CSEC Mathematics

June 2011 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $\frac{2\frac{1}{4}+1\frac{1}{8}}{4\frac{1}{2}}$ expressing your answer as a fraction [3]

$\text{Numerator} = 2\frac{1}{4} + 1\frac{1}{8}$

$= \frac{9}{4} + \frac{9}{8}$

$= \frac{18+9}{8}$

$= \frac{27}{8}$

$\text{Denominator} = 4\frac{1}{2}$

$= \frac{9}{2}$

$\text{Numerator} \div \text{Denominator} = \frac{27}{8} \div \frac{9}{2}$

$= \frac{27}{8} \times \frac{2}{9}$

$= \frac{3}{4}$

$\therefore \frac{2\frac{1}{4}+1\frac{1}{8}}{4\frac{1}{2}} = \frac{3}{4}$

(ii) $3.96 \times 0.25 - \sqrt{0.0256}$ [3]

Using a calculator,

$3.96 \times 0.25 - \sqrt{0.0256} = 0.99 \times 0.16$

$= 0.83$

(b) The table below shows Pamela's shopping bill. Some of the information was not included.

Items	Quantity	Unit Price $	Total Cost $
Rice	$6\frac{1}{2}\,kg$	2.40	W
Potatoes	4 bags	X	52.80
Milk	Y cartons	2.35	14.10
Sub Total			**82.50**
Z % VAT			9.90
TOTAL			**92.40**

Calculate the values of W, X, Y and Z. [5]

$W = 6\frac{1}{2} \times \$2.40$

$= \$15.60$

$X = \frac{\$52.80}{4}$

$= \$13.20$

$Y = \frac{\$14.10}{2.35}$

$= 6$

$Z = \frac{VAT}{Sub\ Total} \times 100$

$= \frac{9.90}{82.50} \times 100$

$= 12$

Total: 11 marks

2. (a) Write as a single fraction in its lowest terms

$$\frac{x-2}{3} + \frac{x+1}{4}$$ [3]

$$\frac{x-2}{3} + \frac{x+1}{4}$$

$$= \frac{4(x-2)+3(x+1)}{12}$$

$$= \frac{4x-8+3x+3}{12}$$

$$= \frac{7x-5}{12}$$

(b) The binary operation $*$ is defined by

$$a * b = (a+b)^2 - 2ab$$

Calculate the value of $3 * 4$. [2]

$a * b = (a+b)^2 - 2ab$

$3 * 4 = (3+4)^2 - 2(3)(4)$

$= (7)^2 - 24$

$= 49 - 24$

$= 25$

(c) Factorise completely

(i) $xy^3 + x^2y$ [2]

$xy^3 + x^2y$

$= xy(y^2 + x)$

(ii) $2mh - 2nh - 3mk + 3nk$ [2]

$2mh - 2nh - 3mk + 3nk$

$= 2h(m - n) - 3k(m - n)$

$= (m - n)(2h - 3k)$

(d) The table below shows corresponding values of the variables x and y, where y varies directly as x.

$\boldsymbol{x}$	2	5	b
$\boldsymbol{y}$	12	a	48

Calculate the values of a and b. [3]

We are given that y varies directly as x.

So, we have,

$y \propto x$

$y = kx$

When $x = 2$ and $y = 12$,

$12 = k(2)$

$k = \frac{12}{2}$

$k = 6$

Hence, the equation can now be expressed as $y = 6x$.

When $x = 5$ and $y = a$,

$a = 6(5)$

$= 30$

When $x = b$ and $y = 48$,

$48 = 6b$

$b = \frac{48}{6}$

$= 8$

$\therefore a = 30$ and $b = 8$

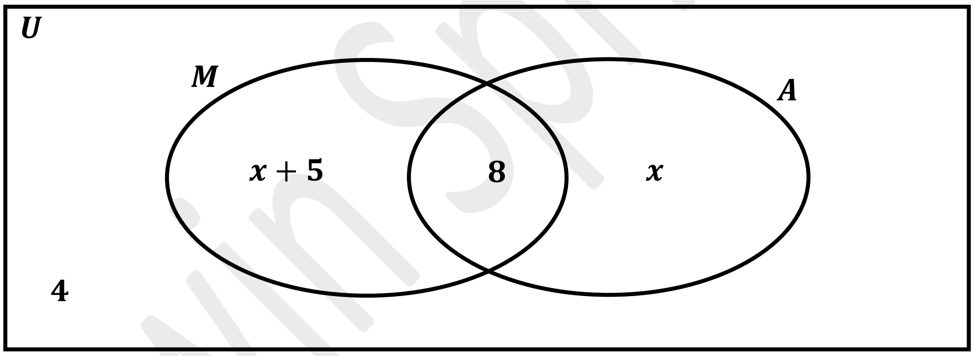
Total: 12 marks

3. (a) The Venn diagram below shows the number of students who study Music and Art in a class of 35 students.

