CSEC Mathematics

January 2012 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $\left(1\frac{3}{4}\right)^2 \div 3\frac{1}{2}$, expressing your answer as a fraction [3]

$$\left(1\frac{3}{4}\right)^2 \div 3\frac{1}{2}$$

$$= \left(\frac{7}{4}\right)^2 \div \frac{7}{2}$$

$$= \frac{49}{16} \div \frac{7}{2}$$

$$= \frac{49}{16} \times \frac{2}{7}$$

$$= \frac{7}{8}$$ (as a fraction in the exact form)

(ii) $\sqrt{0.0529} + 0.216$, expressing your answer in standard form. [3]

Using a calculator,

$\sqrt{0.0529} + 0.216$

$= 0.23 + 0.216$

$= 0.446$

$= 4.46 \times 10^{-1}$ (in standard form)

(b) A typist is paid a basic wage of $22.50 per hour for a 40-hour week.

(i) Calculate the typist's basic weekly wage. [1]

Basic weekly wage

$= \text{Basic Hourly Rate} \times \text{Number of hours in a basic work week}$

$= \$22.50 \times 40$

$= \$900$

Overtime is paid at one and a half times the basic hourly rate.

(ii) Calculate the overtime wage for ONE hour of overtime work. [1]

Overtime Wage For One Hour Of Overtime Work

$= 1\frac{1}{2} \times \text{Basic Hourly Rate}$

$= 1\frac{1}{2} \times \$22.50$

$= \$33.75$

To earn some extra money, the typist decided to work overtime.

Calculate

(iii) the wage she would earn for overtime if she worked for a TOTAL of 52 hours during a given week. [2]

Overtime Wage For 52 hours

$= \text{Number Of Overtime Hours} \times \text{Overtime Rate}$

$= (52 - 40) \times \$33.75$

$= 12 \times \$33.75$

$= \$405$

(iv) the number of overtime hours she must work during a given week to earn a TOTAL wage of $1440. [2]

Basic wage (first 40 hours) $= \$900$

Overtime wage $= \text{Total Wage} - \text{Basic Wage}$

$= \$1440 - \$900$

$= \$540$

Number of Overtime hours worked

$$= \frac{\text{Overtime Wage}}{\text{Overtime Rate}}$$

$$= \frac{\$540}{\$33.75}$$

$= 16$ hours

Total: 12 marks

2. (a) Solve the pair of simultaneous equations [3]

$$3x + 2y = 13$$

$$x - 2y = -1$$

$3x + 2y = 13$ —— Equation 1

$x - 2y = -1$ —— Equation 2

Equation 1 + Equation 2:

$$\begin{array}{rrrl} & 3x & +\ 2y & = \ \ 13 \\ + & x & -\ 2y & = -1 \\ \hline & 4x & & = \ \ 12 \\ \hline \end{array}$$

$$4x = 12$$

$$x = \frac{12}{4}$$

$$x = 3$$

Substituting $x = 3$ into Equation 1 to find y:

$$3(3) + 2y = 13$$

$$9 + 2y = 13$$

$$2y = 13 - 9$$

$$2y = 4$$

$$y = \frac{4}{2}$$

$y = 2$

$\therefore x = 3$ and $y = 2$.

(b) Factorise completely

(i) $x^2 - 16$ [1]

$x^2 - 16$

$= (x)^2 - (4)^2$ [Difference of two squares]

$= (x - 4)(x + 4)$

(ii) $2x^2 - 3x + 8x - 12$ [2]

$2x^2 - 3x + 8x - 12$

$= 2x^2 + 8x - 3x - 12$

$= 2x(x + 4) - 3(x + 4)$

$= (2x - 3)(x + 4)$

(c) Tickets for a football match are sold at \$30 for EACH adult and \$15 for EACH child.

A company bought 28 tickets.

(i) If x of these tickets were for adults, write in terms of x,

(a) the number of tickets for children [1]

The number of tickets for children

= Total number of tickets − Number of tickets for adults

$= 28 - x$

(b) the amount spent on tickets for adults [1]

The amount spent on tickets for adults

= Number of tickets for adults × Cost of 1 adult ticket

$= x \times \$30$

$= \$30x$

(c) the amount spent on tickets for children [1]

The amount spent on tickets for children

= Number of tickets for children × Cost of 1 ticket for a child

$= (28 - x) \times \$15$

$= \$15(28 - x)$

(ii) Show that the TOTAL amount spent on the 28 tickets is $\$(15x + 420)$. [1]

Total amount spent on the 28 tickets

= amount spent on adult tickets + amount spent on children's tickets

$= 30x + 15(28 - x)$

$= 30x + 420 - 15x$

$= 420 + 15x$

$= \$(15x + 420)$

Q.E.D.

(iii) Given that the cost of the 28 tickets was \$660, calculate the number of adult tickets bought by the company. [2]

Total amount spent on the 28 tickets in terms of $x = 15x + 420$

Cost of the 28 tickets $= \$660$

$\therefore 15x + 420 = \$660$

$15x = 660 - 420$

$15x = 240$

$$x = \frac{240}{15}$$

$$x = 16$$

Since adults tickets $= x$,

The number of adult tickets bought $= 16$ tickets.

Total: 12 marks

3. (a) A universal set, U, is defined as:

$U = \{51, 52, 53, 54, 55, 56, 57, 58, 59\}$

A and B are subsets of U, such that:

$A = \{\text{odd numbers}\}$

$B = \{\text{prime numbers}\}$

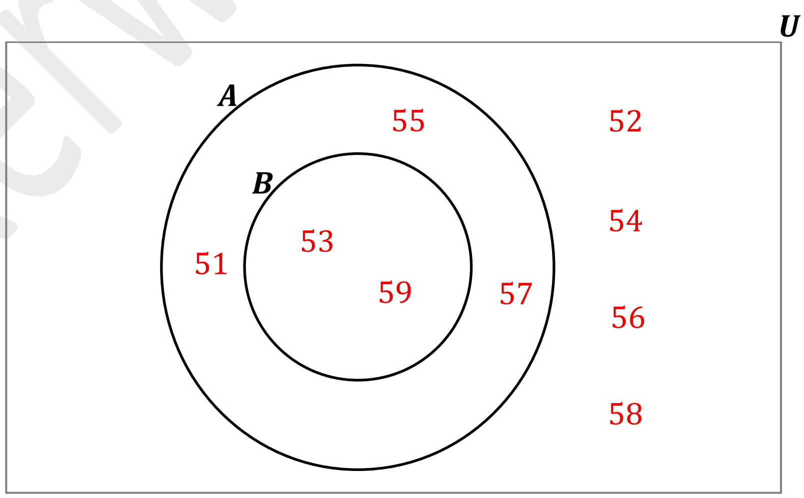
(i) List the members of the set A. [1]

$A = \{51,\ \ 53,\ \ 55,\ \ 57,\ \ 59\}$

(ii) List the members of the set B. [1]

