

CSEC Mathematics
January 2026 - Paper 2
Solutions

Kerwin Springer

SECTION I

Answer ALL questions.

ALL working must be clearly shown.

1. (a) (i) Express $\frac{4}{7} \div 12$ as a single fraction in its LOWEST terms. [1]

$$\begin{aligned} \frac{4}{7} \div 12 \\ &= \frac{\cancel{4}^1}{7} \times \frac{1}{\cancel{12}_3} \\ &= \frac{1}{21} \end{aligned}$$

- (ii) Calculate the difference in value of the underlined digits in the numbers

below:

$\underline{3}201_4$ and $6\underline{3}51_7$ [3]

4^3	4^2	4^1	4^0
<u>3</u>	2	0	1

Converting to base 10:

$$\begin{aligned} 3 \times 4^3 &= 3 \times 64 \\ &= 192 \end{aligned}$$

7^3	7^2	7^1	7^0
6	<u>3</u>	5	1

Converting to base 10:

$$\begin{aligned}
 3 \times 7^2 &= 3 \times 49 \\
 &= 147
 \end{aligned}$$

$$\begin{aligned}
 \text{Difference} &= 192 - 147 \\
 &= 45_{10}
 \end{aligned}$$

(b) By writing each number in the fraction below correct to 1 significant figure, find an INTEGER estimate for the value of

$$\frac{600 - 87.04}{29.6}$$

$$\begin{aligned}
 &\frac{600 - 87.04}{29.6} \\
 &= \frac{600 - 90}{30} \\
 &= \frac{510}{30} \\
 &= 17
 \end{aligned}$$

[2]

(c) Alana and Brentnol share \$17 400 in the ratio Alana : Brentnol = 8:7

(i) Show that Alana receives \$9 280.

[1]

Ratio of money shared is:

Alana : Brentnol

$$8 : 7$$

Total parts = $8 + 7$

$$= 15$$

15 parts = \$17 400

$$1 \text{ part} = \frac{\$17\,400}{15}$$

$$= \$1160$$

Alana receives 8 parts

$$= 8 \times \$1160$$

$$= \$9280$$

Q.E.D.

- (ii) Alana invests her share of money into a business venture, earning simple interest at a rate of 4.5% per annum. She receives \$2 088 in interest.

Determine the number of years that she invested her money.

[2]

Substituting Principal = \$9280, Simple Interest = \$2088 and

Rate = 4.5% into:

$$Time = \frac{Simple\ Interest \times 100}{Principal \times Rate}$$

$$= \frac{2088 \times 100}{9280 \times 4.5}$$

$$= 5\ \text{years}$$

∴ The number of years that she invested her money was 5 years.

Total: 9 marks

2. (a) Factorize the expression $1 - t^2$.

$$\begin{aligned}
 &1 - t^2 \\
 &= (1 + t)(1 - t) \quad \text{[Difference of two squares]}
 \end{aligned}$$

(b) Expand and simplify the following expression.

$$(4r - 5q)(3r + q) + 3qr \quad [2]$$

$$\begin{aligned}
 &(4r - 5q)(3r + q) + 3qr \\
 &= 12r^2 + 4qr - 15qr - 5q^2 + 3qr \\
 &= 12r^2 - 5q^2 - 8qr
 \end{aligned}$$

(c) (i) Solve the inequality $-4p + 3 \geq 19 + 2p$. [2]

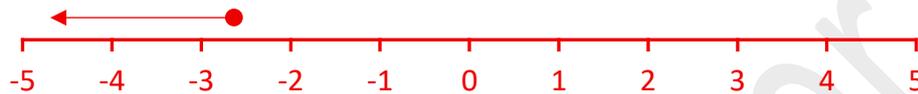
$$\begin{aligned}
 &-4p + 3 \geq 19 + 2p \\
 &-4p - 2p \geq 19 - 3 \\
 &-6p \geq 16 \\
 &p \leq \frac{16}{-6} \\
 &p \leq -\frac{8}{3}
 \end{aligned}$$

(ii) Determine the LARGEST integer value of p that satisfies the inequality in

(c) (i)

[1]

The largest integer value of p that satisfies the inequality $p \leq -\frac{8}{3}$ is -3 .



(d) The diagram below shows a right-angled triangle with its dimensions given

in terms of x .



For the right-angled triangle, show that $x^2 + 4x - 45 = 0$.