U = {students in the class}

M = {students who study Music}

A = {students who study Art}



(i) How many students study neither Art nor Music? [1]

From the Venn diagram, $(M \cup A)' = 4$.

$\therefore$ 4 students neither studied Art nor Music.

(ii) Calculate the value of x. [3]

$n(U) = (x + 5) + 8 + x + 4$

$= 2x + 17$

The total number of students is 35.

So, we have,

$2x + 17 = 35$

$2x = 35 - 17$

$2x = 18$

$x = \frac{18}{2}$

$x = 9$

$\therefore$ The value of $x = 9$.

(iii) Hence, state the number of students who study Music only. [1]

$n(M) = x + 5$

$n(M) = 9 + 5$

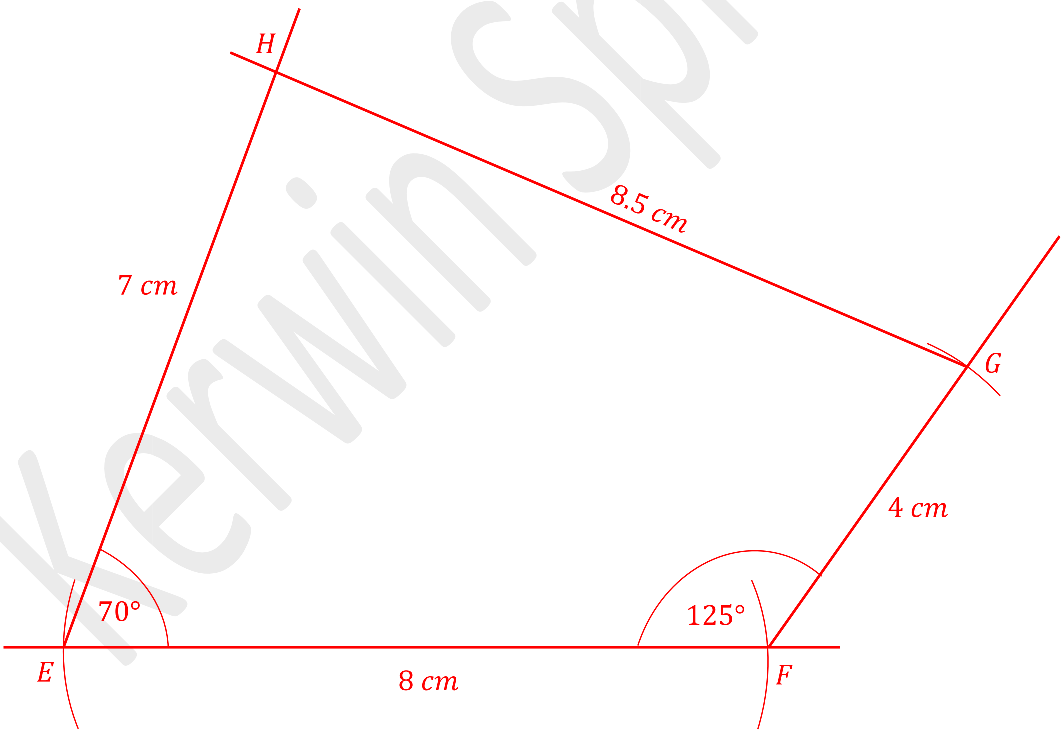
$n(M) = 14$

$\therefore$ The number of students who study Music only is 14 students.

(b)(i) **Using a ruler, pencil, a pair of compasses and a protractor**, draw accurately a quadrilateral $EFGH$ using the following measurements: [5]

$EF = 8\ cm$ $\quad \angle EFG = 125°$ $\quad FG = 4\ cm$ $\quad \angle HEF = 70°$ $\quad EH = 7\ cm$

The construction of the quadrilateral is shown below:



(ii) Measure and state in centimetres, the length of GH. [1]

By measurement using a ruler, $GH = 8.5\ cm$.