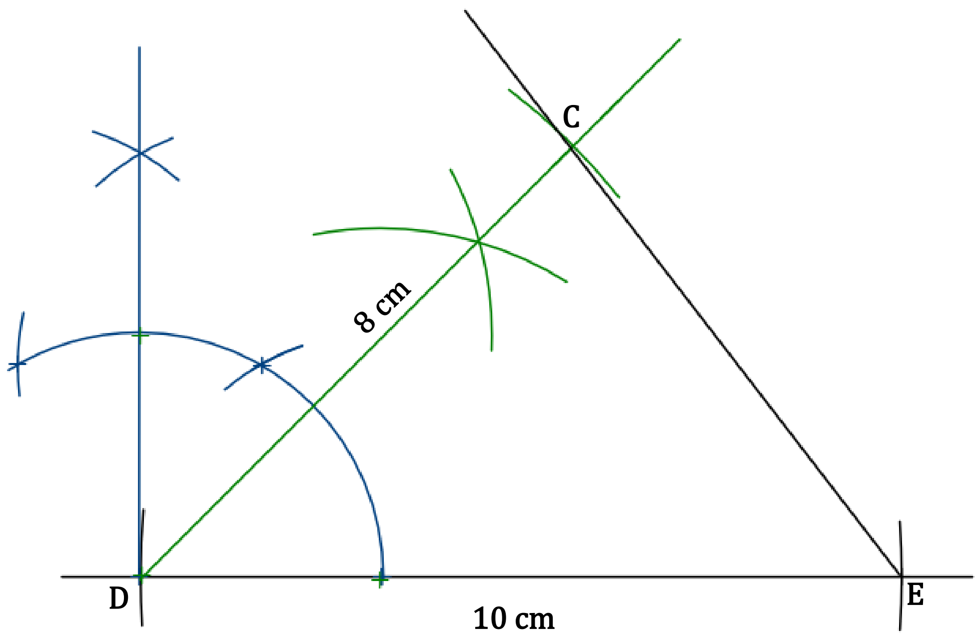
$B = \{53,\ \ 59\}$

(iii) Draw a Venn diagram to represent the sets A, B and U. [3]

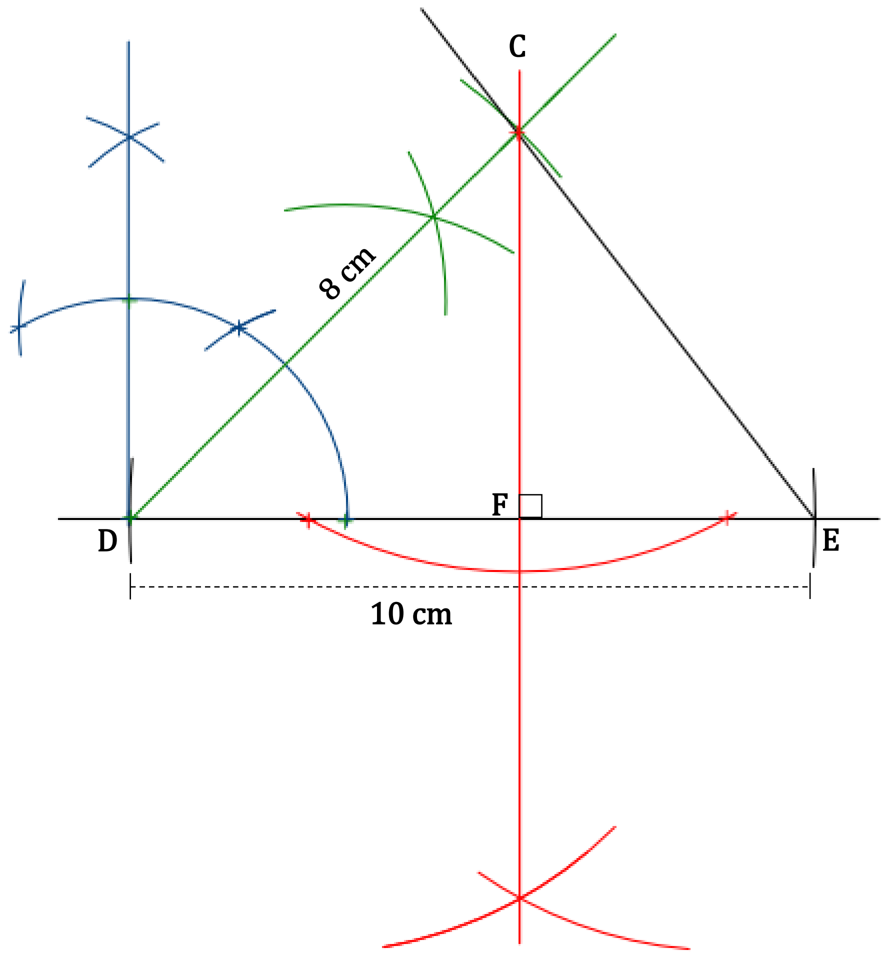


(b) (i) **Using a pair of compasses, a ruler and a pencil,**

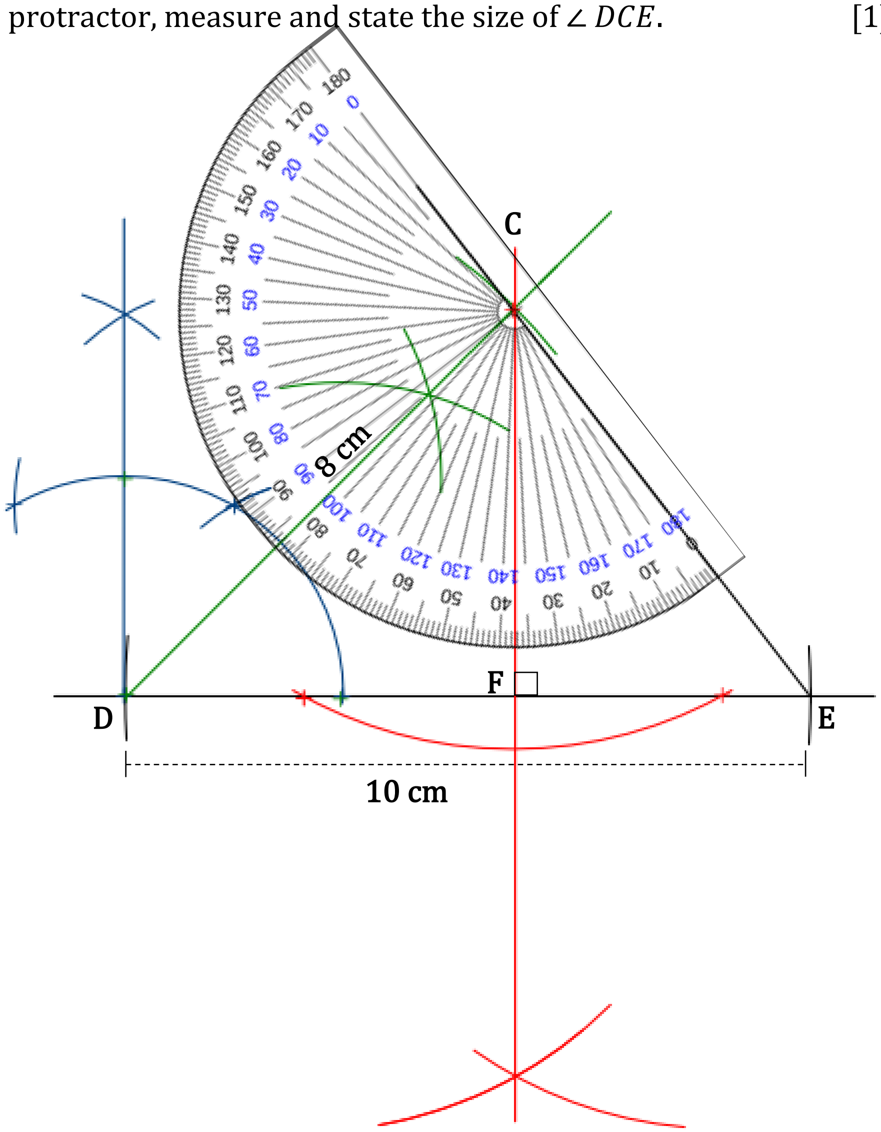
(a) Construct a triangle CDE in which $DE = 10\ cm$, $DC = 8\ cm$ and $\angle CDE = 45°$. [4]



(b) Construct a line, CF, perpendicular to DE such that F lies on DE. [2]



(ii) Using a protractor, measure and state the size of $\angle DCE$. [1]



Reading off the protractor, $\angle DCE = 83°$.

