}{2}[2(-0.5) + (10 - 1)(-0.5)]$

= 5[-1 - 4.5]

= 5(-5.5) = -27.5

12. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

13.

(d) A is false but R is true.

Explanation:

Assertion: Even natural numbers divisible by 5 are 10, 20, 30, 40, ...

They form an A.P. with,

a = 10, d = 10

S_{100} = $\frac{100}{2}[2(10) + 99(10)]$ = 50500

So, reason is correct.

14.

(d) A is false but R is true.

Explanation:

We have,

a_n = a + (n - 1)d

a_{21} - a_7 = {a + (21 - 1)d} - {a + (7 - 1)d} = 84

a + 20d - a - 6d = 84

14d = 84

d = $\frac{18}{14}$ = 6

d = 6

So, A is false but R is true.

15.

(d) A is false but R is true.

Explanation:

We have, common difference of an AP

d = a_n - a_{n-1} is independent of n or constant.

So, A is false but R is true.

16. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

17.

(d) A is false but R is true.

Explanation:

A is false but R is true.

18.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

19.

(d) A is false but R is true.

Explanation:

A is false but R is true.

20.

(c) A is true but R is false.

Explanation:

A is true but R is false.

21. We have, a_n = 5n - 4; a_{12} and a_{15}

Put n = 12

A_{12} = 5(12) - 4

= 60 - 4 = 56

Put n = 15

A_{15} = 5(15) - 4

= 75 - 4 = 71

22. Given:

d = -2, n = 5 and a_n = 0

We know that , a_n = a + (n - 1) d

$0 = a + (n - 1)d$

$[a_n = 0]$

$0 = a + (5 - 1)(-2)$

0 = a + 4 $\times$ - 2

$0 = a - 8$

0 + 8 = a

a = 8

Hence, the value of a is 8.

23. a = -1, $d = \frac{1}{2}$

First term = a = -1

Second term = -1 + d $= -1 + \frac{1}{2} = -\frac{1}{2}$
Third term $= -\frac{1}{2} + d = -\frac{1}{2} + \frac{1}{2} = 0$
Fourth term = 0 + d $= 0 + \frac{1}{2} = \frac{1}{2}$
Hence, the first four terms of the given AP are -1, $-\frac{1}{2}, 0, \frac{1}{2}$.

24. Here the given series is:
$\sqrt{2}, \sqrt{8}, \sqrt{18}$,....
This series can be written as:
$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$,,.....
Therefore an A.P is formed.
Here a = $\sqrt{2}$ and d = $\sqrt{2}$
$\therefore$ Next term, T_4 = a + 3d = $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2} = \sqrt{32}$

25. Here, $a = -37,\ d = -33 - (-37) = -33 + 37 = 4\ and\ n = 12$
Now we know that , $S_n = \frac{n}{2}$ [2a+(n-1)d]
Therefore, $S_{12} = \frac{12}{2}[2 \times (-37) + (12 - 1)4]$
$= 6[-74 + 44]$
= 6 $\times$ (-30)
= -180
Therefore, sum of given A.P is -180.

26. T_n = (5n - 2) (given)
$\Rightarrow T_1 = [(5 \times 1) - 2] = 3$ and $T_2 = [(5 \times 2) - 2] = 8$
Thus we have,
Common difference = $(T_2 - T_1)$ = (8 - 3) = 5

27. First 20 odd natural numbers are :-
1,3,5,7,...... upto 20 terms
So, it is an Arithmetic sequence with a =1,d =2
We know that,sum of n terms of an A.P. is given by:-

$S_n = \frac{n}{2}$ [2a + (n - 1)d]
$= \frac{20}{2}$ [2×1 + (20 -1)×2]
= 10 [2 + 38] = 40×10 = 400

28. $\because \frac{3}{5}$, a, 4 are three consecutive terms of AP
So, d = $a_2 - a_1$ = a - $\frac{3}{5}$ ...(i)
Also d = $a_3 - a_2$ = 4 - a ...(ii)
From equation (i) & (ii)
a - $\frac{3}{5}$ = 4 - a
2a = $4 + \frac{3}{5} = \frac{20+3}{5}$
a = $\frac{23}{10}$ = 2.3

29. We have $a_1 = \frac{1}{2}$, $a_2 = \frac{1}{3}$ and $a_3 = \frac{1}{4}$
$a_2 - a_1 = \frac{-1}{6}$
$a_3 - a_2 = \frac{-1}{12}$
Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

30. The multiples of three are: 3, 6, 9, ...
Clearly, a = 3, d = 3, n = 10,
We have to find S_{10}
We know, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2 \times 3 + (10 - 1)3]$
$= 5(6 + 9 \times 3)$
$= 5(6 + 27)$
$= 5 \times 33$
$= 165$
Therefore, Sum of first 10 multiples of 3 is 165.