Total: 11 marks

4. (a) (i) Solve the inequality: $5 - 2x < 9$ [2]

$5 - 2x < 9$

$-2x < 9 - 5$

$-2x < 4$

$x > \frac{4}{-2}$

$x > -2$

(ii) If x is an integer, determine the SMALLEST value of x that satisfies the inequality in (a)(i) above. [1]

Now $x > -2$ and $x \in \mathbb{Z}$.

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

So, $x_{min} = -1$.

$\therefore$ The smallest integer that satisfies the inequality is the next integer that is greater than -2 which is $x = -1$.

(b) In this question, use $\pi = \frac{22}{7}$.

(i) A piece of wire is bent to form a square of area $121\ cm^2$.

Calculate:

(a) the length of each side of the square

Area of square $= l^2$

We are given that the area of the square $= 121\ cm^2$.

So, we have,

$l^2 = 121$

$l = \sqrt{121}$

$l = 11\ cm$

$\therefore$ The length of each side of the square is $11\ cm$.

(b) the perimeter of the square [3]

Perimeter of the square $= 4 \times l$

$= 4 \times 11$

$= 44\ cm$

$\therefore$ The perimeter of the square is $44\ cm$.

(ii) The same piece of wire is bent to form a circle.

Calculate:

(a) the radius of the circle

Since the circumference of the circle is the same as the perimeter of the square, then circumference of circle $= 44\ cm$

So, we have,

$$2\pi r = 44$$

$$2 \times \frac{22}{7} \times r = 44$$

$$\frac{44}{7} \times r = 44$$

$$r = 44 \div \frac{44}{7}$$

$$r = \frac{44}{1} \times \frac{7}{44}$$

$$r = 7\ cm$$

$\therefore$ The radius of the circle, $r = 7\ cm$.

(b) the area of the circle [4]

Area of circle $= \pi r^2$

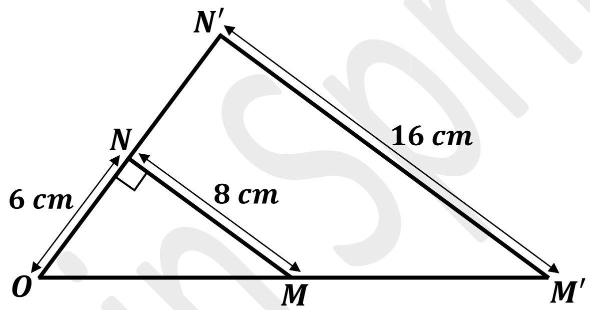
$= \frac{22}{7} \times (7)^2$

$= 154\ cm^2$

$\therefore$ The area of the circle is $54\ cm^2$.

Total: 10 marks

5. (a) The diagram below, **not drawn to scale**, shows ΔOMN, and its image, $\Delta OM'N'$ under an enlargement with centre, O, and scale factor, k. Angle $OMN = 90°$.



Using the dimensions shown on the diagram, calculate

(i) the value of k, the scale factor of the enlargement [1]

Scale factor $= \frac{Image\ Length}{Object\ Length}$

$= \frac{N'M'}{NM}$

$= \frac{16}{8}$

$= 2$

(ii) the length of OM [1]

Using Pythagoras' Theorem,

$OM^2 = ON^2 + NM^2$

$= (6)^2 + (8)^2$

$= 36 + 64$

$= 100$

$OM = \sqrt{100}$

$= 10\ cm$

(iii) the length of OM' [2]

The scale factor of enlargement is 2.

So,

$\frac{OM'}{OM} = 2$

$\frac{OM'}{10} = \frac{2}{1}$

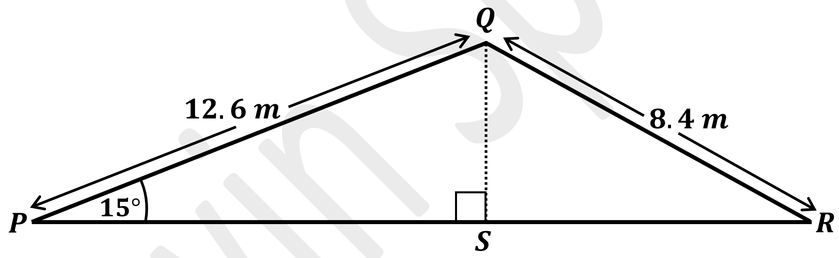
$OM' = 2 \times 10$

$OM' = 20\ cm$

$\therefore$ The length of OM' is $20\ cm$.