[3]

According to Pythagoras' Theorem,

$$c^2 = a^2 + b^2$$

$$(2x + 3)^2 = (12)^2 + (x)^2$$

$$4x^2 + 12x + 9 = 144 + x^2$$

$$4x^2 - x^2 + 12x + 9 - 144 = 0$$

$$3x^2 + 12x - 135 = 0$$

Dividing throughout by 3

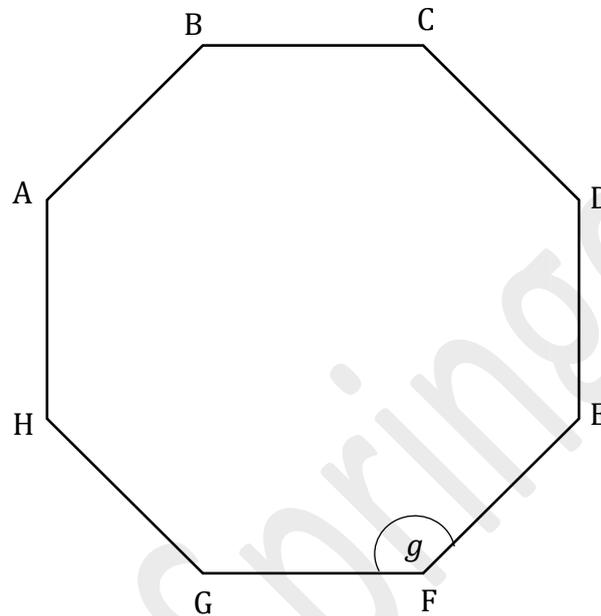
$$x^2 + 4x - 45 = 0$$

Q.E.D.

Total: 9 marks

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3. (a) $ABCDEFGH$ is a **regular** octagon, with one of its internal angles marked g , as shown in the diagram below.

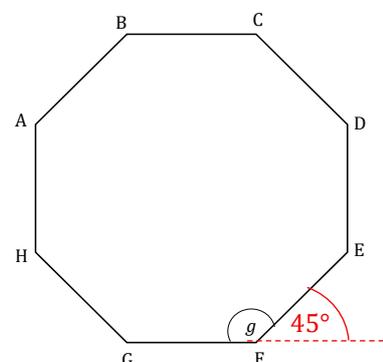


- (i) Complete the statement below in relation to the diagram shown above.

The regular octagon has 8 lines of symmetry and rotational symmetry of order 8. [2]

- (ii) Calculate the value of Angle g . [2]

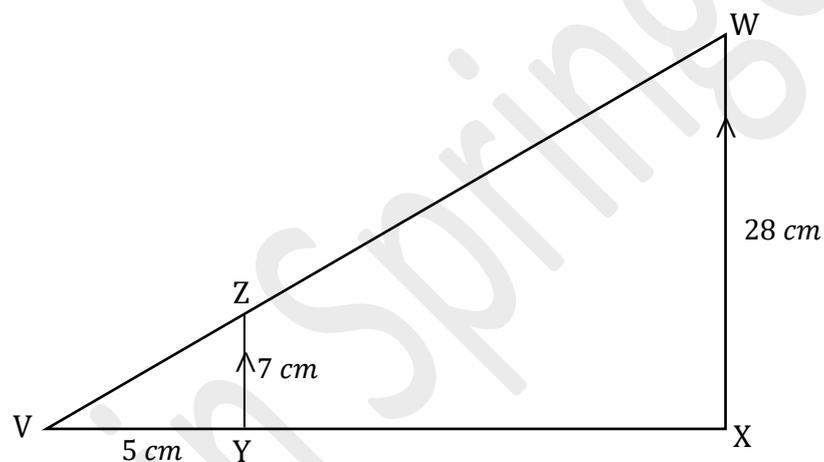
$$\begin{aligned}
 \text{External angles in a polygon} &= \frac{360}{n} \\
 &= \frac{360}{8} \\
 &= 45^\circ
 \end{aligned}$$



Since angles on a straight line add up to 180° ,

$$\begin{aligned}\text{Angle } g &= 180^\circ - 45^\circ \\ &= 135^\circ\end{aligned}$$

(b) The diagram below shows two similar, right-angled triangles drawn from a common vertex, V . The lines ZY and WX are parallel. Also, the lines $VY = 5 \text{ cm}$, $ZY = 7 \text{ cm}$ and $WX = 28 \text{ cm}$.



(i) Calculate the length of YX .