Section B

31. The first 2-digit multiple of 7 is 14, and the next 11 two-digit multiples of 7 are: 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91

To find the common difference, we subtract the first term from the second term:

21 - 14 = 7

$\mathbf{S} = (\frac{n}{2})(2a + (n - 1)d)$

where S is the sum, n is the number of terms, a is the first term, and d is the common difference.

Substituting the values we have:

$S = (\frac{12}{2})(2(14) + (12 - 1)(7))$

S = 6(28 + 77)

S = 6(105)

S = 630

Therefore, the sum of the first twelve 2-digit multiples of 7 is 630.

32. Let 'a' be first term and 'd' be common difference of an A.P

Given,

4^{th} term = - 15

a + 3d = - 15 [using a_n = a + (n - 1)d]

a = - 3d - 15 ...(1)

9^{th} term = - 30

a + 8d = - 30

- 3d - 15 + 8d = - 30 [using 1]

5d = - 15

d = - 3

Putting this value in (1) we get

a = - 3 (- 3) - 15

= 9 - 15 = - 6

Also we know,

Sum of n terms, $\mathrm{S_n} = \frac{\mathrm{n}}{2}[2\mathrm{a} + (\mathrm{n} - 1)\mathrm{d}]$

$\mathrm{S}_{17} = \frac{17}{2}[2(-6) + (17 - 1)(-3)]$

$= \frac{17}{2}[-12 - 48]$

$= \frac{17}{2}[-60]$

= 17 (- 30)

= - 510

33. First term = a = 114

Common difference = a_2 - a_1

d = 109 - 114

d = -5

Let n^{th} term of A.P = 0

a + (n - 1)d = 0

114 + (n - 1)(-5) = 0

114 - 5n + 5 = 0

119 - 5n = 0

-5n = - 119

n = $\frac{119}{5}$

n = 23.4

Therefore, 24th term of given A.P. is first negative term.

34. We have, a = 5, d = 3

n^{th} term = 50

We know that, $a_n = a + (n - 1)d$

a + (n - 1)d = 50

5 + (n - 1)3 = 50

3n - 3 = 45

3n = 45 +3

3n = 48

n = $\frac{48}{3}$

n = 16

Sn = $\frac{n}{2}$ [2a + (n - 1)d]

= $\frac{16}{2}$ [2 $\times$ 5 + (16 - 1)3]

= 8 [10 + 45]

= 8 $\times$ 55

= 440

35. Here it is given that a = 24, d = 21 – 24 = –3, Sn = 78, We need to find n.

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

So,$78 = \frac{n}{2}[48 + (n-1)(-3)] = \frac{n}{2}[51 - 3n]$

or $3n^2 - 51n + 156 = 0$

$n^2 - 17n + 52 = 0$

(n – 4)(n – 13) = 0

n = 4 or 13

Both values of n are admissible. So, the number of terms is either 4 or 13

36. Let the first term be a and common difference be d.

a_n = a + (n -1)d)

As per given condition

a_9= -32

a + 8d = - 32.........(i)

$a_{11} + a_{13}$ = -94

a + 10d + a+ 12d = - 94

2a + 22d = -94

a + 11d = - 47 .....(ii)

Solving equations (i) and (ii)

(a + 11d) - (a + 8d) = -47 - (-32)

a + 11d -a - 8d = -47 + 32

3d = -15

d = -5

Common difference d = - 5

37. From the given numbers, we have

$a_2 - a_1 = -1 - 1 = -2$

$a_3 - a_2 = -3 - (-1) = - 3 + 1 = -2$

$a_4 - a_3 = -5 - (-3) = - 5 + 3 = -2$

i.e., $a_{k+1} - a_k$ is the same every time

So, the given list of numbers forms an AP with the common difference d = –2

The next two terms are:

-5 + (-2) = -7 and

-7 + (-2) = -9

38. AP: 3, 6, 9, 12 ,..., 99

a = 3, d = 6 - 3 = 3, a_n = 99

$a_n = a + (n - 1)d$

99 = 3 + (n - 1) × 3

96 = (n - 1) × 3

(n - 1) = 32

n = 33

39. Given,

AP: 65, 61, 57, 53

a = 65 and d = 61 - 65 = -4

Consider the n^{th} term of the A.P. as the first negative term

i.e., $a_n < 0$

We know that n^{th} term of an AP,

$a^n = a + (n - 1)d$

Here, [a + (n - 1)d] < 0

⇒ 65 + (n - 1) (-4) < 0

⇒ 65 - 4n + 4 < 0

⇒ 69 - 4n < 0

⇒ 4n > 69

⇒ n > $\frac{69}{4}$

⇒ n > 17.25

⇒ n = 18

So the 18^{th} term is the first negative term of the given AP.