(b) The diagram below, **not drawn to scale**, shows ΔPQR, which represents the cross section of a roof. QS is perpendicular to PSR.

$PQ = 12.6$ metres $\quad QR = 8.4$ metres $\quad \angle QPR = 15°$



Using the dimensions shown on the diagram, calculate, correct to 3 significant figures:

(i) the length of QS [2]

Consider ΔPQS.

$\sin Q\hat{P}R = \frac{opp}{adj}$

$\sin Q\hat{P}R = \frac{QS}{PQ}$

$\sin 15° = \frac{QS}{12.6}$

$QS = 12.6 \times \sin 15°$

$QS = 3.26\ m$ (to 3 significant figures)

$\therefore$ The length of QS is $3.26\ m$.

(ii) the measure of $\angle RQS$ [3]

Consider ΔRQS.

$\cos R\hat{Q}S = \frac{adj}{hyp}$

$\cos R\hat{Q}S = \frac{QS}{PS}$

$\cos R\hat{Q}S = \frac{3.26}{8.4}$

$R\hat{Q}S = \cos^{-1}\left(\frac{3.26}{8.4}\right)$

$R\hat{Q}S = 67.2°$ (to 3 significant figures)

$\therefore$ The measure of $\angle RQS$ is $67.2°$.

(iii) the area of ΔPQR [3]

The sum of angles in a triangle add up to 180°.

$P\hat{Q}S = 180° - (90° + 15°)$

$= 180° - 105°$

$= 75°$

Now,

$P\hat{Q}R = R\hat{Q}S + P\hat{Q}S$

$= 67.2° + 75°$

$= 142.21°$

Hence,

Area of $\Delta PQR = \frac{1}{2}(QP)(QR)\sin P\hat{Q}R$

$= \frac{1}{2}(12.6)(8.4)\sin 142.21°$

$= 32.4\ cm^2$ (to 3 significant figures)

$\therefore$ The area of ΔPQR is $32.4\ cm^2$.

Total: 12 marks

STUDENT HUB

6. (a) The functions f and g are defined by

$$f(x) = 6x + 8 \quad ; \quad g(x) = \frac{x-2}{3}$$

(i) Calculate the value of $g\left(\frac{1}{2}\right)$. [2]

$$g(x) = \frac{x-2}{3}$$

$$g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-2}{3}$$

$$= \frac{-\frac{3}{2}}{3}$$

$$= -\frac{3}{2} \div 3$$

$$= -\frac{3}{2} \times \frac{1}{3}$$

$$= -\frac{1}{2}$$

(ii) Write an expression for $gf(x)$ in its simplest form. [2]

$gf(x) = g[f(x)]$

$= g(6x + 8)$

$= \frac{(6x+8)-2}{3}$

$= \frac{6x+8-2}{3}$

$= \frac{6x+6}{3}$

$= \frac{3(2x+2)}{3}$

$= 2x + 2$

$\therefore gf(x) = 2x + 2$

(iii) Find the inverse function $f^{-1}(x)$. [2]

$f(x) = 6x + 8$

Let $y = f(x)$.

$y = 6x + 8$

Interchange variables x and y.

$x = 6y + 8$

Make y the subject.

