CSEC Mathematics

June 2018 – Paper 2

Solutions

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, evaluate EACH of the following, giving your answer to two decimal places.

(i) $73.18 - 5.23 \times 9.34$ [1]

$73.18 - 5.23 \times 9.34$

$= 73.18 - (5.23 \times 9.34)$

$= 73.18 - 48.8482$

$= 24.3318$

$= 24.33$ (to 2 decimal places)

(ii) $\frac{3.1^2}{6.17} + 1.12$ [1]

$\frac{3.1^2}{6.17} + 1.12$

$= \frac{9.61}{6.17} + 1.12$

$= 1.5575 + 1.12$

$= 2.6775$

$= 2.68$ (to 2 decimal places)

(b) Jenny works at Sammy's Restaurant and is paid according to the rates in the following table.

Jenny's weekly wage agreement
Basic wage \$600.00 PLUS \$0.90 for each customer served

In a week when Jenny serves n customers, her weekly wage, W_j, in dollars, is given by the formula

$$W_j = 600 + 0.90n$$

(i) Determine Jenny's weekly wage if she served 230 customers. [1]

$W_j = 600 + 0.90n$

When $n = 230$,

$W_j = 600 + (0.9 \times 230)$

$W_j = 600 + 207$

$W_j = 807$

$\therefore$ Jenny's weekly wage is \$807 if she serves 230 customers.

(ii) In a **good** week, Jenny's wage is $1 000.00 or more. What is the LEAST number of customers that Jenny must serve in order to have a **good** week? [2]

$W_J = 600 + 0.9n$ and $W_J \geq 1000$

Hence,

$600 + 0.9n \geq 1000$

$0.9n \geq 1000 - 600$

$0.9n \geq 400$

$n \geq \frac{400}{0.9}$

$n \geq 444.4$

We need to round up in order for Jenny to earn at least $1000.

∴ The least number of customers Jenny must serve is 445.

(iii) At the same restaurant, Shawna is paid a weekly wage of $270.00 plus $1.50 for each customer she serves.

If W_s is Shawna's weekly wage, in dollars, write a formula for calculating Shawna's weekly wage when she serves m customers. [1]

W_s = Shawna's salary and m = number of customers

$\therefore W_s = 270 + 1.5m$

(iv) In a certain week, Jenny and Shawna received the same wage for serving the same number of customers.

How many customers did they EACH serve? [3]

$W_J = 600 + 0.9n$ and $W_s = 270 + 1.5m$

If the number of customers served by both Jenny and Shawna is the same, then $n = m$.

Then,

$600 + 0.9n = 270 + 1.5n$

$600 - 270 = 1.5n - 0.9n$

$330 = 0.6n$

$n = \frac{330}{0.6}$

$n = 550$

$\therefore$ Jenny and Shawn each served 550 customers.

Total: 9 marks

2. (a) Factorize, completely, EACH of the following expressions.

(i) $1 - 4h^2$ [1]

$1 - 4h^2 = (1 - 2h)(1 + 2h)$ [difference of two squares]

(ii) $pq - q^2 - 3p + 3q$ [2]

$pq - q^2 - 3p + 3q$

$= q(p - q) - 3(p - q)$

$= (q - 3)(p - q)$

(b) Solve EACH of the following equations.

(i) $\frac{3}{2}y = 12$ [1]

$\frac{3}{2}y = 12$

$3y = 12 \times 2$

$3y = 24$

$y = \frac{24}{3}$

$y = 8$

(ii) $2x^2 + 5x - 3 = 0$ [2]

$2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x(x + 3) - 1(x + 3) = 0$

$(2x - 1)(x + 3) = 0$

Either $2x - 1 = 0$ or $x + 3 = 0$

$2x = 1$ $x = -3$

$x = \frac{1}{2}$

(c) The quantities F, m, u, v and t are related according to the formula

$$F = \frac{m(v-u)}{t}$$

(i) Find the value of F when $m = 3, u = -1, v = 2$ and $t = 1$. [1]

$F = \frac{m(v-u)}{t}$

When $m = 3, u = -1, v = 2$ and $t = 1$

$F = \frac{3(2-(-1))}{1}$

$= 3(2 + 1)$

$= 3(3)$

$= 9$

$\therefore$ The value of $F = 9$.