Total: 12 marks

4. (a) The following is an extract from a bus schedule. The bus begins its journey at Belleview, travels to Chagville and ends its journey at St. Andrews.

Town	Arrive	Depart
Belleview	—	6:40 a.m.
Chagville	7:35 a.m.	7:45 a.m.
St. Andrews	8:00 a.m.	—

(i) How long did the bus spend at Chagville? [1]

Time spent at Chagville

= Departure Time from Chagville − Arrival Time at Chagville

$= 7{:}45 - 7{:}35$

$$\begin{array}{r} 7 : 45 \\ -\ 7 : 35 \\ \hline 0 : 10 \\ \hline \end{array}$$

∴ The time spent at Chagville was 10 minutes.

(ii) How long did the bus take to travel from Belleview to Chagville? [1]

Time taken to travel from Belleview to Chagville

= Arrival time to Chagville − Departure time from Belleview

$= 7{:}35 - 6{:}40$

Recall: 1 hour = 60 minutes

∴ 7: 35 is equivalent to 6 hours and 95 minutes.

$$\begin{array}{rccc} & 6 & : & 95 \\ - & 6 & : & 40 \\ \hline & 0 & : & 55 \\ \hline \end{array}$$

∴ To travel from Belleview to Chagville, the bus took 55 minutes.

(iii) The bus travelled at an average speed of 54 km/hour from Belleview to Chagville. Calculate, in kilometres, the distance from Belleview to Chagville. [2]

Speed = 54 km/hour

Time = 55 minutes

$= \frac{55}{60}\ hour$

$= \frac{11}{12}\ hour$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= 54 \times \frac{11}{12}$$

$$= \frac{99}{2}$$

$$= \frac{99}{2}$$

$$= 49.5\ km$$

(b) Water is poured into a cylindrical bucket with a base area of 300 cm^2. If 4.8 litres of water was poured into the bucket, what is the height of the water in the bucket? [3]

$$\text{Base Area} = 300\ cm^2$$

$$\text{Volume} = 4.8 \text{ litres}$$

$$= (4.8 \times 1000)\ cm^3 \qquad [\text{ Since 1 litre} = 1000\ cm^3 \text{ }]$$

$$= 4800\ cm^3$$

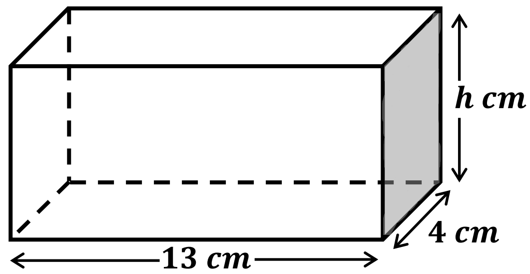
$$Volume = Base\ Area \times Height$$

$$\text{Height} = \frac{Volume}{Base\ Area}$$

$$= \frac{4800}{300}$$

$$= 16\ cm$$

(c) The diagram below, **not drawn to scale**, shows a cuboid with length 13 cm, width 4 cm and height h cm.



(i) State, in terms of h, the area of the shaded face of the cuboid. [1]

Area of the shaded face of the cuboid $= w \times h$

$= 4 \times h$

$= 4h\ cm^2$

(ii) Write an expression, in terms of h, for the volume of the cuboid. [1]

Volume of the cuboid $= l \times w \times h$

$= 13 \times 4 \times h$

$= 52h\ cm^3$

(iii) If the volume of the cuboid is 286 cm^3, calculate the height, h, of the cuboid. [2]

Volume of the cuboid in terms of $h = 52h\ cm^3$

Volume of the cuboid $= 286\ cm^3$

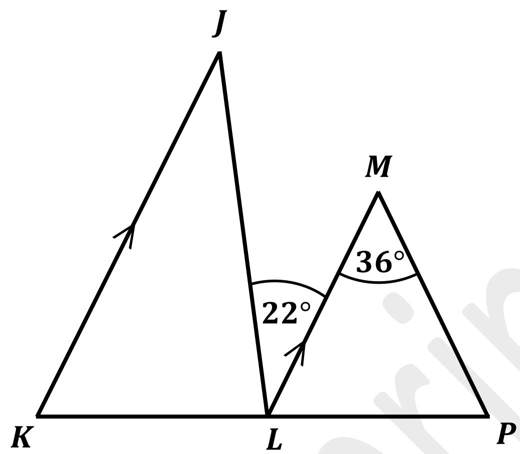
$$52h = 286$$

$$h = \frac{286}{52}$$

$$h = 5.5\ cm$$

Total: 11 marks

5. (a) The diagram below, not drawn to scale, shows TWO triangles, JKL and MLP, with JK parallel to ML. $LM = MP$, KLP is a straight line, angle $JLM = 22°$ and angle $LMP = 36°$.



Calculate, giving reasons for your answers, the measure of EACH of the following:

(i) $\angle MLP$

Since $LM = MP$, this suggests that triangle LMP is an isosceles triangle.

Since there are 180° in a triangle and $M\hat{L}P = L\hat{P}M$,

$$M\hat{L}P = \frac{180° - 36°}{2}$$

$$= \frac{144°}{2}$$

$$= 72°$$

(ii) $\angle LJK$

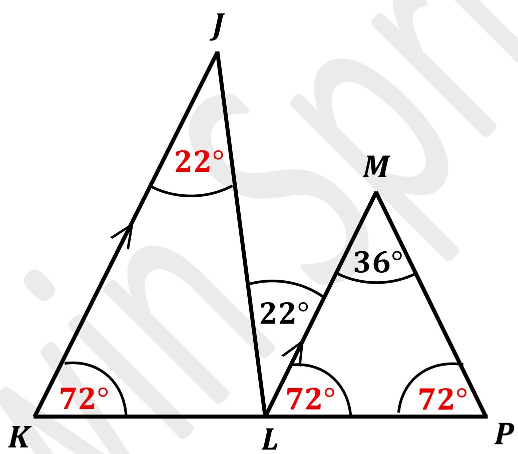
$L\hat{J}K = J\hat{L}M$ [Since alternate angles are equal]

$= 22°$

(iii) $\angle JKL$

$J\hat{K}L = M\hat{L}P$ [Since corresponding angles are equal]

$= 72°$



(iv) $\angle KLJ$ [4]