[3]

Finding the ratio:

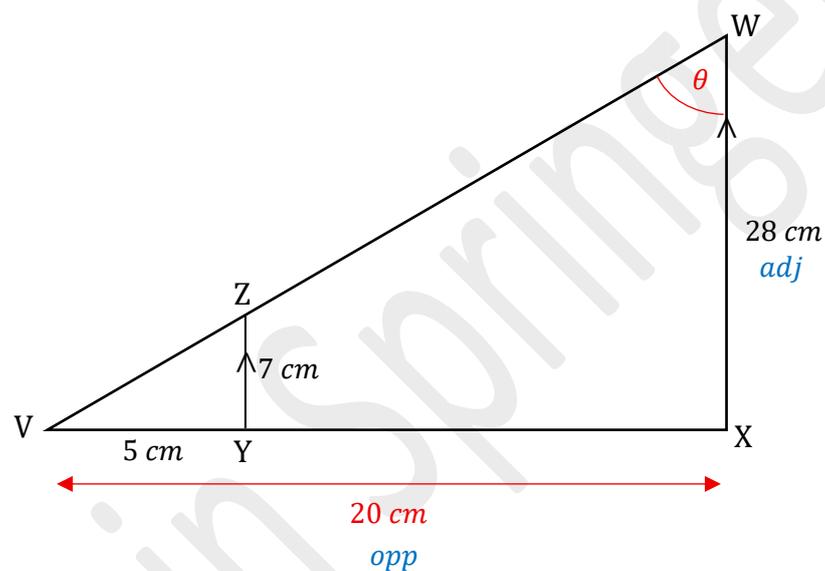
$$\begin{aligned}\frac{WX}{ZY} &= \frac{28}{7} \\ &= 4\end{aligned}$$

$$\begin{aligned}\therefore VX &= 5 \text{ cm} \times 4 \\ &= 20 \text{ cm}\end{aligned}$$

$$\begin{aligned}
 YX &= VX - VY \\
 &= 20\text{cm} - 5\text{cm} \\
 &= 15\text{cm}
 \end{aligned}$$

(ii) Determine the magnitude of Angle VWX .

[2]



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{20}{28}$$

$$\theta = \tan^{-1}\left(\frac{20}{28}\right)$$

$$\theta = 35.5^\circ$$

\therefore Angle VWX is 35.5° .

Total: 9 marks

4. A straight line, L , whose equation is $3x + 2y = c$, passes through the point $(-2, 8)$.

(a) Show that the value of c is 10. [1]

Substituting the point $(-2, 8)$ into $3x + 2y = c$ to find c :

$$3(-2) + 2(8) = c$$

$$-6 + 16 = c$$

$$10 = c$$

$$c = 10$$

Q.E.D.

(b) Determine the gradient of L . [2]

The equation of L is $3x + 2y = 10$.

$$2y = 10 - 3x$$

$$y = \frac{10 - 3x}{2}$$

$$y = \frac{10}{2} - \frac{3}{2}x$$

$$y = 5 - \frac{3}{2}x$$

$$y = -\frac{3}{2}x + 5$$

Which is of the form

$$y = mx + c$$

Where the gradient, $m = -\frac{3}{2}$.

(c) The line with equation $3x + 4y = 8$ intersects the line L at the point T .

Determine the coordinates of the point T .

[3]

$$3x + 2y = 10 \quad \text{--- Equation 1}$$

$$3x + 4y = 8 \quad \text{--- Equation 2}$$

Equation 1 - Equation 2:

$$\begin{array}{r} 3x + 2y = 10 \\ - 3x + 4y = 8 \\ \hline -2y = 2 \end{array}$$

$$-2y = 2$$

$$y = \frac{2}{-2}$$

$$y = -1$$

Substituting $y = -1$ into Equation 1 to find x :