(ii) Make v the subject of the formula. [2]

$F = \frac{m(v-u)}{t}$

$\frac{F}{1} = \frac{m(v-u)}{t}$

Cross-multiplying gives:

$Ft = m(v - u)$

$Ft = mv - mu$

$Ft + mu = mv$

$mv = Ft + mu$

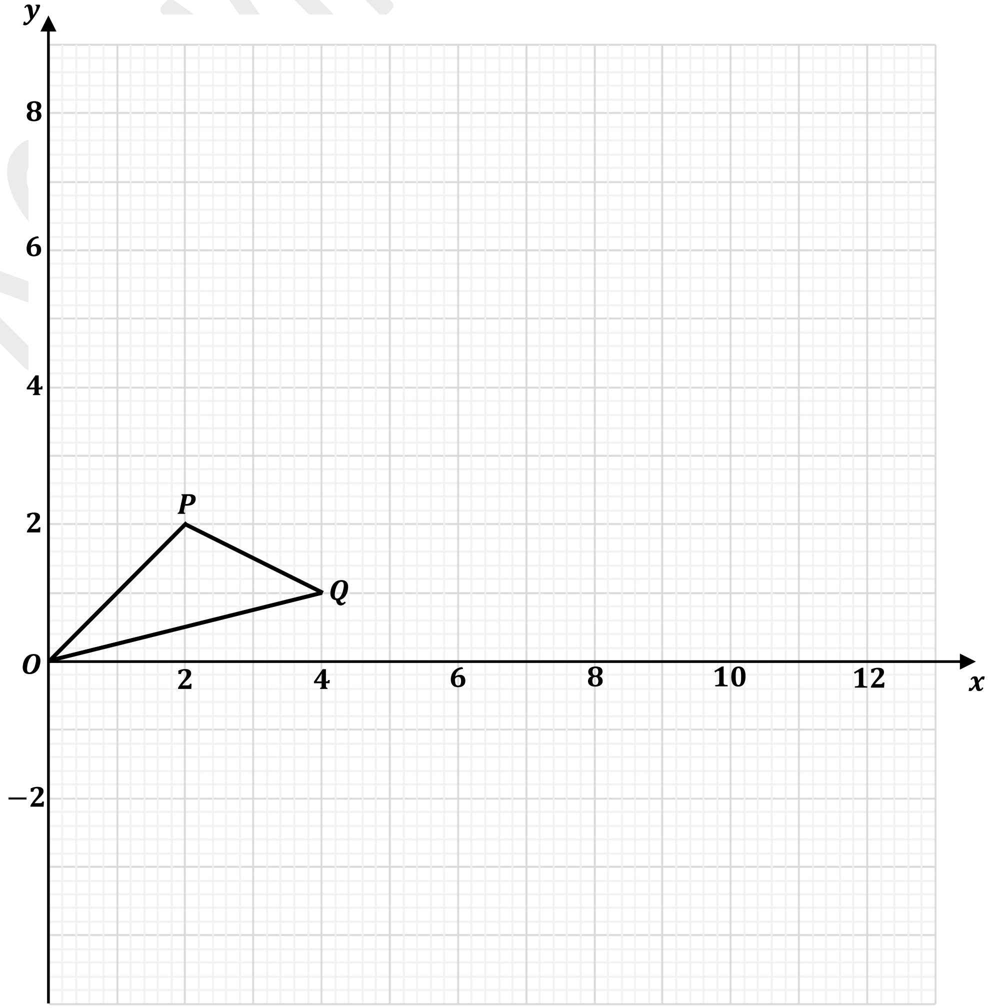
$v = \frac{Ft+mu}{m}$

Total: 9 marks

3. (a) Using a ruler, a pencil and a pair of compasses, construct the triangle ABC,

such that $AB = 8\ cm$, $\angle BAC = 30°$ and $AC = 10\ cm$. [4]

(b) The diagram below shows the triangle OPQ.



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(i) State the coordinates of the point Q. [1]

(ii) The line PQ is mapped to $P'Q'$ by an enlargement, centre O and scale factor 3. On the diagram above, draw the line $P'Q'$. [2]

(iii) The ΔOPQ undergoes a reflection in the line $y = 0$ to produce the image $O"P"Q"$. On the diagram above, draw the $\Delta O"P"Q"$. [2]

Total: 9 marks

4. (a) The function f with domain, $A = \{1, 2, 3\}$ is given by

$$f(x) = \frac{1}{2}x - 3.$$

(i) What is the value of $f(1)$? [1]

$f(x) = \frac{1}{2}x - 3$

When $x = 1$,

$f(1) = \frac{1}{2}(1) - 3$

$= \frac{1}{2} - 3$

$= -\frac{5}{2}$

(ii) Find the value of x for which $f(x) = -2$. [1]

$f(x) = -2$

So, we have,

$\frac{1}{2}x - 3 = -2$

$\frac{1}{2}x = -2 + 3$

$\frac{1}{2}x = 1$

$x = 1 \times 2$

$x = 2$

(iii) An ordered pair for the function is expressed in the form (a, b). Using your answers to (a)(i) and (a)(ii), or otherwise, list the ordered pairs for the function, f. [2]

From (a)(i), $f(1) = -\frac{5}{2}$

From (a)(ii), $f(2) = -2$

$f(3) = \frac{1}{2}(3) - 3$

$= \frac{3}{2} - 3$

$= -\frac{3}{2}$

So, the list of ordered pairs is $\left\{\left(1, -\frac{5}{2}\right), (2, -2), \left(3, -\frac{3}{2}\right)\right\}$.