40. Let the first term and the common difference of the AP be a and d respectively.

Second term = 14 ...... Given

⇒ a + (2 - 1)d = 4 .... $\because a_n = a + (n-1)d$

⇒ a + d = 14 .... (1)

Third term = 18 ...... Given

⇒ a + (3 - 1)d = 18 $\because a_n = a + (n-1)d$

⇒ a + 2d = 18 ..... (2)

Solving equation (1) and equation (2), we get

a = 1

d = 2

Now, sum of first 51 terms

$S51 = \frac{51}{2}[2(1) + (51-1)2] = \frac{51}{2}[2 + 100]$

$= \frac{51}{2} \times 102 = 51 \times 51 = 2601$

Section C

41. $a_3 = 16$

⇒ a + 2d = 16 ..... (i)

$a_7 = a_5 + 12$

⇒ a + 6d = a + 4d + 12

⇒ 2d = 12

⇒ d = 6

Put the value of d in eq. (i)

$a + 2 \times 6 = 16$

⇒ a = 16 - 12

⇒ a = 4

4, 10, 16....

42. S_1 = 1 + 2 + 3 + ...n

S_2 = 1 + 3 + 5 + ...upto n terms

S_3 = 1 + 4 + 7 + ...upto n terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_1 = \frac{n}{2}[2(1) + (n-1)1]$

$S_1 = \frac{n}{2}[2 + n - 1]$

or, $S_1=\frac{n(n+1)}{2}$
Also, $S_2=\frac{n}{2}[2\times 1+(n-1)2]$
$S_2=\frac{n}{2}[2+2n-2]$
$=\frac{n}{2}[2n]=n^2$
and $S_3=\frac{n}{2}[2\times 1+(n-1)3]$
$S_3=\frac{n}{2}[2+3n-3]$
$=\frac{n(3n-1)}{2}$
Now, $S_1+S_3=\frac{n(n+1)}{2}+\frac{n(3n-1)}{2}$
$=\frac{n[n+1+3n-1]}{2}$
$=\frac{n[4n]}{2}$
$= 2n^2 = 2S_2$

Hence Proved.

43. Let the first term and the common difference of the AP be a and d respectively.
Given that, $a_{17} = a_{10}+7$
$\Rightarrow$ a + (17 - 1) d = a + (10 - 1)d + 7 [$\because a_n = a +(n - 1)d$]
$\Rightarrow$ a + 16d = a + 9d + 7
$\Rightarrow$ 16d - 9d = 7
$\Rightarrow$ 7d = 7
$\Rightarrow d=\frac{7}{7}=1$
Hence, the common difference is 1.

44. The given n odd natural numbers are,
1, 3, 5, ... 2n - 1
Clearly, it is an AP with
First term (a) = 1
Common difference (d) = 3 - 1 = 2
Number of terms = n
Sum of n terms $=\frac{n}{2}[2a+(n-1)d]$
$=\frac{n}{2}[2\times 1+(n-1)\times 2]$
$=\frac{n}{2}[2+(2n-2)]$
$=n^2$

45. Here it is given that first car goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by 1/2 km in each succeeding hour.
Suppose the second car overtakes the first car after t hours. Then, the two cars travel the same distance in t hours.Distance travelled by the first car in t hours = 10×t km.
|Distance travelled by the second car in t hours= Sum of t terms of an A.P. with first term 8 and common difference 1/ 2
$=\frac{t}{2}\left\{2\times 8+(t-1)\times\frac{1}{2}\right\}=\frac{t(t+31)}{4}$
When the second car overtakes the first car, we have
$10t=\frac{t(t+31)}{4}$
$\Rightarrow 40t=t^2+31t$
$\Rightarrow t^2-9t=0$
$\Rightarrow t(t-9)=0$
$\Rightarrow t=9$
Since t can not be equal to zero, so we get t=9. Hence, it is proved that the second car will overtake the first car in 9 hours.