$$3x + 2(-1) = 10$$

$$3x - 2 = 10$$

$$3x = 10 + 2$$

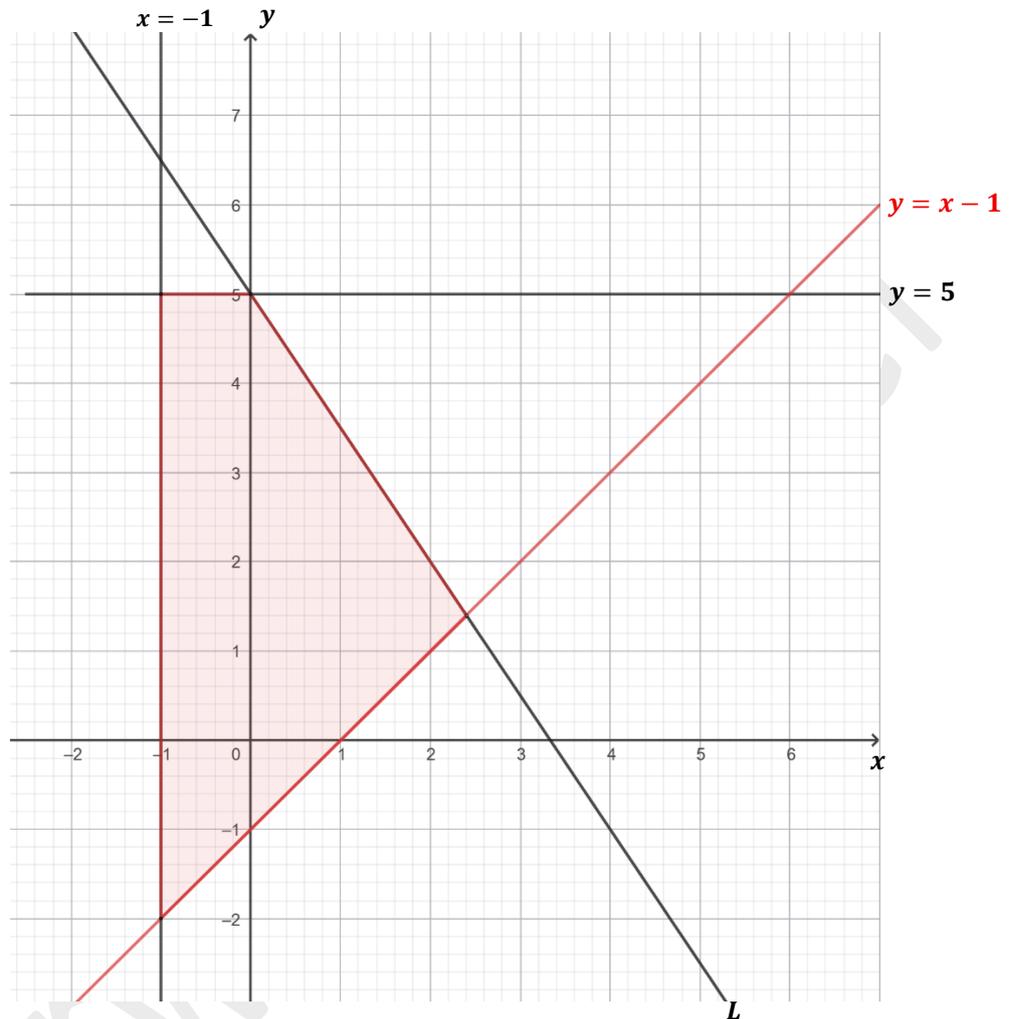
$$3x = 12$$

$$x = \frac{12}{3}$$

$$x = 4$$

\therefore The coordinates of the point T is $(4, -1)$

(d) The diagram below shows the graph of L and two other lines, $x = -1$ and $y = 5$.



On the graph above, draw the line $y = x - 1$ and shade the region that satisfies the inequalities listed below.

$$x \geq -1$$

$$y \leq 5$$

$$y \geq x - 1$$

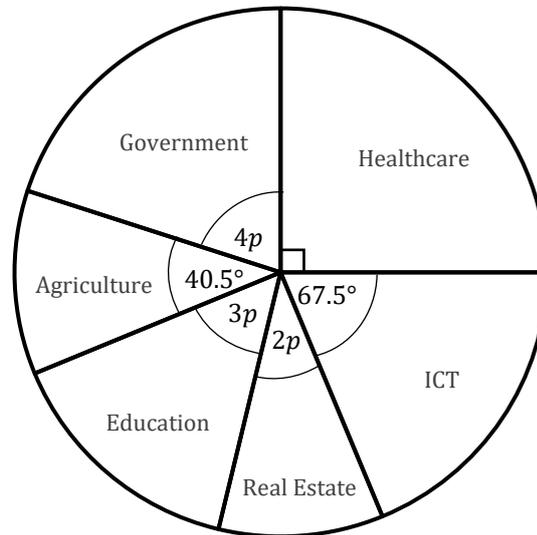
$$3x + 2y \leq 10$$

[3]

The line and shaded region were shown on the graph above.

Total: 9 marks

5. (a) Some Grade 11 students were surveyed to determine the job sector in which they are most likely to seek employment after graduating. Their choices are represented on the pie chart shown below.