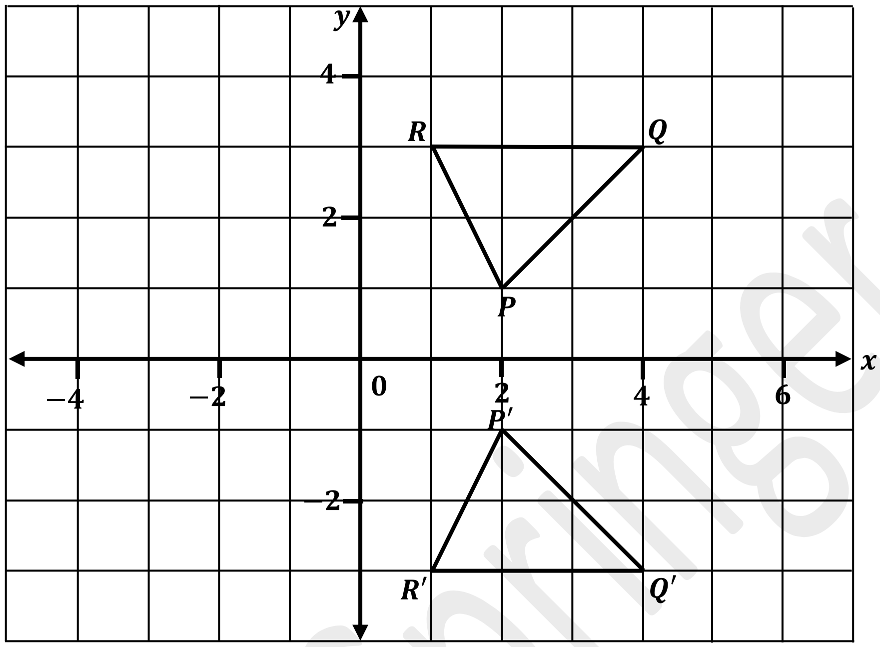
Since there are 180° in a triangle,

$K\hat{L}J = 180° - (72° + 22°)$

$= 180° - 94°$

$= 86°$

(b) The diagram below shows a triangle, PQR, and its image, $P'Q'R'$.



(i) State the coordinates of P and Q. [2]

Reading off the diagram, $P = (2, 1)$ and $Q = (4, 3)$

(ii) Describe fully the transformation that maps triangle PQR onto triangle $P'Q'R'$. [2]

Triangle PQR is mapped onto triangle $P'Q'R'$ by reflection in the x-axis.

(iii) Write down the coordinates of the images of P and Q under the translation $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$. [2]

$P = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $Q = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$$P'' = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

$$Q'' = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$\therefore P''$ (The image of P) will have the coordinates $(5, -5)$ and Q'' (The image of Q) will have the coordinates $(7, -3)$

Total: 10 marks

6. The table below shows corresponding values of x and y for the function

$y = x^2 - 2x - 3$, for integer values of x from -2 to 4.

x	-2	-1	0	1	2	3	4
y	5		-3	-4		0	5

(a) Copy and complete the table. [2]