46. Given that,
Yasmeen, during the first month, saves = 32 Rs
During the second month, saves = 36 Rs
During the third month, saves = 40 Rs
Let Yasmeen saves Rs 2000 during the n months.
Here, we have arithmetic progression 32, 36, 40, …
First term, a = 32

common difference, d = 36 – 32 = 4

Total money save by her in n months = Sum of this AP upto n terms

$2000 = \frac{n}{2}[2a + (n-1)d]$ $\left(\text{using } S_n = \frac{n}{2}[2a + (n-1)d]\right)$

4000 = n[2(32) + (n - 1)4]

4000 = n[64 + 4n - 4)

4000 = n[4n + 60]

$4n^2 + 60n - 4000 = 0$

$n^2 + 15n - 1000 = 0$

$n^2 + 40n - 25n - 1000 = 0$

(n - 25)(n + 40) = 0

n = 25 or n = - 40

but n = - 40 as no of terms and months cannot be negative

So it would take 25 months to save Rs. 2000

47. The first 15 multiples of 8 are 8, 16, 24, 32,....

Here, $a_2 - a_1 = 16 - 8 = 8$

$a_3 - a_2 = 24 - 16 = 8$

$a_4 - a_3 = 32 - 24 = 8$

i.e. $a_{k-1} - a_k$ is the same everytime.

So, the above list of numbers forms an AP.

Here, a = 8

d = 8

n = 15

$\therefore$ Sum of first 15 multiples of 8 $= S_{15}$

$= \frac{15}{2}[2a + (15-1)d]$ .... $\because S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{15}{2}[2a + 14d]$

= 15 (a + 7d)

$= (15)(8 + 7 \times 8)$

= (15) (8 + 56)

= (15) (64)

= 960

48. Let the first term of the Arithmetic progression be 'a'.

and the common difference be 'd'.

24^{th} term of the Arithmetic progression, $t_{24} = a + (24 - 1)d = a + 23d$

10^{th} term of the A.P., $t_{10} = a + (10 - 1)d = a + 9d$

72^{nd} term of the A.P., $t_{72} = a + (72 - 1)d = a + 71d$

15^{th} term of the A.P., $t_{15} = a + (15 - 1)d = a + 14d$

$t_{24} = 2 \times t_{10}$

$\Rightarrow$ a + 23d = 2(a + 9d)

$\Rightarrow$ a + 23d = 2a + 18d

$\Rightarrow$ 23d - 18d = 2a - a

$\Rightarrow$ 5d = a

And, $t_{72} = a + 71d$ (substitute value of a)

= 5d + 71d

= 76d

= 20d + 56d

$= 4 \times 5d + 4 \times 14d$

= 4(5d + 14d)

= 4(a + 14d)

$= 4t_{15}$

Therefore, $t_{72} = 4t_{15}$

Hence proved.

49. All the two-digit natural numbers divisible by 4 are 12, 16, 20, 24, ..., 96

Here, $a_1 = 12$

$a_2 = 16$

$a_3 = 20$

$a_4 = 24$

$\therefore a_2 - a_1 = 16 - 12 = 4$

$a_3 - a_2 = 20 - 16 = 4$

$a_4 - a_3 = 24 - 20 = 4$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ...$ (= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 12, d = 4, l = 96

Let the number of terms be n.

Then, $l = a + (n - 1)d \Rightarrow 96 = 12 + (n - 1)4$

$\Rightarrow 96 - 12 = (n - 1) 4 \Rightarrow 84 = (n - 1)4$

$\Rightarrow (n - 1) 4 = 84 \Rightarrow n - 1 = \frac{84}{4}$

$\Rightarrow n - 1 = 21 \Rightarrow n = 21 + 1 \Rightarrow n = 22$

$\therefore S_n = \frac{n}{2}(a + l) = \frac{22}{2}(12 + 96) = (11)(108) = 1188$

50. Given sequence is,

3, 6, 12, 24,......

First term of this A.P is $a_1 = 3a_1 = 3$

Second term of this A.P is $a_2 = 6a_2 = 6$

Third term of this A.P is $a_3 = 12a_3 = 12$

The condition for an sequence to be an A.P is their must be a common difference (i.e., $d = a_{n+1} - a_n$)

putting n = 1 in above equation

$d = a_2 - a_1 = 6 - 3 = 3$

putting n = 2 in above equation

$d = a_3 - a_2 = 12 - 6 = 6d = a_3 - a_2 = 12 - 6 = 6$

as we can see we get two values of d,

But in an A.P their must be a single value of d which means for this sequence we can't define a common difference.

Hence this sequence does not form an A.P.