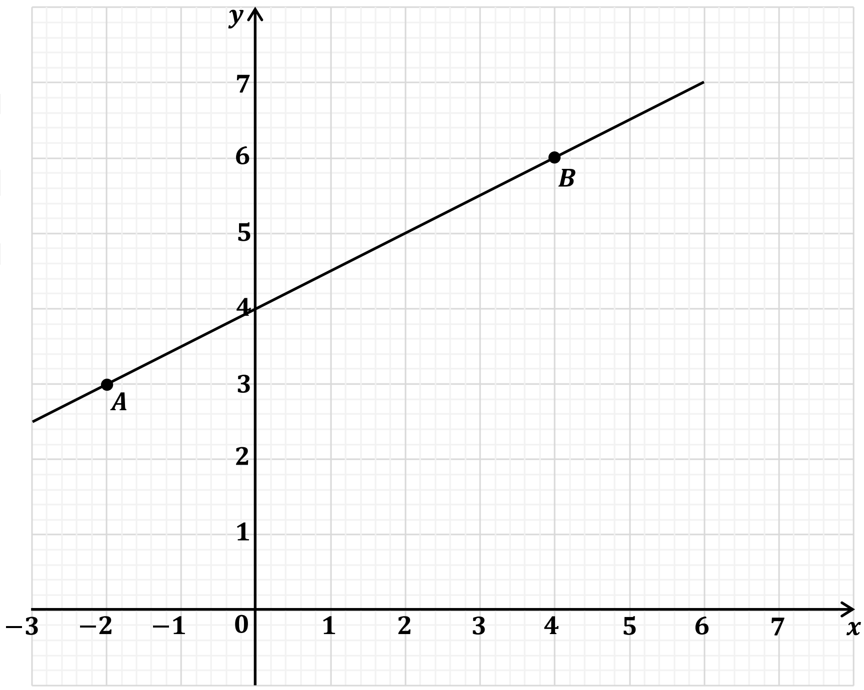
$x - 8 = 6y$

$\frac{x-8}{6} = y$

$y = \frac{x-8}{6}$

$\therefore f^{-1}(x) = \frac{x-8}{6}$

(b) The diagram below shows the line segment which passes through the points A and B.



Determine

(i) the coordinates of A and B [2]

From the graph,

The coordinate of A is $(-2, 3)$.

The coordinate of B is $(4, 6)$.

(ii) the gradient of the line segment AB [2]

Points are $A(-2, 3)$ and $B(4, 6)$.

Now,

Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-3}{4-(-2)}$

$= \frac{3}{6}$

$= \frac{1}{2}$

$\therefore$ The gradient of the line segment AB is $\frac{1}{2}$.

(iii) the equation of the line which passes through A and B. [2]

Substituting $m = \frac{1}{2}$ and point $A(-2, 3)$ into $y - y_1 = m(x - x_1)$ gives,

$y - 3 = \frac{1}{2}\left(x - (-2)\right)$

$y - 3 = \frac{1}{2}(x + 2)$

$y - 3 = \frac{1}{2}x + 1$

$y = \frac{1}{2}x + 1 + 3$

$y = \frac{1}{2}x + 4$

$\therefore$ The equation of the line passing through A and B is $y = \frac{1}{2}x + 4$.

Total: 12 marks

7. The table below shows the distribution of the masses of 100 packages.

Mass (kg)	No. of Packages	Cumulative Frequency
1 – 10	12	12
11 – 20	28	40
21 – 30	30	
31 – 40	22	
41- 50	8	

(a) Copy and complete the table to show the cumulative frequency for the distribution. [2]

(b) **Using a scale of 2 cm to represent 10 kg on the x-axis** and **1 cm to represent 10 packages on the y-axis**, draw the cumulative frequency curve for the data. [5]

(c) Estimate from the graph

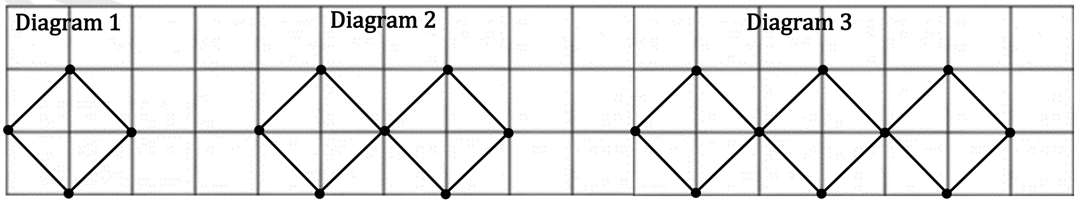
(i) the median mass of the packages [2]

(ii) the probability that a package, chosen at random, has a mass which is LESS than 35 kg. [3]

Total: 12 marks

8. **An answer sheet is provided for this question.**

The figure below shows the first three diagrams in a sequence. Each diagram is made up of sticks joined at the ends by thumb tacks. The sticks are represented by lines and the thumb tacks by dots. In each diagram, there are *t* thumb tacks and *s* sticks.