(iv) Explain why $f(x) \neq 5$ for the function specified **on page 13.** [1]

If $f(x) = 5$, then

$\frac{1}{2}x - 3 = 5$

$\frac{1}{2}x = 5 + 3$

$\frac{1}{2}x = 8$

$x = 8 \times 2$

$x = 16$

But 16 is out of the domain, A, of the function which includes $\{1, 2, 3\}$.

$\therefore f(x)$ cannot be equal to 5.

(b) (i) Solve the inequalities

(a) $3x - 1 < 11$ [1]

$3x - 1 < 11$

$3x < 11 + 1$

$3x < 12$

$x < \frac{12}{3}$

$x < 4$

(b) $2 \leq 3x - 1$ [1]

$2 \leq 3x - 1$

$2 + 1 \leq 3x$

$3 \leq 3x$

$\frac{3}{3} \leq x$

$1 \leq x$

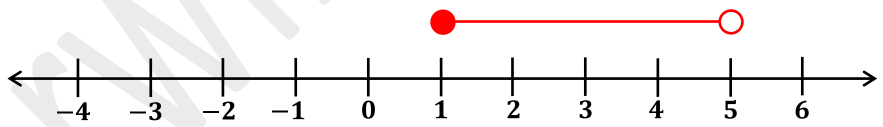
$x \geq 1$

(ii) Represent the solution to $2 \leq 3x - 1 < 11$ on the number line shown below. [2]

The two inequalities are $x < 4$ and $x \geq 1$.

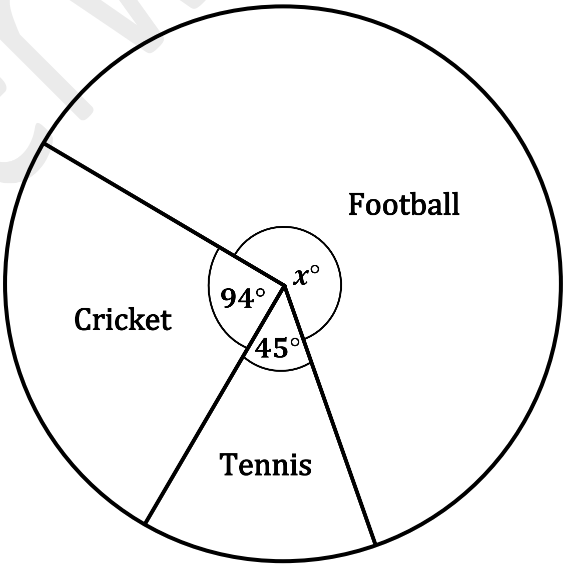
The solution to $2 \leq 3x - 1 < 11$ is $1 \leq x < 4$.

The inequality $1 \leq x < 4$ is represented on the number line below:



Total: 9 marks

5. (a) Students in a group were asked to name their favourite sport. Their responses are shown on the pie chart below.



(i) Calculate the value of x. [1]

The sum of the angles in a circle add up to 360°.

$x = 360° - (94° + 45°)$

$= 221°$

(ii) What percentage of the students chose cricket? [1]

The percentage of students who chose cricket

$= \frac{Angle\ corresponding\ to\ cricket}{360°} \times 100\%$

$= \frac{94°}{360°} \times 100\%$

$= 26.11\%$ (to 2 decimal places)

(iii) Given that 40 students chose tennis, calculate the TOTAL number of students in the group. [2]