Section D

51. i. Money saved on 1st day = ₹ 27.5

$\because$ Sehaj increases his saving by a fixed amount of ₹ 2.5

$\therefore$ His saving form an AP with a = 27.5 and d = 2.5

$\therefore$ Money saved on 10th day,

$a_{10} = a + 9d = 27.5 + 9(2.5)$

= 27.5 + 22.5 = ₹ 50

ii. $a_{25} = a = 24d$

= 27.5 + 24(2.5)

= 27.5 + 60 = ₹ 87.5

iii. Total amount saved by Sehaj in 30 days.

= $\frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

= 15(55 + 29(2.5)

= ₹1912.5

OR

Let $S_n = 387.5$, a = 27.5 and d = 2.5

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$
$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$
$\Rightarrow$ 775 = 55n + 2.5n^2 - 2.5n
$\Rightarrow$ 25n^2 + 525n = 7750 = 0
$\Rightarrow$ n^2 + 21n - 310 = 0
$\Rightarrow$ (n + 31)(n - 10) = 0
$\Rightarrow$ n = -31 reject n = 10 accept
So in 10 years Sehaj saves ₹ 387.5.

52. i. The distance covered by Dinesh to pick up the first flower plant and the second flower plant,
= 2 $\times$ 10 + 2 $\times$ (10 + 5) = 20 + 30
therefore, the distance covered for planting the first 5 plants
= 20 + 30 + 40 + ... 5 terms
This is in AP where the first term a = 20
and common difference d = 30 - 20 = 10
We know that: $S_n = \frac{n}{2}[2a + (n-1)d]$
so, the sum of 5 terms
$S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times 80 = 200 \text{ m}$
hence, Dinesh will cover 200 m to plant the first 5 plants.
ii. As a = 20 ,d = 10 and here n = 10
so, $S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130 = 650 \text{ m}$
hence Ramesh will cover 650 m to plant all 10 plants.
iii. Total distance covered by Ramesh = 650 m
$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{650}{10} = 65 \text{ minutes}$
Time taken to plant all 10 plants=15 $\times$ 10 = 150 minutes
Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

53. i. a = 3, d = 2
Pair of shoes in 6^{th} row = 3 + (5)2 = 13
ii. Difference of pair of shoes in 6^{th} and 1^{st} row = 13 - 3 = 10
iii. a. n = 15
Total pair of shoes in 15 rows = $\frac{n}{2}[2a + (n-1)d]$
$= \frac{15}{2}[2 \times 3 + 14 \times 2]$
$= \frac{15}{2} \times 34 = 255$
OR
b. Pair of shoes in 4^{th} row = 3 + (3)2 = 9
Money earned = 500 $\times$ 9 = ₹ 4500

54. i. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We have, a_3 = 600 and
a_3 = 600
$\Rightarrow$ 600 = a + 2d
$\Rightarrow$ a = 600 - 2d ...(i)
$\Rightarrow$ a_7 = 700
$\Rightarrow$ a_7 = 700
$\Rightarrow$ 700 = a + 6d
$\Rightarrow$ a = 700 - 6d ...(ii)
From (i) and (ii)
600 - 2d = 700 - 6d
$\Rightarrow$ 4d = 100
$\Rightarrow$ d = 25

ii. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We know that first term = a = 550 and common difference = d = 25
a_n = 1000
$\Rightarrow$ 1000 = a + (n - 1)d
$\Rightarrow$ 1000 = 550 + 25n - 25
$\Rightarrow$ 1000 - 550 + 25 = 25n
$\Rightarrow$ 475 = 25n
$\Rightarrow$ n = $\frac{475}{25}$ = 19

iii. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
The production in the 10th term is given by $a_{10.}$ Therefore, production in the 10th year = a_{10} = a + 9d = 550 + 9 $\times$ 25 = 775.
So, production in 10th year is of 775 TV sets.
OR
Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$
$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$
$\Rightarrow$ S_7 = 4375

55. i. a = 1000
d = 100
S_n = 1,18,000
t_{30} = a + 29 d
= 1000 + 29 $\times$ 100
= 1000 + 2900
t_{30} = 3900
i.e., he will pay ₹ 3900 in 30^{th} installment.

ii. S_n = $\frac{n}{2}$ {2a + (n - 1)d}
S_{30} = $\frac{30}{2}$ {2 $\times$ 1000 + (30 - 1) $\times$ 100}
S_{30} = 15 {2000 + 2900}
S_{30} = 15 $\times$ 4900
S_{30} = 73,500
i.e., he will pay ₹ 73500 in 30 installments.