On the answer sheet provided:

(a) Draw the FOURTH diagram in the sequence. [2]

(b) (i) How many sticks are in the SIXTH diagram? [1]

(ii) How many thumb tacks are in the SEVENTH diagram? [1]

(c) Complete the table by inserting the missing values at the rows marked (i) and (ii).

	No. of sticks s	**Rule Connecting** t **and** s	**No. of Thumb Tacks** t	
	4	$1 + \left(\frac{3}{4} \times 4\right)$	4	
	8	$1 + \left(\frac{3}{4} \times 8\right)$	7	
	12	$1 + \left(\frac{3}{4} \times 12\right)$	10	
(i)	52	__________	__________	[2]
(ii)	__________	__________	55	[2]

(d) Write the rule, in terms of s and t, to show how t is related to s. [2]

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Solve the pair of simultaneous equations:

$y = x^2 - x + 3$

$y = 6 - 3x$ [5]

(b)(i) Express the function $f(x) = 4x^2 - 8x - 2$ in the form $a(x + h)^2 + k$,

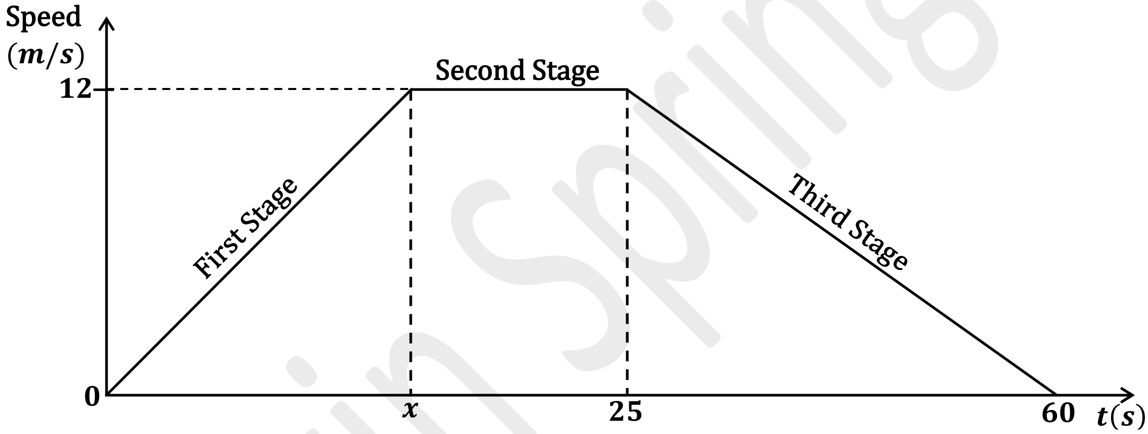
where a, h and k are constants. [2]

State

(ii) the minimum value of $f(x)$ [1]

(iii) the value of x for which $f(x)$ is a minimum. [1]

(c) The speed-time graph below, **not drawn to scale**, shows the three-stage journey of a racing car over a period of 60 seconds.



During the FIRST stage of the journey, the car increased its speed from 0 m/s to 12 m/s in x seconds accelerating at 0.6 m/s^2.

(i) Calculate the value of x. [2]

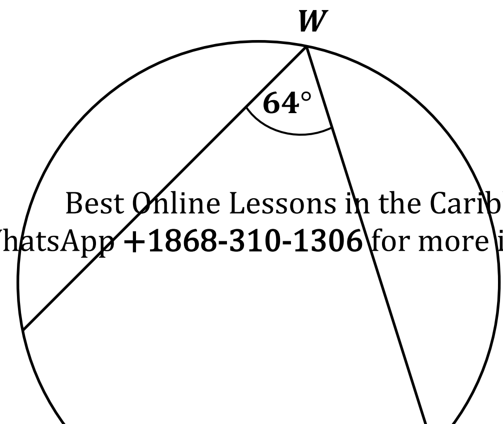
(ii) What is the gradient of the graph during the SECOND stage? Explain, in one sentence, what the car is doing during this stage. [2]

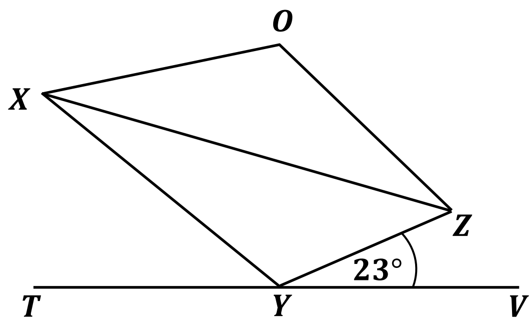
(iii) Calculate the distance travelled by the car on the THIRD stage of the journey. [2]

Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) In the diagram below, **not drawn to scale**, W, X, Y and Z are points on the circumference of a circle, centre O. TYV is a tangent to the circle at Y, $\angle XWZ = 64°$ and $\angle ZYV = 23°$.