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

$y = x^2 - 2x - 3$

When $x = -1$,

$y = (-1)^2 - 2(-1) - 3$

$= 1 + 2 - 3$

$= 0$

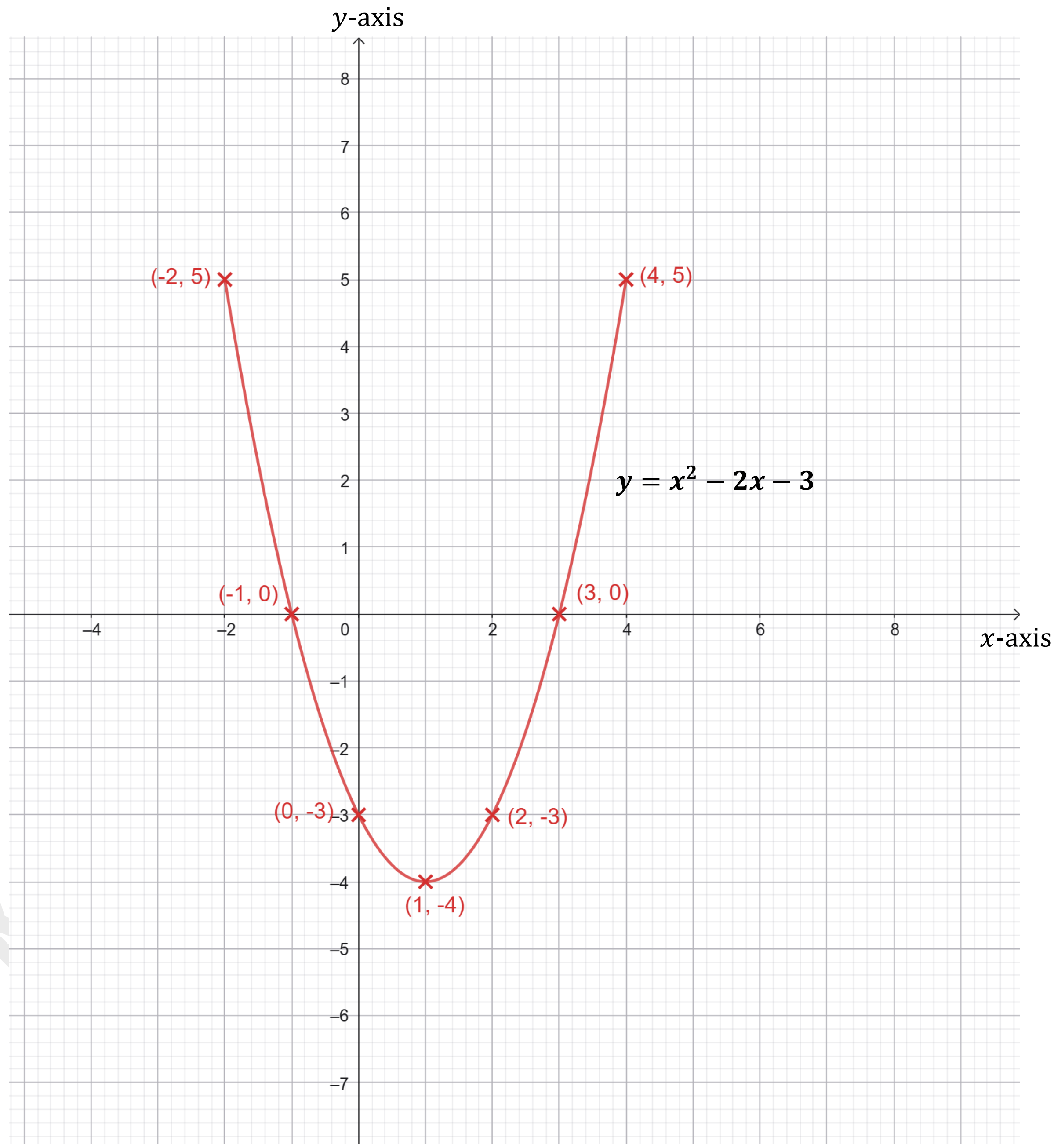
When $x = 2$,

$y = (2)^2 - 2(2) - 3$

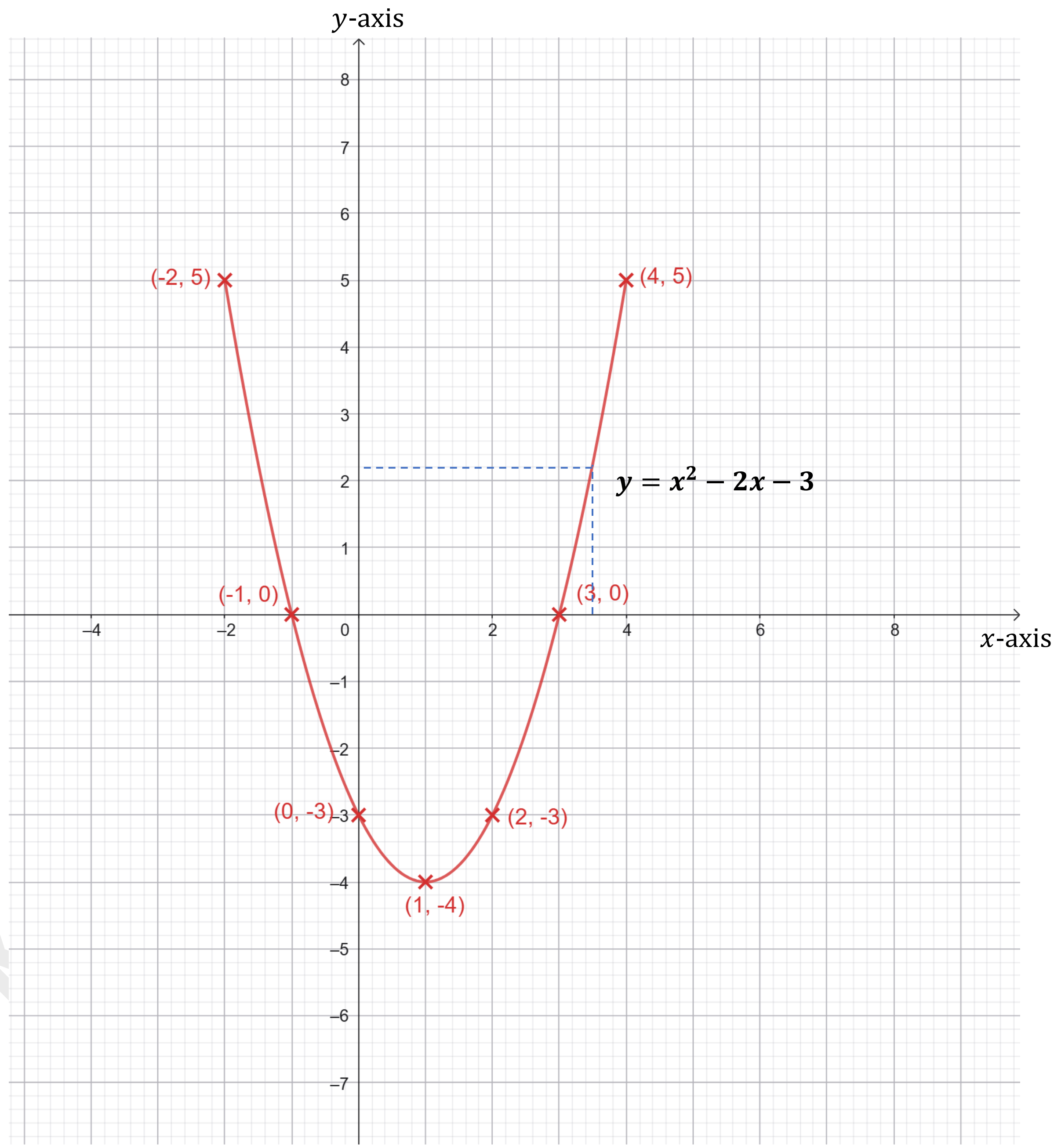
$= 4 - 4 - 3$

$= -3$

(b) **Using a scale of 2 cm to represent 1 unit on the x-axis, and 1 cm to represent 1 unit on the y-axis**, plot the points whose x and y values are recorded in your table, and draw a smooth curve through your points. [4]



(c) Using your graph, estimate the value of y when $x = 3.5$. Show on your graph how the value was obtained. [2]



Reading off the graph, when $x = 3.5$, $y = 2.2$

(d) Without further calculations,

(i) write the equation of the axis of symmetry of the graph [1]

The equation of the axis of symmetry is $x = 1$.

(ii) estimate the minimum value of the function y [1]

The minimum value of the function y is $y = -4$.

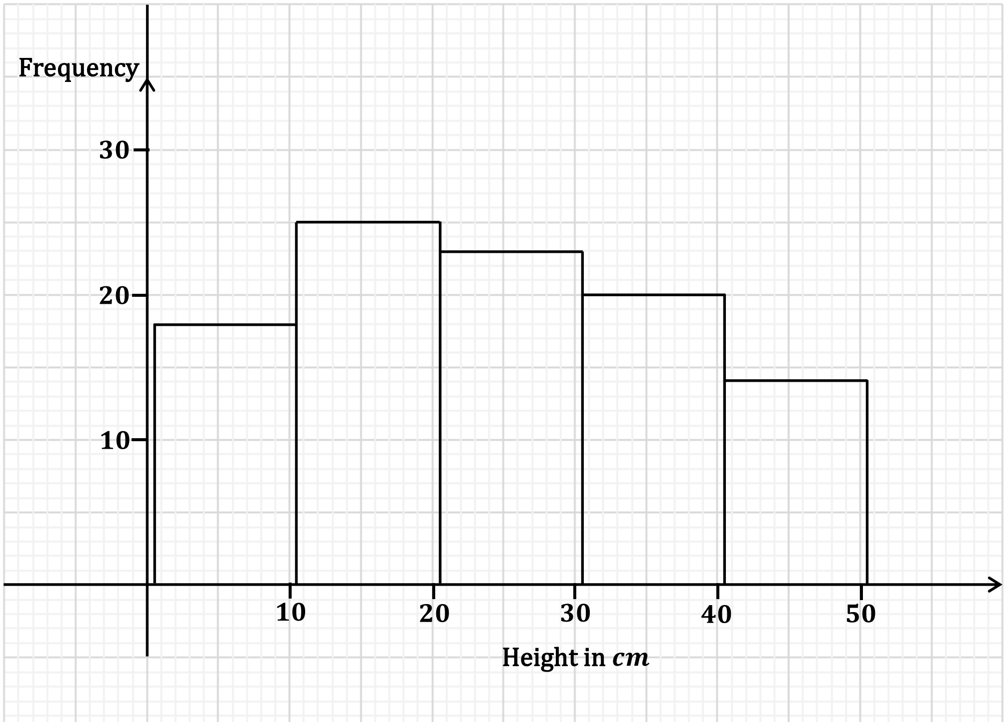
(iii) state the values of the solutions of the equation: $x^2 - 2x - 3 = 0$. [1]

The values of the solutions of the equation $x^2 - 2x - 3 = 0$ occur at the points where the graph cuts the x-axis or where $y = 0$.

Reading off the graph, the solutions of $x^2 - 2x - 3 = 0$ are $x = -1$ and $x = 3$.

Total: 11 marks

7. The histogram below shows the distribution of heights of seedlings in a sample.



(a) Copy and complete the frequency table for the data in the sample. [3]

Height in *cm*	Class Boundaries	Class Midpoint, x	Frequency, f
$1-10$	$0.5 \le x < 10.5$	5.5	18
$11-20$	$10.5 \le x < 20.5$	15.5	25
$21-30$	$20.5 \le x < 30.5$	25.5	23
$31-40$	$30.5 \le x < 40.5$	35.5	20
$41-50$	$40.5 \le x < 50.5$	45.5	14

(b) Determine

(i) the modal class interval [1]

The modal class interval $= 11 - 20$ because it has the highest frequency.