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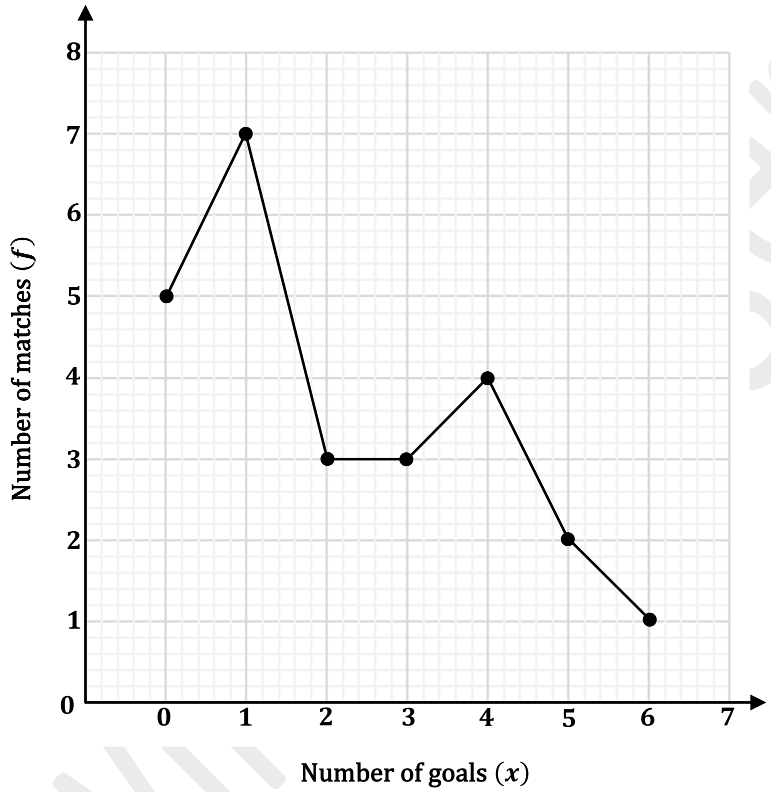
(b) The diagram below shows a frequency polygon of the number of goals scored by a football team in 25 matches.



(i) Complete the following table using the information in the diagram. [1]

Number of matches (f)	5	7		3	4		1
Number of goals scored (x)	0	1	2	3	4	5	6

(ii) What is the modal number of goals scored by the team? [1]

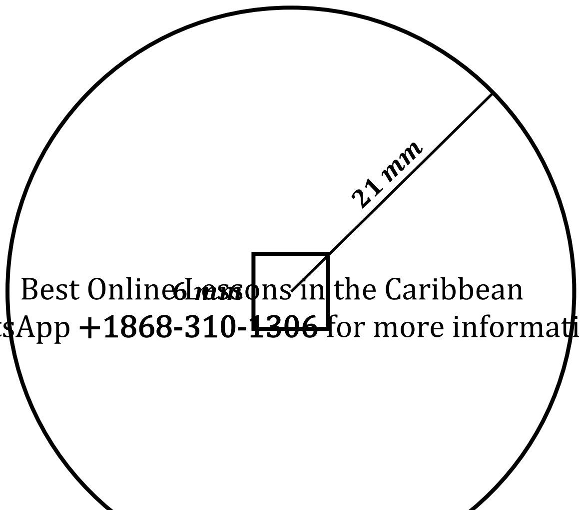
(iii) Determine the median number of goals scored by the team. [1]

(iv) Calculate the mean number of goals scored by the team. [2]

Total: 9 marks

6. (a) **In this question, take the value of π to be $\frac{22}{7}$.**

The diagram below, **not drawn to scale**, shows the cross-section of a circular metal disc of radius 21 mm. A square hole with sides 6 mm is located at the centre of the disc.



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Calculate

(i) the circumference of the disc [1]

Circumference of the disc $= 2\pi r$

$= 2 \times \frac{22}{7} \times 21$

$= 132\ mm$

$\therefore$ The circumference of the disc is $132\ mm$.

(ii) the area, in mm^2, of the cross-section of the disc [2]

Area of whole disc $= \pi r^2$

$= \frac{22}{7} \times (21)^2$

$= 1386\ mm^2$

Area of square centre $= s \times s$

$= 6 \times 6$

$= 36\ mm^2$

Area of cross-section = Area of whole disc − Area of the square centre

$= 1386 - 36$

$= 1350\ mm^2$

$\therefore$ The area of the cross-section of the disc is $1350\ mm^2$.

(iii) Given that the thickness of the disc is 2 mm, calculate the **maximum** number of discs that can be constructed from 1 000 cm^3 of available material.

($1\ cm^3 = 1\ 000\ mm^3$) [3]

Volume of 1 disc = Cross-sectional area × thickness

$= 1350 \times 2$

$= 2700\ mm^3$

Converting to cm^3:

$1000\ mm^3 = 1\ cm^3$

$1\ mm^3 = \frac{1}{1000}\ cm^3$

$2700\ mm^3 = \frac{2700}{1000}\ cm^3$

$= 2.7\ cm^3$

Number of discs that can be made $= \frac{Volume\ of\ metal}{Volume\ of\ 1\ disc}$

$= \frac{1000}{2.7}$

$= 370.4$ discs (to 1 decimal place)

$\therefore$ 370 completed discs can be made.

(b) A globe is a scaled spherical representation of the earth. The actual length of the equator (LL) is 40 000 km and is represented on the globe by a piece of string of length 160 cm.