iii. S_n = $\frac{n}{2}$ {a + l}
1,18,000 = $\frac{40}{2}$ {1000 + l}
1,18,000 = 20,000 + 20 l
98,000 = 20 l
l = 4900
i.e., the last installment will be of ₹ 4900.
OR
t_{10} = a + 9d
= 2000 + 9 $\times$ 100

t_{10} = 2000 + 900

t_{10} = ₹ 2900

56. Clearly, the amount of installment in the first month is 1000, which increases by Rs100 every month therefore, installment amount in second month=Rs 1100, third month=Rs1200, fourth month=1300 ..... which forms an AP, with first term, a = 1000 and common difference, d=1100-1000= 100

(i) Amount Paid in 4th installment will be 1400.

(ii) Now, amount paid in the 30th installment,

$a_{30} = 1000 + (30 - 1)100 = 3900 \ \{a_n = a + (n - 1)d\}$

(iii) Amount paid in 30 installments,

$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100] = 73500$

(iv) Hence, the remaining amount of the loan that he has to pay = 118000 - 73500 = 44500 Rupees

57. i. The distance covered by Dinesh to pick up the first flower plant and the second flower plant,
    = 2 × 10 + 2 × (10 + 5) = 20 + 30
    therefore, the distance covered for planting the first 5 plants
    = 20 + 30 + 40 + ... 5 terms
    This is in AP where the first term a = 20
    and common difference d = 30 - 20 = 10
    ii. We know that a = 20, d = 10 and number of terms = n = 5 so,
    $S_n = \frac{n}{2}[2a + (n - 1)d]$
    So, the sum of 5 terms
    $S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times$ 80 = 200 m
    Hence, Dinesh will cover 200 m to plant the first 5 plants.
    iii. As a = 20, d = 10 and here n = 10
    So, $S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130$ = 650 m
    So, hence Ramesh will cover 650 m to plant all 10 plants.
    OR
    Total distance covered by Ramesh 650 m
    Time = $\frac{\text{distance}}{\text{speed}} = \frac{650}{10}$ = 65 minutes
    Time taken to plant all 10 plants = 15 × 10 = 150 minutes
    Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

58. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We have,a_3 = 600 and a_7 = 700 ⇒ a + 2d = 600 and a + 6d = 700. Solving these equations, we get;a = 550 and d = 25.
    1. We have, a = 550
       ∴ Production in the first year is of 550 TV sets.
    2. The production in the 10th term is given by a_{10}.
       Therefore, production in the 10th year = a_{10} = a + 9d = 550 + 9 × 25 = 775. So, production in 10th year is of 775 TV sets.
    3. Total production in 7 years
       = Sum of 7 terms of the A.P. with first term a (= 550) and common difference d (= 25).
       $= \frac{7}{2}\{2 \times 550 + (7 - 1) \times 25\}$
       $= \frac{7}{2}(1100 + 150) = 4375.$

59. The number of rose plants in the 1^{st}, 2^{nd}, ... are 23,21,19,...........5
a = 23, d = 21 - 23 = - 2 , a_n = 5

∴ a_n = a + (n - 1)d

or, 5 = 23 + (n - 1)(-2)

or, 5 = 23 -2n + 2

or, 5 = 25 -2n

or, 2n = 20

or, n =10

Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10-1)(-2)]$

$= 5[46 - 20 + 2]$

S_{10} = 5(46 - 18)

= 5 (28)

S_{10} = 140

60. i. Let 1^{st} year production of TV = x

Production in 6^{th} year = 16000

t_6 = 16000

t_9 = 22,600

t_6 = a + 5d

t_9 = a + 8d

16000 = x + 5d ...(i)
22600 = x + 8d ...(ii)
-   -   -
-6600 = - 3d

d = 2200

Putting d = 2200 in equation ...(i)

16000 = x + 5 $\times$ (2200)

16000 = x + 11000

x = 16000 - 11000

x = 5000

$\therefore$ Production during 1^{st} year = 5000

ii. Production during 8th year is (a + 7d) = 5000 + 7(2200) = 20400

iii. Production during first 3 year = Production in (1^{st} + 2^{nd} + 3^{rd}) year