Calculate the

- (i) Percentage of students who are likely to seek employment within the agriculture and ICT sectors. [2]

Total degrees of the Agriculture and ICT sectors

$$= 40.5^\circ + 67.5^\circ$$

$$= 108^\circ$$

% of students in the Agriculture and ICT sectors

$$= \frac{108^\circ}{360^\circ} \times \frac{100}{1}$$

$$= 30\%$$

(ii) Value of p .

$$4p + 3p + 2p + 40.5^\circ + 67.5^\circ + 90^\circ = 360^\circ$$

$$9p + 198^\circ = 360^\circ$$

$$9p = 360^\circ - 198^\circ$$

$$9p = 162^\circ$$

$$p = \frac{162^\circ}{9}$$

$$p = 18^\circ$$

(b) Liz has a set of red, yellow, and white buttons in a sack. She chooses a button at random. The probability that she chooses a yellow button is 0.3. The probability that she chooses a white button is 0.1.

(i) Determine the probability that Liz chooses a red button. [2]

Since the total probability of all possible outcomes in a sample space is 1,

$$\begin{aligned} \text{Probability (Red)} &= 1 - (0.3 + 0.1) \\ &= 1 - (0.4) \\ &= 0.6 \end{aligned}$$

- (ii) If there were 80 buttons in the sack originally, determine the number of buttons that were red or yellow. [2]

For mutually exclusive events,

$$\begin{aligned}
 P(R \cup Y) &= P(R) + P(Y) \\
 &= 0.6 + 0.3 \\
 &= 0.9
 \end{aligned}$$

Number of buttons that were red or yellow

$$\begin{aligned}
 &= \frac{9}{10} \times \frac{80}{1} \\
 &= 72 \text{ buttons}
 \end{aligned}$$

Total: 9 marks

6. (a) A map is drawn to a scale of 1 : 20 000. On the map, 1 *cm* represents *n* km.

Determine the

- (i) Value of *n*. [1]

Scale is 1 : 20 000.

So, we have

$$1 \text{ cm} = 20\,000 \text{ cm}$$

Recall:

$$100\,000 \text{ cm} = 1 \text{ km}$$

$$1 \text{ cm} = \frac{1}{100\,000} \text{ km}$$

$$\begin{aligned} 20\,000 \text{ cm} &= \frac{1}{100\,000} \times \frac{20\,000}{1} \text{ km} \\ &= 0.2 \text{ km} \end{aligned}$$

$$\therefore n = 0.2 \text{ km}$$

- (ii) Distance on the map, in *cm*, which corresponds to an actual distance of

4.8 km. [1]

On the map, 1 *cm* = 0.2 *km*.

$$0.2 \text{ km} = 1 \text{ cm}$$

$$1 \text{ km} = \frac{1}{0.2} \text{ cm}$$

$$4.8 \text{ km} = \frac{1}{0.2} \times \frac{4.8}{1} \text{ cm}$$

$$= 24 \text{ cm}$$

- (iii) Actual area, in square kilometres, of a lake which has an area of 12 cm^2 on the map. [2]

Since the scale is 1 : 20 000, we have

$$1 \text{ unit} = 20\,000 \text{ units}$$

$$1 \text{ square unit} = (20\,000)^2 \text{ square units}$$

$$12 \text{ cm}^2 \text{ on the map} = 12 \times (20\,000)^2$$

$$= 4.8 \times 10^9 \text{ cm}^2$$

Now,

$$100\,000 \text{ cm} = 1 \text{ km}$$

$$(100\,000)^2 \text{ cm}^2 = 1 \text{ km}^2$$

$$1 \text{ cm}^2 = \frac{1}{(100\,000)^2} \text{ km}^2$$

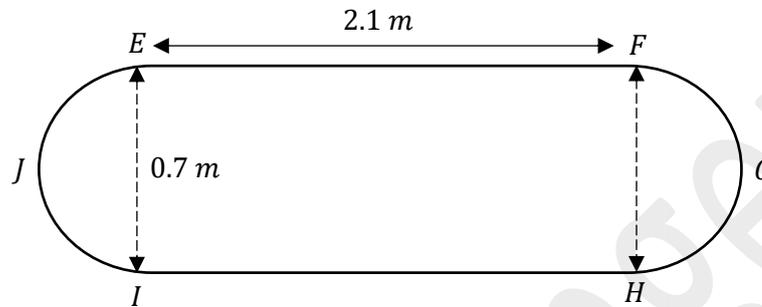
$$4.8 \times 10^9 \text{ cm}^2 = \frac{1}{(100\,000)^2} \times 4.8 \times 10^9 \text{ km}^2$$

$$= \frac{12}{25} \text{ km}^2$$

$$= 0.48 \text{ km}^2$$

\therefore The actual area of a lake which has an area of 12 cm^2 on the map is 0.48 km^2 .