(i) What length of string would represent an actual distance of 500 km on the globe? [1]

(ii) The distance between Palmyra (P) and Quintec (Q) is represented on the globe by a string of length 25 cm. Calculate the value of PQ, the actual distance, in km, between P and Q. [2]

Total: 9 marks

7. A sequence of figures is made from squares of unit length. The first three figures in the sequence are shown below.

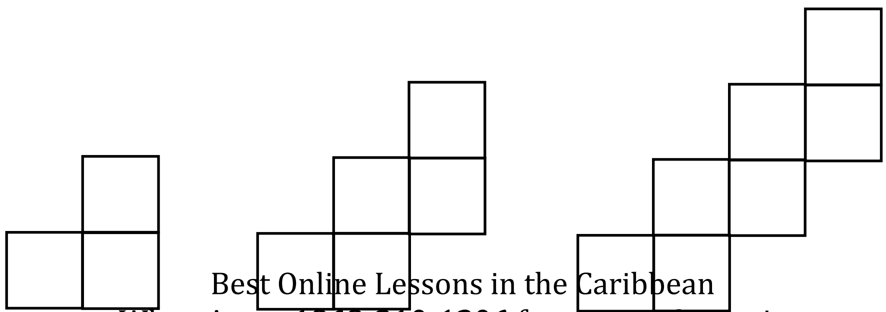


Figure 1 **Figure 2** **Figure 3**

(a) Draw Figure 4 of the sequence. [2]

(b) Study the pattern of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures **on page 22**. Some rows have not been included in the table.

Complete the rows numbered (i), (ii) and (iii).

	Figure	Number of Squares in Figure (S)	Perimeter of Figure (P)	
	1	3	8	
	2	5	12	
	3	7	16	
(i)	4			[2]
	⋮	⋮	⋮	
(ii)		43		[2]
(iii)	n			[2]

(c) Determine the relationship between the number of squares, S, and the perimeter, P, of a figure. [2]

Total: 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The diagram **on page 25** shows six points of the function $y = 3x + \frac{1}{x}$. The coordinates of these six points are given in the table below.

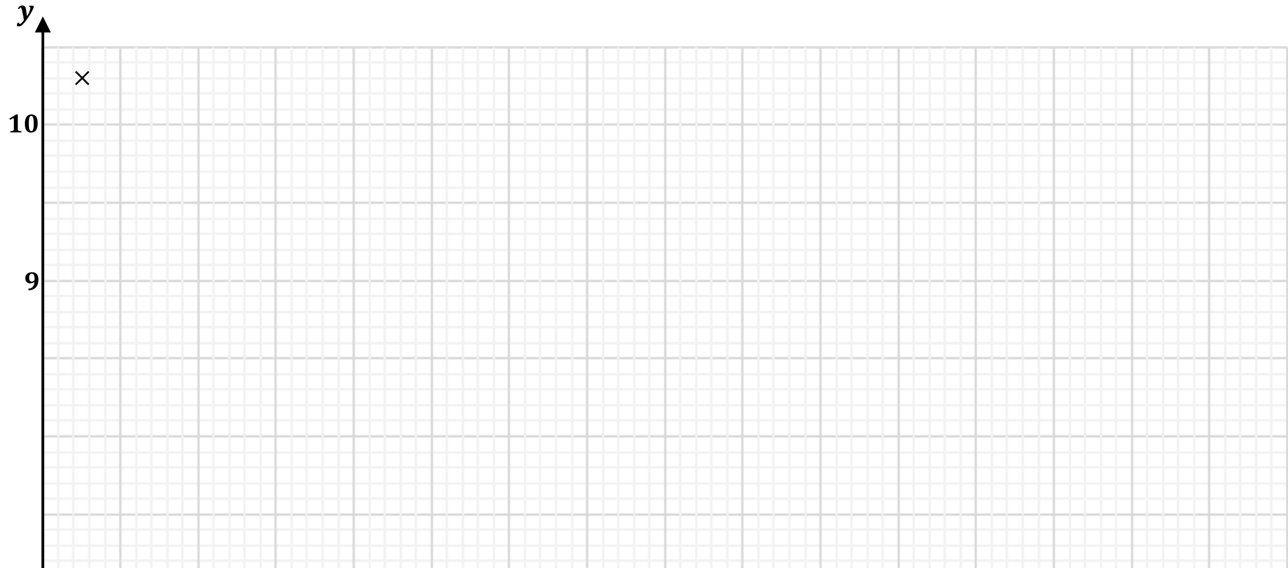
x	0.1	0.2	0.5	1	1.5	2	2.2	2.5
y	10.3	5.6		4	5.2		7.1	7.9