(ii) the number of seedlings in the sample [2]

The number of seedlings $= 18 + 25 + 23 + 20 + 14$

$= 100$ seedlings

(iii) the mean height of the seedlings [4]

$$Mean,\ \ \bar{x} = \frac{\sum fx}{\sum f}$$

Height in cm	Class Boundaries	Class Midpoint, x	Frequency, f	$f \times x$
$1 - 10$	$0.5 \leq x < 10.5$	5.5	18	99
$11 - 20$	$10.5 \leq x < 20.5$	15.5	25	387.5
$21 - 30$	$20.5 \leq x < 30.5$	25.5	23	586.5
$31 - 40$	$30.5 \leq x < 40.5$	35.5	20	710
$41 - 50$	$40.5 \leq x < 50.5$	45.5	14	637
			$\sum f = 100$	$\sum fx = 2420$

$$\therefore Mean,\ \ \bar{x} = \frac{2420}{100}$$

$$= 24.2 \text{ cm}$$

(iv) the probability that a seedling chosen at random has a height that is GREATER than 30 cm. [2]

$$P(\text{Seedlings with height} > 30) = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{34}{100}$$

$$= 0.34$$

Total: 12 marks

8. **An answer sheet is provided for this question.**

Sarah is making a pattern of squares using straws. She uses four straws for the sides and two longer straws for the diagonals. The first three figures in her sequence of shapes are shown below:

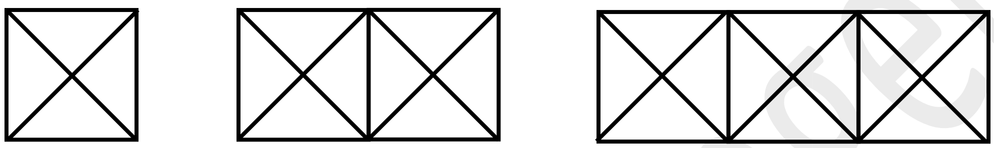
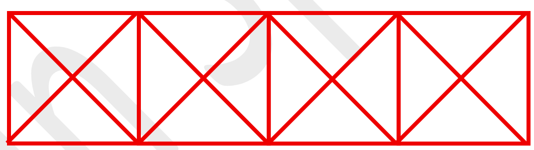


Figure 1 **Figure 2** **Figure 3**

(a) **On your answer sheet**, draw Figure 4, the FOURTH shape in the pattern. [2]



(b) **On your answer sheet**, complete the TWO rows in the table for [4]

(i) Figure 4

(ii) Figure 10

		Total Number of Straws	
	Figure	Formula	Number
	1	$1(6) - 0$	6
	2	$2(6) - 1$	11
	3	$3(6) - 2$	16
(i)	4	$4(6) - 3$	21
(ii)	10	$10(6) - 9$	51

(c) Which figure in the sequence uses 106 straws? [2]

Based on the pattern seen in the table above, we can deduce that:

Total number of straws $=$ (Figure number $\times$ (6)) $-$ (Figure number $-$ 1)

$\therefore$ For the n^{th} term,

Total number of straws $= (\text{n} \times (6)) - (\text{n} - 1)$

If the total number of straws $= 106$, then, we can solve for n:

$106 = (\text{n} \times (6)) - (\text{n} - 1)$

$106 = (6\text{n}) - (\text{n} - 1)$

$106 = 6\text{n} - \text{n} + 1$

$$106 = 5n + 1$$

$$106 - 1 = 5n$$

$$105 = 5n$$

$$5n = 105$$

$$n = \frac{105}{5}$$

$$n = 21$$

∴ Figure 21 has 106 straws.

(d) Obtain an expression in $\boldsymbol{n}$, for the total number of straws used in the $\boldsymbol{nth}$ pattern. [2]

For the n^{th} term,

$$\text{Total number of straws} = (n \times (6)) - (n - 1)$$

$$= 6n - (n - 1)$$

$$= 6n - n + 1$$

$$= 5n + 1$$

∴ The total number of straws used in the n^{th} pattern is $5n + 1$.

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a)(i) Make x the subject of the formula

$$y = \frac{2x+3}{x-4}$$ [2]

$$y = \frac{2x+3}{x-4}$$

$$y\,(x-4) = 2x+3$$

$$xy - 4y = 2x + 3$$

$$xy - 2x = 4y + 3$$

$$x(y-2) = 4y + 3$$

$$x = \frac{4y+3}{y-2}$$

(ii) Hence, determine the inverse of $f(x) = \frac{2x+3}{x-4}$, where $x \neq 4$. [2]

$$f(x) = \frac{2x+3}{x-4}$$

Let $y = f(x)$

$$y = \frac{2x+3}{x-4}$$

Make x the subject of the formula (done in part (a)(i)):

$$x = \frac{4y+3}{y-2}$$

Interchange x and y:

$$y = \frac{4x+3}{x-2}$$

$$\therefore f^{-1}(x) = \frac{4x+3}{x-2}$$

(iii) Find the value of x for which $f(x) = 0$. [2]

$$f(x) = \frac{2x+3}{x-4}$$

When $f(x) = 0$,

$$\frac{2x+3}{x-4} = 0$$

$$\frac{2x+3}{x-4} \times (x-4)^2 = 0 \times (x-4)^2$$

$$(2x+3)(x-4) = 0$$

Either

$2x + 3 = 0$ or $x - 4 = 0$

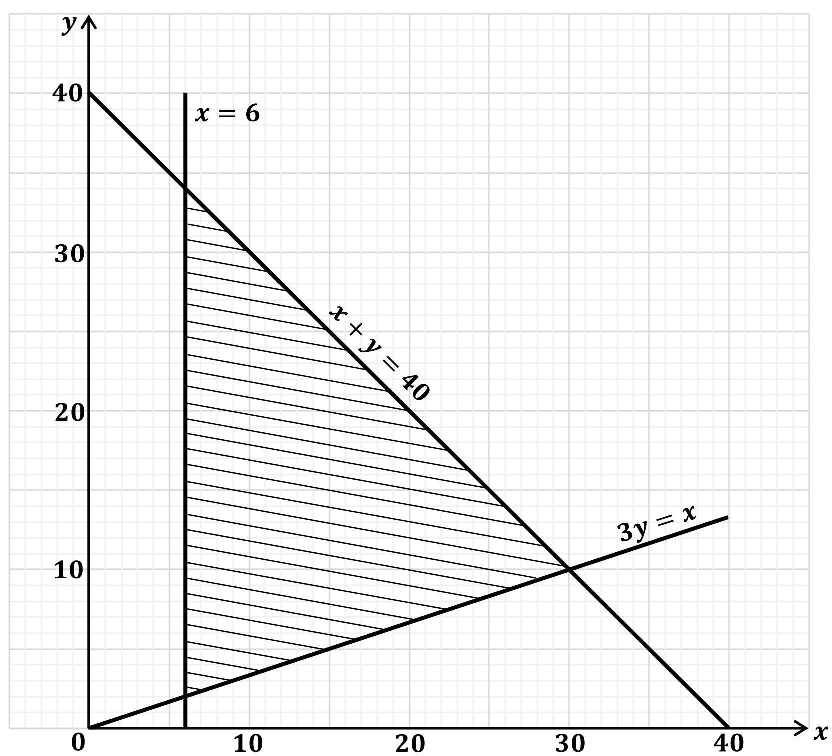
$2x = -3$ or $x = 4$

$x = -\frac{3}{2}$ Invalid

$\therefore x = -\frac{3}{2}$ only

(b) The diagram below shows the graphs of three lines and a shaded region defined by three inequalities associated with these lines.