(b) The diagram below shows a running belt, $EFGHIJE$, that revolves around a running board on a treadmill. In the diagram, EJI and HGF are semicircles with a diameter of 0.7m and $EFHI$ is a rectangle.



[Use $\pi = \frac{22}{7}$]

- (i) Calculate the length of the belt. [2]

Note: Both semi-circles when put together make a full circle.

Length of the belt = (Circumference of circle) + 2(length of rectangle)

$$= (\pi \times \text{diameter}) + 2(\text{length of rectangle})$$

$$= \left(\frac{22}{7} \times 0.7\right) + 2(2.1)$$

$$= 2.2 + 4.2$$

$$= 6.4 \text{ m}$$

- (ii) Glenda uses the equipment to exercise 4 times a week and runs at 9 km/h for 20 minutes each time.

Calculate the number of complete revolutions the running belt makes in one week during Glenda's exercise routine.

[Assume that the running belt moves at the same speed as Glenda]

[3]

From the question, Glenda's speed = 9 km/h
= 9000 m/h

From the question, Glenda ran for 20 minutes each time.

Since 60 minutes = 1 hour

$$20 \text{ minutes} = \left(\frac{1}{60} \times \frac{20}{1} \right) \text{ hour}$$

$$= \frac{1}{3} \text{ hour}$$

Substituting Speed = 9000 m/h and Time = $\frac{1}{3}$ hour to find the distance she ran during each of her exercises:

$$\begin{aligned} \text{Distance} &= \text{Speed} \times \text{Time} \\ &= 9000 \text{ m/h} \times \frac{1}{3} \text{ h} \\ &= 3000 \text{ m} \end{aligned}$$

Since she exercised 4 times a week, then the total distance she ran in one week will be

$$= 3000 \text{ m} \times 4$$

$$= 12\,000 \text{ m}$$

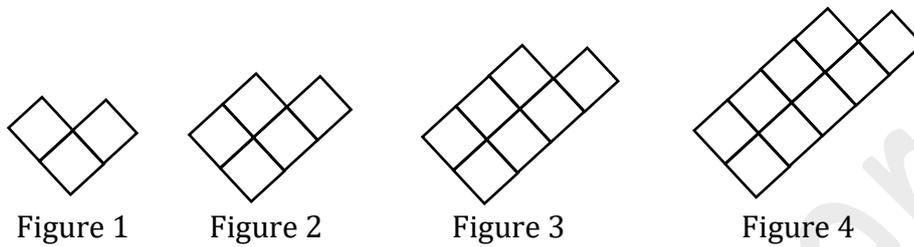
In part (i), we found that the length of the belt = 6.4 m.

$$\begin{aligned} \text{Number of revolutions} &= \frac{\text{Total distance she ran in one week}}{\text{Length of the belt}} \\ &= \frac{12\,000 \text{ m}}{6.4 \text{ m}} \\ &= 1875 \end{aligned}$$

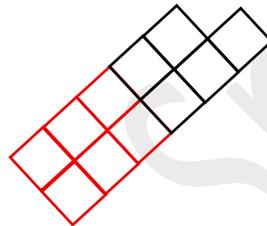
∴ The number of complete revolutions the running belt makes in one week during Glenda's exercise routine is 1875.

Total: 9 marks

7. The diagram below shows the first 4 figures in a sequence of figures made up of squares of unit length.



(a) Complete the diagram below to show Figure 5. [2]



(b) The number of squares, Q , the number of sticks, S , and the perimeter of each figure, P , follow a pattern. The values for Q , S and P for the first 4 figures are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.

Complete the rows marked (i), (ii) and (iii) in the table below.

Figure Number (D)	Number of Squares (Q)	Number of Sticks (S)	Perimeter (P)
1	3	10	8
2	5	15	10
3	7	20	12
4	9	25	14
(i) 5	_____	_____	16
⋮	⋮	⋮	⋮
(ii) _____	47	120	_____
⋮	⋮	⋮	⋮
(iii) n	_____	_____	_____

(iii) Looking at Figures 1, 2, 3 and 4, we can deduce that:

Number of Squares, $Q = 2n + 1$

Number of Sticks, $S = 5n + 5$

Perimeter, $P = 2n + 6$

Explanation for Q :

Figure	Number of Squares (Q)	Number of Sticks (S)	Perimeter (P)
1	3	10	8
2	5	15	10
3	7	20	12
4	9	25	14

The number of squares increase by 2.