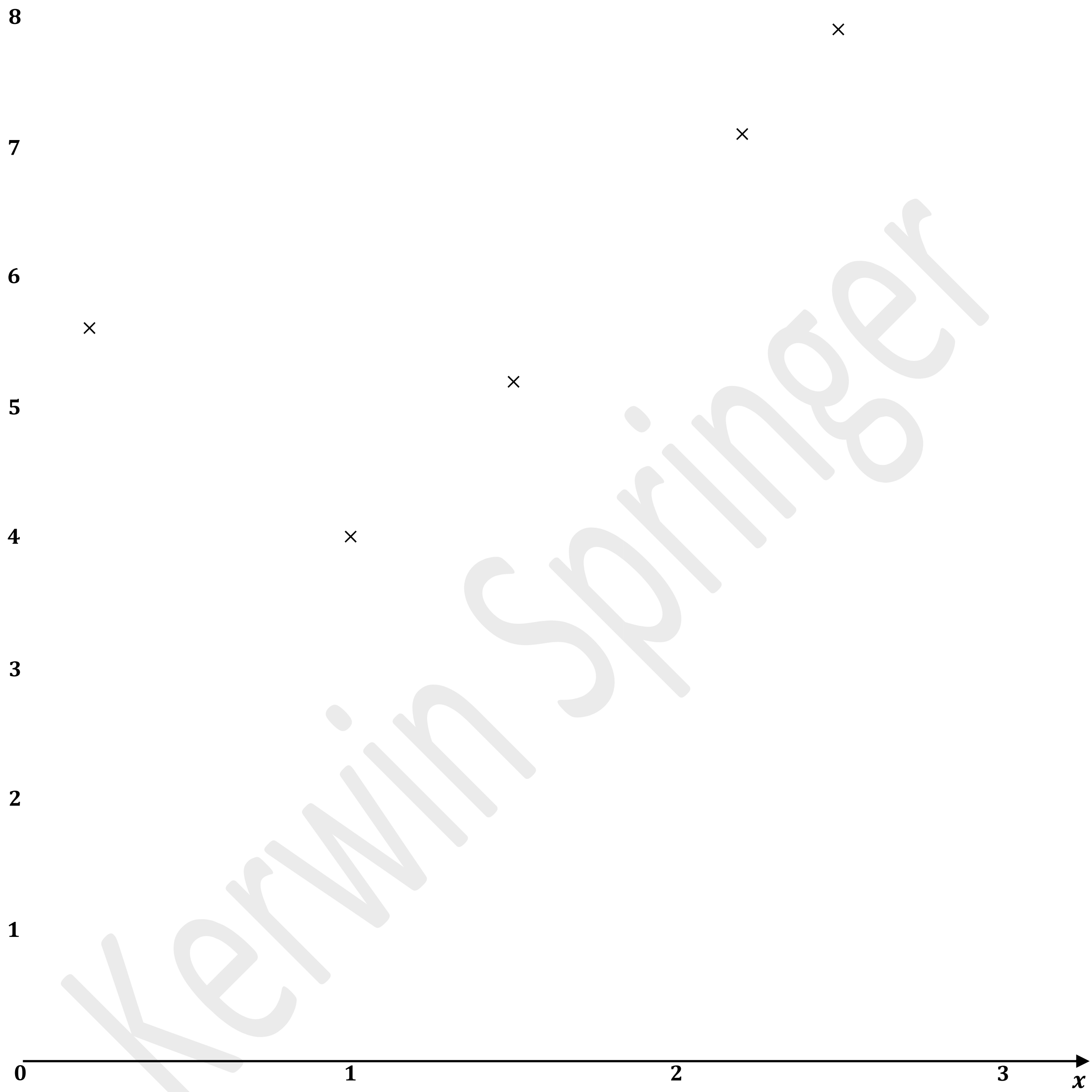
(i) Complete the table above by calculating and inserting the missing values of y. [2]

(ii) On the diagram **on page 25**, the ordered pairs shown in the table have been plotted except for the missing ones. Using your answers in (a)(i), plot the missing points and connect all the points with a smooth curve. [2]