The inequality associated with the line $3y = x$ is $3y \geq x$.



(i) State the other TWO inequalities which define the shaded region. [2]

The other two inequalities which define the shaded region are:

$x \geq 6$ and $x + y \leq 40$

The function $p = 4x + 3y$ satisfies the solution set represented by the closed triangular region.

(ii) Identify the three pairs of (x, y) values for which p has a maximum or a minimum value. [3]

Reading off the graph, the three pairs of coordinates are:

$(6, 2)$, $(6, 34)$ and $(30, 10)$.

(iii) Which pair of (x, y) values makes p a maximum?

Justify your answer. [4]

$p = 4x + 3y$

When $x = 6$ and $y = 2$,

$p = 4(6) + 3(2)$

$= 24 + 6$

$= 30$

When $x = 6$ and $y = 34$,

$p = 4(6) + 3(34)$

$= 24 + 102$

$= 126$

When $x = 30$ and $y = 10$,

$p = 4(30) + 3(10)$

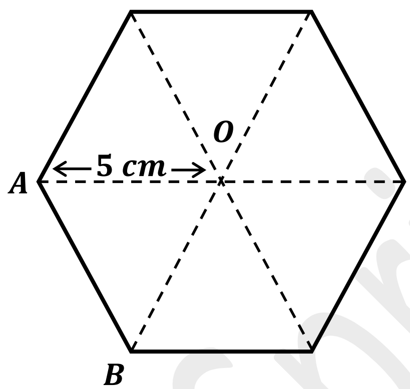
$= 120 + 30$

$= 150$

$\therefore$ The pair of coordinates that make p a maximum is $(30, 10)$.

Total: 15 marks

10. (a) The diagram below, **not drawn to scale**, shows a regular hexagon with centre, O, and $AO = 5\ cm$.



(i) Determine the size of angle AOB. [2]

A regular hexagon is made up of 6 identical triangles.

$$\therefore A\hat{O}B = \frac{360°}{6}$$

$$= 60°$$

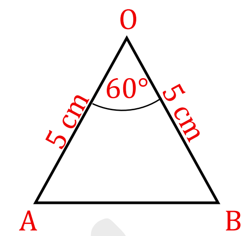
(ii) Calculate, to the nearest whole number, the area of the hexagon. [3]

We can find the area of one of the triangles and then multiply it by 6 to find the area of the hexagon.

Area of one triangle $= \frac{1}{2}ab \sin C$

$= \frac{1}{2}(5)(5) \sin 60°$

$= 10.8\ cm^2$ (to 3 s.f.)

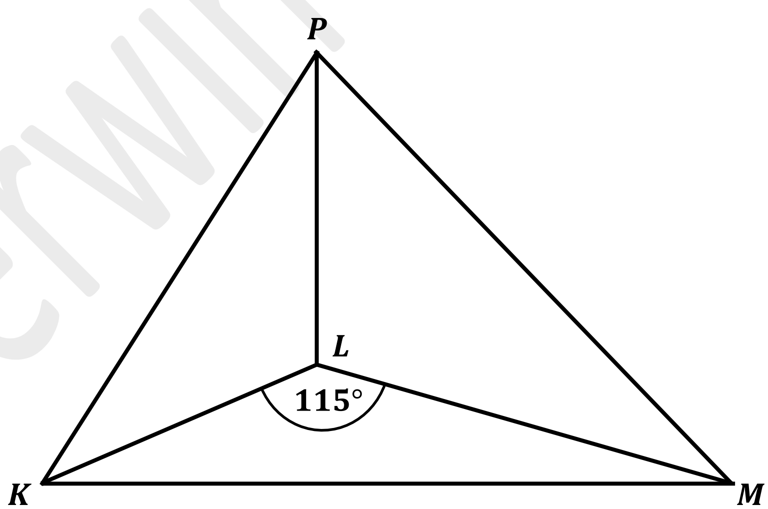


Area of 6 triangles $= 10.8\ cm^2 \times 6$

$= 65\ cm^2$ (to the nearest whole number)

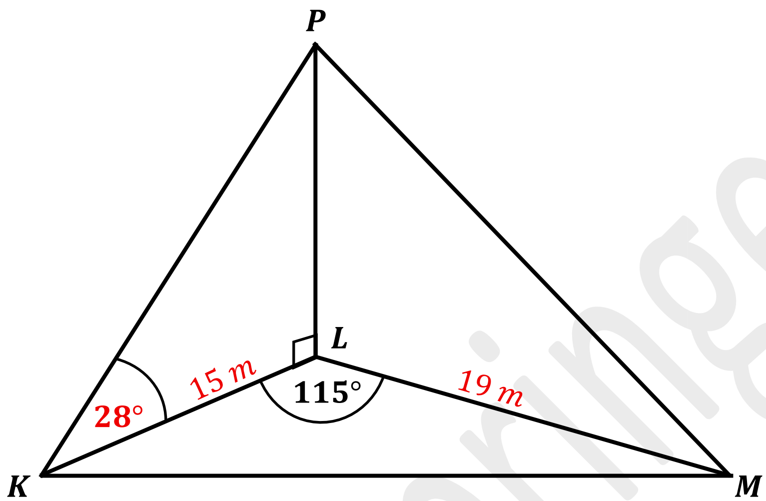
$\therefore$ The area of the hexagon is $65\ cm^2$ (to the nearest whole number).

(b) The diagram below, **not drawn to scale**, shows a vertical pole, PL, standing on a horizontal plane, KLM. The angle of elevation of P from K is 28°, $KL = 15\ m$, $LM = 19\ m$ and $\angle KLM = 115°$.



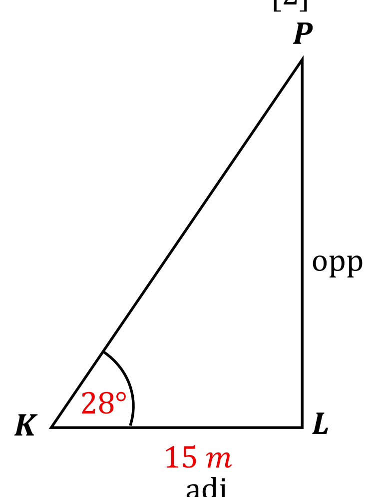
(i) Copy the diagram. Show the angle of elevation, 28° and ONE right angle.