Production in 1^{st} year = 5000

Production in 2^{nd} year = 5000 + 2200

= 7200

Production in 3^{rd} year = 7200 + 2200

= 9400

$\therefore$ Production in first 3 year = 5000 + 7200 + 9400

= 21,600

OR

Let in n^{th} year production was = 29,200

t_n = a + (n - 1)d

29,200 = 5000 + (n - 1) 2200

29,200 = 5000 + 2200n - 2200

29200 - 2800 = 2200n

26,400 = 2200n

$\therefore n = \frac{26400}{2200}$

n = 12

i.e., in 12^{th} year, the production is 29,200

Section E

61. Let n be the required number of terms.

Here d = -3

$\therefore 105 = \frac{\mathrm{n}}{2}[54 + (\mathrm{n}-1)(-3)]$

$\Rightarrow 3n^2 - 57n + 210 = 0$ or $n^2 - 19n + 70 = 0$

$\Rightarrow$ n = 14 or n = 5

a_{10} = 0 or 10^{th} term is zero

62. All integers between 100 and 550, which are divisible by 9

= 108, 117, 126,.........., 549

First term (a) = 108

Common difference(d) = 117 - 108 = 9

Last term(a_n) = 549

$\Rightarrow$a + (n - 1)d = 549

$\Rightarrow$108 + (n - 1)(9) = 549

$\Rightarrow$108 + 9n - 9 = 549

$\Rightarrow$9n = 549 + 9 - 108

$\Rightarrow$ 9n = 450

$\Rightarrow \quad n=\frac{450}{9}=50$

Sum of 50 terms = $\frac{n}{2}[a+a_n]$

$=\frac{50}{2}[108+549]$

$=25\times 657$

= 16425

Now, sum of all integers between 100 and 550 which are not divisible by 9

= Sum of all integers between 100 and 550 - Sum of all integers between 100 and 550 which are divisible by 9

= [101 + 102 + 130 +......+ 549] - 16425

= $\frac{549\times 550}{2}-\frac{100\times 101}{2}$ - 16425

= 150975 - 5050 - 16425

= 129500

63. The given AP is 3, 8, 13, ..., 253

Here, a = 3

d = 8 - 3 = 5

l = 253

Let the number of terms of the AP be n.

Term, nth term = l

$\Rightarrow$ 3 + (n - 1)5 = 253 $\because a_n=a+(n-1)d$

$\Rightarrow$ (n - 1)5 = 253 - 3

$\Rightarrow$ (n - 1)5 = 250

$\Rightarrow n-1=\frac{250}{5}$

$\Rightarrow$ n - 1 = 50

$\Rightarrow$ n = 50 + 1

$\Rightarrow$ n = 51

So, there are 51 terms in the given AP.

Now, 20th term from the last term

= (51 - 20 + 1)th term from the beginning

= 32th term from the beginning

= 3 + (32 - 1)5 $\because a_n=a+(n-1)d$

= 3 + 155

= 158

Hence, the 20th term from the last term of the given AP is 158.

Aliter. Let us write the given AP in the reverse order.

Then the AP becomes 253, 248, 243, ..., 3

Here, a = 253

d = 248 - 253 = -5

Therefore, required term

= 20th term of the AP

= 253 + (20 - 1) (-5) $\because a_n=a+(n-1)d$

= 253 - 95

= 158

Hence, the 20th term from the last term of the given AP is 158.

64. $S_7 = \frac{7}{2}[2a + 6d] = -21$ ...(i)

$S_{17} = \frac{17}{2}[2a + 16d] = -221$ ...(ii)

Solving (i) and (ii) } d = -2 and a = 3

$\therefore S_n = \frac{n}{2}[6 + (n - 1)(-2)]$

$= \frac{n}{2}(8 - 2n)$ or $(4n - n^2)$

65. $S_n = 3n^2 + 5n$

$S_1 = 3(1)^2 + 5(1) = 3 + 5 = 8 = a_1$

$S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$

Now, $a_2 = S_2 - S_1 = 22 - 8 = 14$

and, $a_2 - a_1 = 14 - 8 = 6 = d$

Thus, we have a = 8 and d = 6

$\therefore a_n = a + (n - 1)d = 8 + (n - 1)(6)$

$\Rightarrow a_k = 8 + (k - 1)(6)$

$\Rightarrow 164 = 8 + 6k - 6$

$\Rightarrow 6k = 162$

$\Rightarrow k = 27$

66. Let first instalment(a) = Rs a

Let common difference = Rs d

Given,

Amount of 40 instalments = Rs 3600

$\Rightarrow \frac{40}{2}[2a + (40 - 1)d] = 3600$

$\Rightarrow 20[2a + 39d] = 3600$

$\Rightarrow 2a + 39d = 180$..............(i)

and, Amount of 30 instalments = Rs 2400

$\Rightarrow \frac{30}{2}[2a + (30 - 1)d] = 2400$

$\Rightarrow 15[2a + 29d] = 2400$

$\Rightarrow 2a + 29d = 160$..............(ii)