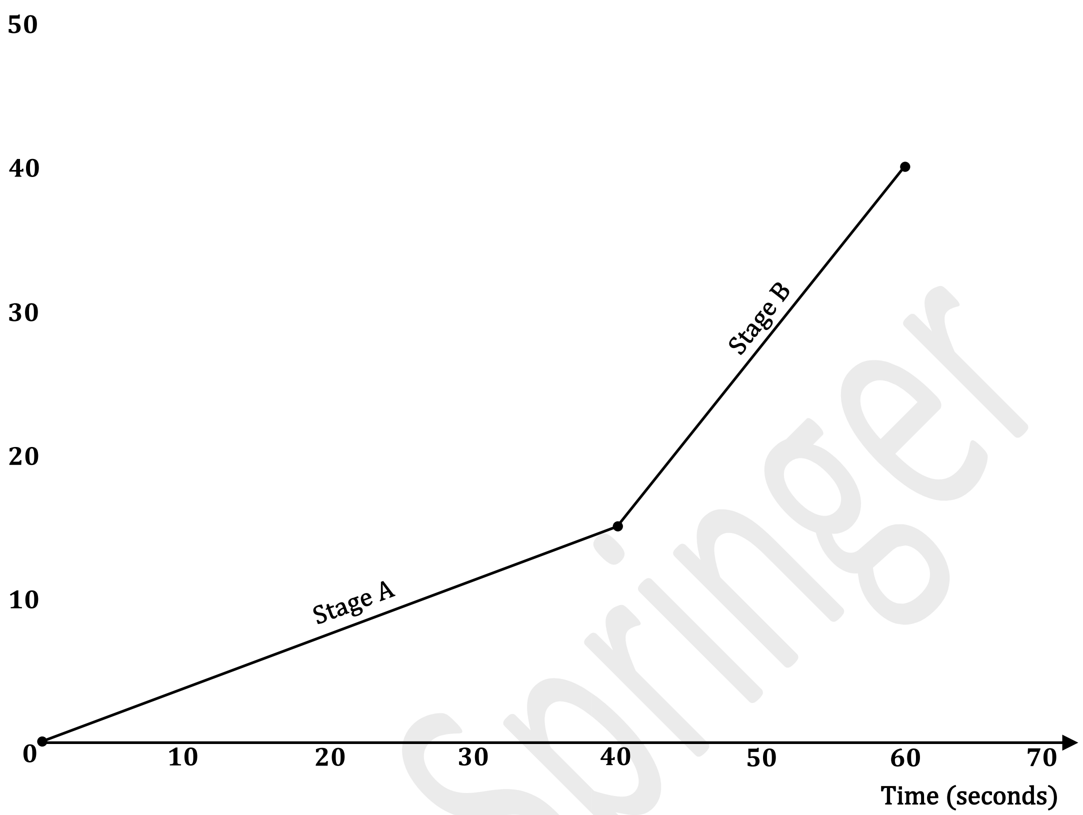
(iii) By drawing an appropriate straight line on the diagram **on page 25**, find approximate solutions to the equation [3]

$$3x + \frac{1}{x} = 6$$





(b) The speed-time graph below shows information on the first 60 seconds of a car's journey.



(i) Calculate the acceleration, in ms^{-2}, of the car during Stage B. [1]

(ii) Calculate the average speed of the car during Stage B. [3]

(iii) At time $t = 60$ seconds, the car starts to slow down with a uniform deceleration of 2.5 ms^{-2}.

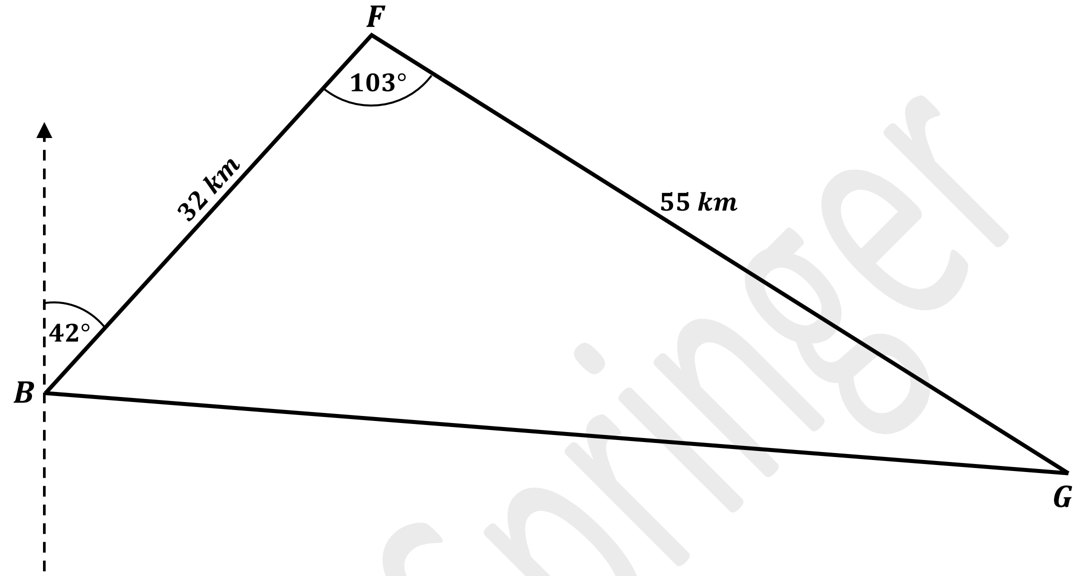
Determine how long it will take the car to come to rest. [1]

Total: 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below, **not drawn to scale**, shows the relative positions of three

reservoirs B, F and G, all on level ground. The distance $BF = 32$ km, $FG =$ 55 km, $\angle BFG$ is 103° and F is on a bearing of 042° from B.