∴ In terms of n , we can deduce that $Q = 2n + k$ (where k is a constant)

Substituting $n = 1$ and $Q = 3$ into:

$$Q = 2n + k$$

$$3 = 2(1) + k$$

$$3 = 2 + k$$

$$3 - 2 = k$$

$$1 = k$$

$$k = 1$$

We can also substitute the following into $Q = 2n + k$ to verify that $k = 1$

for all figures:

$$n = 2 \text{ and } Q = 5$$

$$n = 3 \text{ and } Q = 7$$

$$n = 4 \text{ and } Q = 9$$

$$\therefore Q = 2n + 1$$

Explanation for S :

Figure	Number of Squares (Q)	Number of Sticks (S)	Perimeter (P)
1	3	10	8
2	5	15	10
3	7	20	12
4	9	25	14

The number of sticks increase by 5.

\therefore In terms of n , we can deduce that $S = 5n + k$ (where k is a constant)

Substituting $n = 1$ and $S = 10$ into:

$$S = 5n + k$$

$$10 = 5(1) + k$$

$$10 = 5 + k$$

$$10 - 5 = k$$

$$5 = k$$

$$k = 5$$

We can also substitute the following into $S = 5n + k$ to verify that $k = 5$ for all figures:

$$n = 2 \text{ and } S = 15$$

$$n = 3 \text{ and } S = 20$$

$$n = 4 \text{ and } S = 25$$

$$\therefore S = 5n + 5$$

Explanation for P :

Figure	Number of Squares (Q)	Number of Sticks (S)	Perimeter (P)
1	3	10	8
2	5	15	10
3	7	20	12
4	9	25	14

The Perimeter increase by 2.

\therefore In terms of n , we can deduce that $P = 2n + k$ (where k is a constant)

Substituting $n = 1$ and $P = 8$ into:

$$P = 2n + k$$

$$8 = 2(1) + k$$

$$8 = 2 + k$$

$$8 - 2 = k$$

$$6 = k$$

$$k = 6$$

We can also substitute the following into $P = 2n + k$ to verify that $k = 6$

for all figures:

$$n = 2 \text{ and } P = 10$$

$$n = 3 \text{ and } P = 12$$

$$n = 4 \text{ and } P = 14$$

$$\therefore P = 2n + 6$$

(i) When $n = 5$,

$$\begin{aligned} \text{Number of Squares, } Q &= 2n + 1 \\ &= 2(5) + 1 \\ &= 10 + 1 \\ &= 11 \end{aligned}$$

$$\begin{aligned}
 \text{Number of Sticks, } S &= 5n + 5 \\
 &= 5(5) + 5 \\
 &= 25 + 5 \\
 &= 30
 \end{aligned}$$

(ii) Finding the value of n :

$$\begin{aligned}
 \text{Number of Squares, } Q &= 2n + 1 \\
 47 &= 2n + 1 \\
 47 - 1 &= 2n \\
 46 &= 2n \\
 \frac{46}{2} &= n \\
 23 &= n \\
 n &= 23
 \end{aligned}$$

\therefore Figure Number, $D = 23$

$$\begin{aligned}
 \text{Perimeter, } P &= 2n + 6 \\
 &= 2(23) + 6 \\
 &= 46 + 6 \\
 &= 52
 \end{aligned}$$

(c) Mala says that she can make one of the figures with EXACTLY 502 squares.

Explain why she is incorrect.

[1]

$$\begin{aligned}
 \text{Number of Squares, } Q &= 2n + 1 \\
 502 &= 2n + 1 \\
 502 - 1 &= 2n \\
 501 &= 2n
 \end{aligned}$$

$$\frac{501}{2} = n$$

$$n = \frac{501}{2}$$

Since n is not an integer, then she is incorrect and cannot make one of the figures with exactly 502 squares.

Total: 10 marks

Kerwin Springer

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS.

8. (a) The functions f and g are defined as follows

$$f: x \rightarrow x^2 + 3, x \geq 0$$

$$g: x \rightarrow 2x + 2, x \in R$$

(i) Calculate the value of $g(-1)$. [1]

$$g(x) = 2x + 2$$

$$g(-1) = 2(-1) + 2$$

$$= -2 + 2$$

$$= 0$$

(ii) Write down an expression for $f^{-1}(x)$ [2]

$$f(x) = x^2 + 3$$

$$\text{Let } y = f(x)$$

$$y = x^2 + 3$$

Make x the subject of the formula:

$$y = x^2 + 3$$

$$y - 3 = x^2$$

$$x^2 = y - 3$$

$$x = \pm\sqrt{y - 3}$$

Interchange x and y :

$$y = \pm\sqrt{x - 3}$$

$$\therefore f^{-1}(x) = \pm\sqrt{x - 3} \quad (\text{where } x > 3)$$

(b) (i) Derive a simplified expression for $gf(x)$. [2]

$$f(x) = x^2 + 3$$

$$g(x) = 2x + 2$$

$$gf(x) = g[f(x)]$$

$$= g[x^2 + 3]$$

$$= 2(x^2 + 3) + 2$$

$$= 2x^2 + 6 + 2$$

$$= 2x^2 + 8$$

(ii) Given that $fg(x) = 4x^2 + 8x + 7$, solve the following equation.