Subtracting (ii) from (i),

$2a + 39d - 2a - 29d = 180 - 160$

$\Rightarrow 10d = 20$

$\Rightarrow \quad d = \frac{20}{10} = 2$

Putting value of d in eq(i),

$2a + 39 \times 2 = 180$

$\Rightarrow 2a + 78 = 180$

$\Rightarrow 2a = 180 - 78$

$\Rightarrow 2a = 102$

$\Rightarrow a = \frac{102}{2} = 51$

Therefore, Value of first instalment = Rs 51

67. Let $S_n = -5 + (-8) + (-11) + \ldots + (-230)$

Clearly, the terms of the sum form an A.P.

with, a = –5

d = –8 –(–5) = –8 + 5 = –3

l = –230

Let the number of terms of the AP be n

We know that

$l = a + (n - 1)d$

$\Rightarrow -230 = -5 + (n - 1)(-3)$

$\Rightarrow (n - 1)(-3) = -230 + 5$

$\Rightarrow$ (n - 1) (-3) = -225

$\Rightarrow n-1=\frac{-225}{-3}=75$

$\Rightarrow$ n = 75 + 1

$\Rightarrow$ n = 76

Again, we know that

$S_n=\frac{n}{2}(a+l)$

$\Rightarrow S_{76}=\frac{76}{2}[(-5)+(-230)]$

$\Rightarrow S_{76}=38(-235)$

$\Rightarrow S_{76}=-8930$

Hence, the required sum is -8930.

68. Here it is given that sum of the 4th and the 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44.

We know that general term of an AP is as, a_n=a+(n-1)d.

Now, a_4=a+(4-1)d

$\Longrightarrow a_4=a+3d$

And a_8=a+(8-1)d

$\Longrightarrow a_8=a+7d$

Also, a_6=a+(6-1)d

$\Longrightarrow a_6=a+5d$.

And,a_{10}=a+(10-1)d

$\Longrightarrow a_{10}=a+9d$.

Given that $a_4+a_8=24$

$\Rightarrow$a+3d+a+7d=24

$\Rightarrow$2a+10d=24...(i)

Also, $a_6+a_{10}=44$

$\Rightarrow$a+5d+a+9d=44

$\Rightarrow$2a+14d=44...(ii)

Subtracting (i) from (ii), we get

4d=20

$\Rightarrow$d=5

Substituting in (i), we get a=-13

We get first term a=-13 and common difference d=5.

So, the sum of the first 10 terms is

$S_{10}=\frac{10}{2}[2(-13)+(10-1)(5)]$

$\Longrightarrow S_{10}=5[-26+9(5)]$

$\Longrightarrow S_{10}=5[-26+45]$

$\Longrightarrow S_{10}=5(19)$

$\Longrightarrow S_{10}=95$.

Thus, sum of first ten terms is 95

69. a_2 = 29 $\Rightarrow$ a + d = 29 ...(i)

a_4 = 51 $\Rightarrow$ a + 3d = 51 ...(ii)

Solving (i) and (ii)

d = 11 and a = 18

Now, a_n = 425 $\Rightarrow$ a + (n - 1)d = 425

18 + (n - 1)11 = 425 $\Rightarrow$ n = 38

$\therefore$ Number of terms = 38

$S_{38}=\frac{n}{2}[2a+(n-1)d]$

$=\frac{38}{2}[2\times18+37\times11]$

= 8417

70. It is given that the gap between two consecutive rungs is 25 cm and the top and bottom rungs are 2.5 metre i.e., 250 cm apart.

$\therefore$ Number of rungs = $\frac{250}{25}$ + 1 = 10 + 1 = 11.

It is given that the rungs are decreasing uniformly in length from 45 cm at the bottom to 25 cm at the top.

Therefore, lengths of the rungs form an A.P. with first term a = 45 cm and 11^{th} term l = 25 cm. n = 11

$\therefore$ Length of the wood required for rungs = Sum of 11 terms of an A.P. with first term 45 cm and last term is 25 cm

$= \frac{11}{2}$ (45 + 25) cm $\left[\because S_n = \frac{n}{2}(a+l)\right]$

$= \frac{11}{2}$ (70) cm

= 11 (35) cm

= 385 cm

Length of the wood required for rungs = $\frac{385}{100}$ = 3.85 metres ($\because$100 cm = 1 m)

The length of the wood required for the rungs is 3.85 metres.