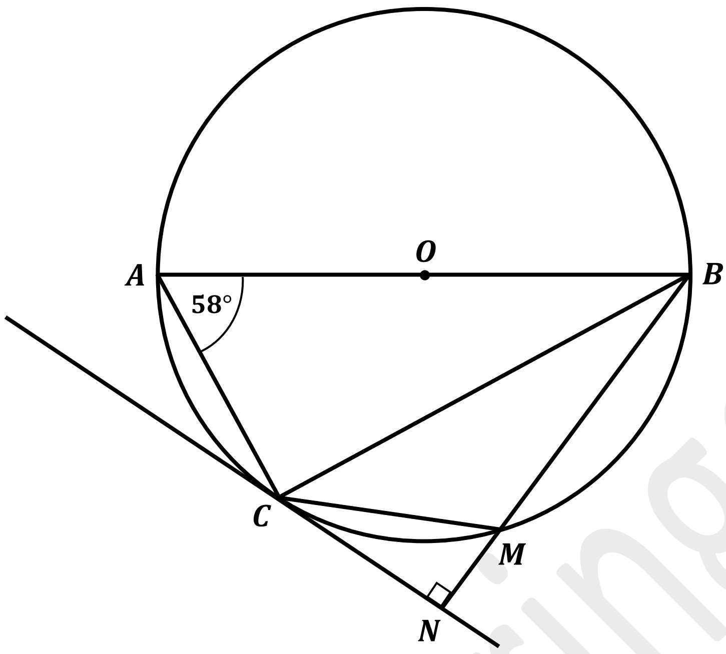


(i) Determine the bearing of B from F. [1]

(ii) Calculate the distance BG, giving your answer to one decimal place. [2]

(iii) Calculate, to the nearest degree, the bearing of G from B. [3]

(b) The diagram below, **not drawn to scale**, shows a circle, with centre O. The points A, B, C and M are on the circumference. The straight line CN is a tangent to the circle at the point C and is perpendicular to BN.



Determine, giving a reason for your answer,

(i) $A\hat{B}C$ [2]

(ii) $C\hat{M}B$ [2]

(iii) $N\hat{C}M$ [2]

Total: 12 marks

VECTORS AND MATRICES

10. (a) A transformation, T, is defined by the matrix

$$T = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$$

The point $A(-2, 3)$ is mapped on to the point $A'(a, b)$ under T.

(i) Find the value of a and of b. [2]

(ii) Determine the transformation matrix that maps A' to A. [2]

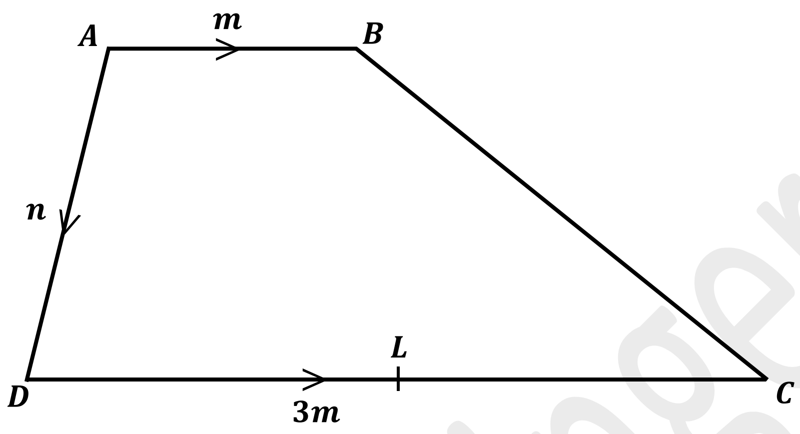
(iii) Another transformation, P, is defined by the matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$.

(a) Find the single 2×2 matrix that represents the combined transformation of T followed by P. [2]

(b) Hence, find the image of the point $(1, 4)$ under this combined transformation. [1]

(b) The diagram below, **not drawn to scale**, shows a quadrilateral $ABCD$ in which

$\overrightarrow{AB} = \boldsymbol{m}$, $\overrightarrow{DC} = 3\boldsymbol{m}$ and $\overrightarrow{AD} = \boldsymbol{n}$.



(i) Complete the statement below on the geometric properties of the following vectors.

$\overrightarrow{AB}$ and $\overrightarrow{DC}$ are ………………………………………………. and

$|\overrightarrow{AB}|$ is ………………………………………………. times $|\overrightarrow{CD}|$. [2]

(ii) Express $\overrightarrow{BC}$ in terms of m and n. [1]

(iii) L is the midpoint of $\overrightarrow{CD}$. Find $\overrightarrow{BL}$ in terms of m and n. [2]

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.