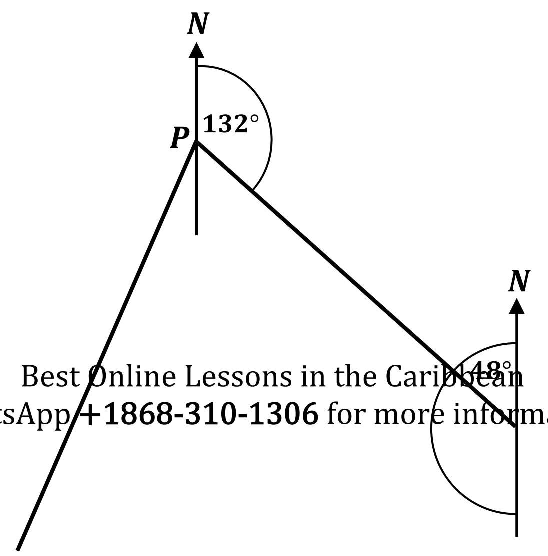
Calculate, **giving reasons for your answer**, the measure of angle

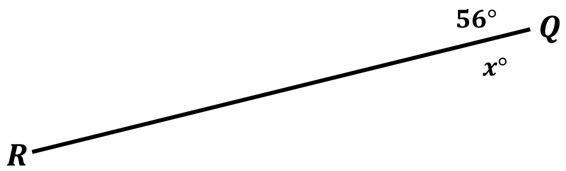
(i) XYZ [2]

(ii) YXZ [2]

(iii) OXZ [3]

(b) The diagram below, **not drawn to scale**, shows the route of an aeroplane flying from Portcity (P) to Queenstown (Q) and then to Riversdale (R). The bearing of Q from P is 132° and the angle PQR is 56°.





(i) Calculate the value of x, as shown in the diagram. [2]

(ii) The distance from Portcity (P) to Queenstown (Q) is 220 kilometres and the distance from Queenstown to Riversdale (R) is 360 kilometres. Calculate the distance RP. [3]

(iii) Determine the bearing of R from P. [3]

Total: 15 marks

VECTORS AND MATRICES

11. (a) Determine the inverse of the matrix $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$. [2]

(b) The transformation, $M = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}$, maps the points R and T onto R' and T' such that:

$R(7, 2) \longrightarrow R'(2, -7)$ and

$T(-5, 4) \longrightarrow T'(4, 5)$

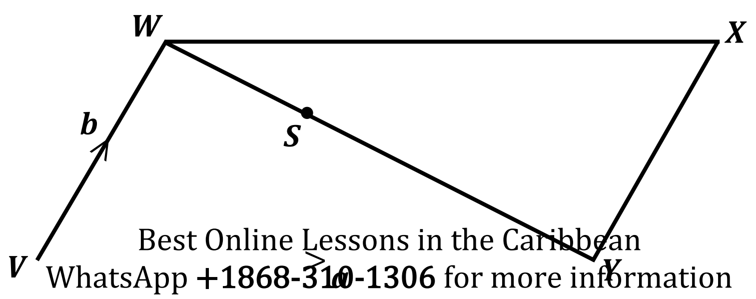
(i) Determine the values of a and b. [2]

(ii) Describe fully the transformation, M. [3]

(c) $WXYV$ is a parallelogram in which

$\overrightarrow{VY} = \boldsymbol{a}$ and $\overrightarrow{VW} = \boldsymbol{b}$.

S is a point on WY such that $WS : SY = 1 : 2$.



(i) Write in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, an expression for:

(a) $\overrightarrow{WY}$

(b) $\overrightarrow{WS}$

(c) $\overrightarrow{SX}$ [5]

(ii) R is the mid-point of VW. Prove that R, S and X are collinear. [3]

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.