[2]



(ii) Calculate, giving your answer to 2 significant figures, the measure of

(a) PL [2]



$$\tan\theta = \frac{opp}{adj}$$

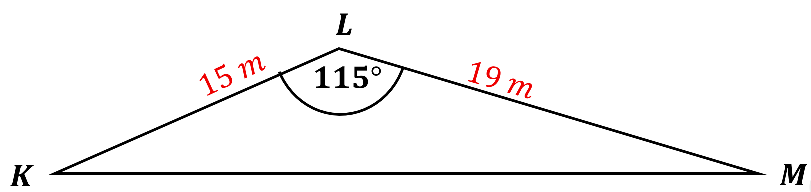
$$\tan 28° = \frac{PL}{15}$$

$$\tan 28° \times 15 = PL$$

$$PL = 15\tan 28°$$

$PL = 8.0\ m$ [to 2 significant figures]

(b) KM [3]



Using the cosine rule,

$KM^2 = KL^2 + LM^2 - 2(KL)(LM)\cos(K\hat{L}M)$

$KM^2 = (15)^2 + (19)^2 - 2(15)(19)\cos(115°)$

$KM^2 = 225 + 361 - 570\cos(115°)$

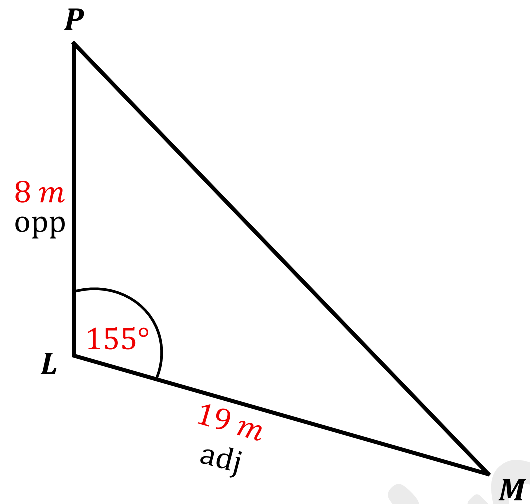
$KM^2 = 826.89$

$KM = \sqrt{826.89}$

$KM = 28.76$

$KM = 29\ m$ [to 2 significant figures]

(c) the angle of elevation of P from M [3]



$$\tan\theta = \frac{opp}{adj}$$

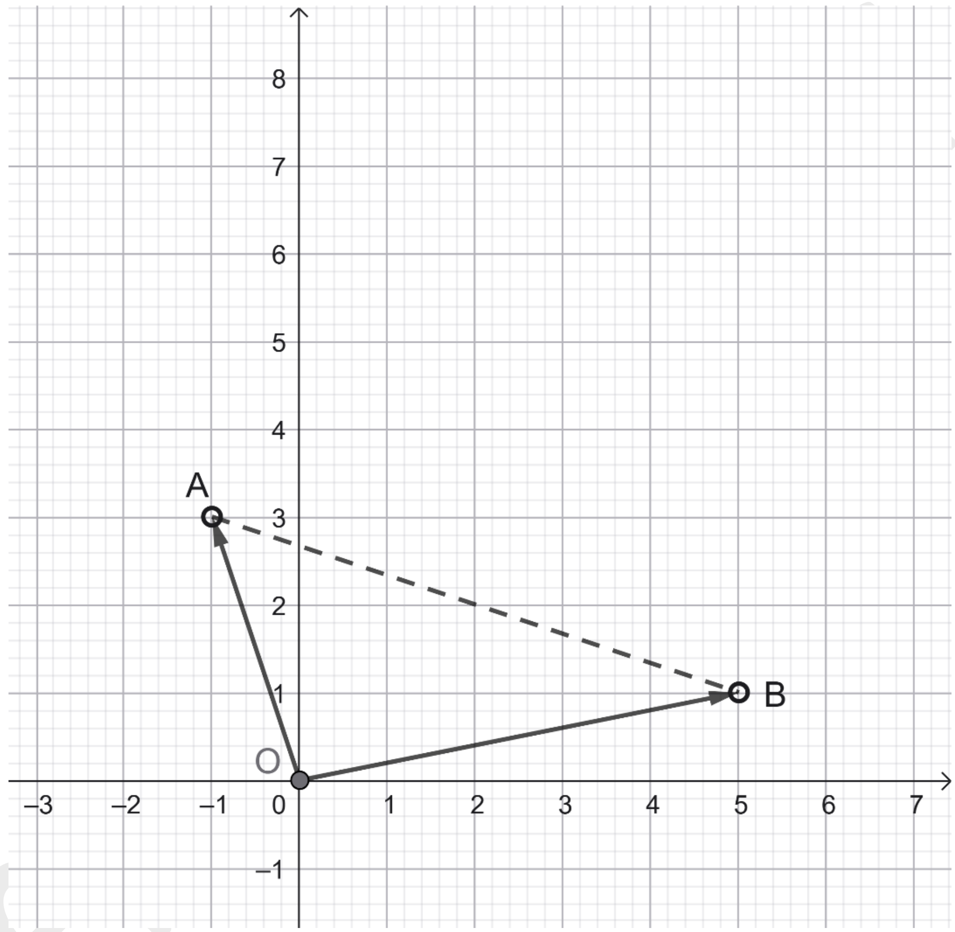
$$\tan P\hat{M}L = \frac{8}{19}$$

$$P\hat{M}L = \tan^{-1}\left(\frac{8}{19}\right)$$

$P\hat{M}L = 23°$ [to 2 significant figures]

Total: 15 marks

11. (a) The diagram below shows two position vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



(i) Write as a column vector, in the form $\begin{pmatrix} x \\ y \end{pmatrix}$:

(a) $\overrightarrow{OA}$ [1]

$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

(b) $\overrightarrow{OB}$ [1]

$\overrightarrow{OB} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(c) $\overrightarrow{BA}$ [2]

$$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$$
$$= -\begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} -5 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$
$$= \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

(ii) Given that G is the mid-point of the line AB, write as a column vector in the form $\begin{pmatrix} x \\ y \end{pmatrix}$:

(a) $\overrightarrow{BG}$ [1]

$$\overrightarrow{BG} = \frac{1}{2}\overrightarrow{BA}$$

$$= \frac{1}{2}\begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

(b) $\overrightarrow{OG}$ [1]

$$\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BG}$$

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

(b) L and M are two matrices where

$$L = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \text{ and } M = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$$

Evaluate

(i) $L + 2M$ [2]

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} + 2\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} -2 & 6 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 8 \\ 1 & 8 \end{pmatrix}$$

(ii) LM [2]

$$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \times -1) + (2 \times 0) & (3 \times 3) + (2 \times 2) \\ (1 \times -1) + (4 \times 0) & (1 \times 3) + (4 \times 2) \end{pmatrix}$$

$$= \begin{pmatrix} -3 + 0 & 9 + 4 \\ -1 + 0 & 3 + 8 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 13 \\ -1 & 11 \end{pmatrix}$$

(c) The matrix, Q, is such that $Q = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$.

(i) Find Q^{-1}. [2]

Q is of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a = 4, b = 2, c = 1, d = 1$.

$$\therefore Q^{-1} = \frac{1}{|Q|} Adj\ Q$$

$$= \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{(4)(1) - (2)(1)} \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$$

$$= \frac{1}{4 - 2} \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{2}{2} \\ -\frac{1}{2} & \frac{4}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -1 \\ -\frac{1}{2} & 2 \end{pmatrix}$$

(ii) Using a matrix method, find the values of x and y in the equation

$$\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}.$$ [3]

$4x + 2y = 8$ — Equation 1

$x + y = 3$ — Equation 2

Rearranging Equation 2 to get Equation 3:

$x = 3 - y$ — Equation 3

Substituting Equation 3 into Equation 1:

$4(3 - y) + 2y = 8$

$12 - 4y + 2y = 8$

$12 - 2y = 8$

$12 - 8 = 2y$

$4 = 2y$

$\frac{4}{2} = y$

$2 = y$

$y = 2$

Substituting $y = 2$ into Equation 3 to find x:

$x = 3 - (2)$

$x = 3 - 2$

$x = 1$

$\therefore x = 1$ and $y = 2$.

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.