$$fg(x) = 2gf(x) + 15 \quad [3]$$

Substituting $fg(x) = 4x^2 + 8x + 7$ and $gf(x) = 2x^2 + 8$ into:

$$fg(x) = 2gf(x) + 15$$

$$4x^2 + 8x + 7 = 2(2x^2 + 8) + 15$$

$$4x^2 + 8x + 7 = 4x^2 + 16 + 15$$

$$4x^2 + 8x + 7 = 4x^2 + 31$$

$$4x^2 - 4x^2 + 8x + 7 = 31$$

$$8x + 7 = 31$$

$$8x = 31 - 7$$

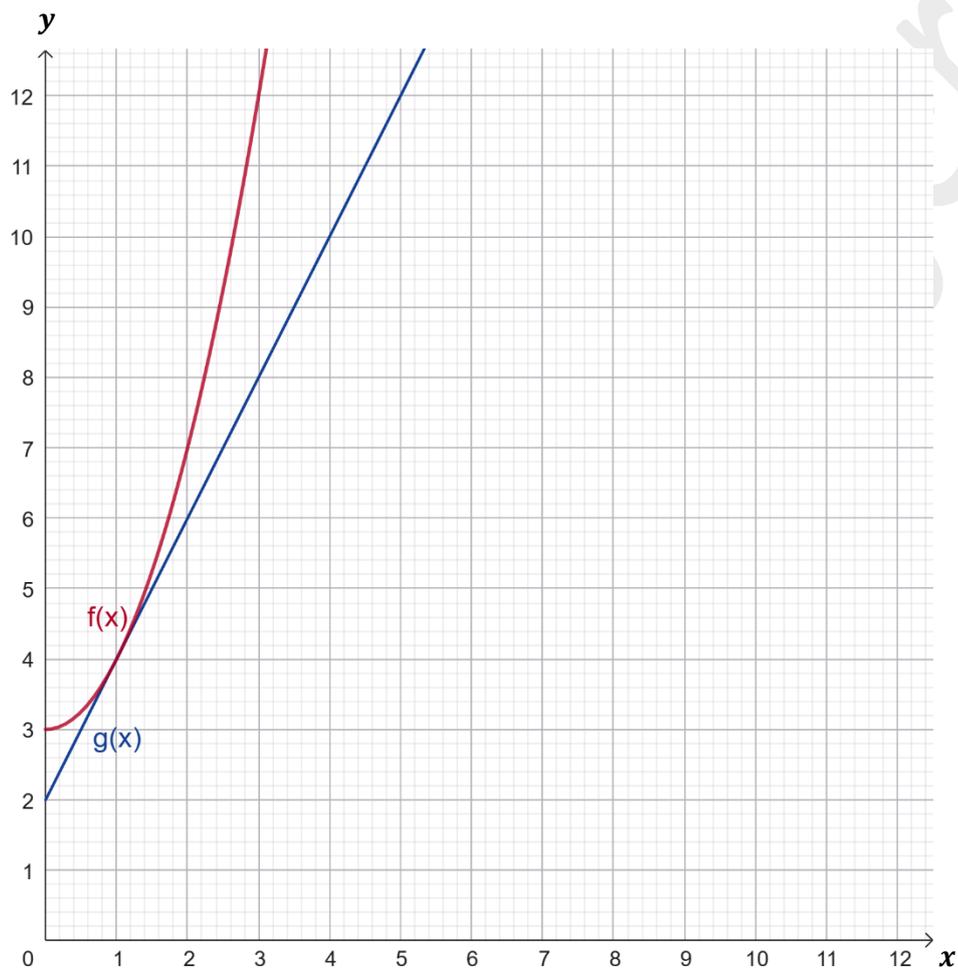
$$8x = 24$$

$$x = \frac{24}{8}$$

$$x = 3$$

(c) On the grid provided in the answer booklet, plot the graph of the functions f and g , for $x \geq 0$. [4]

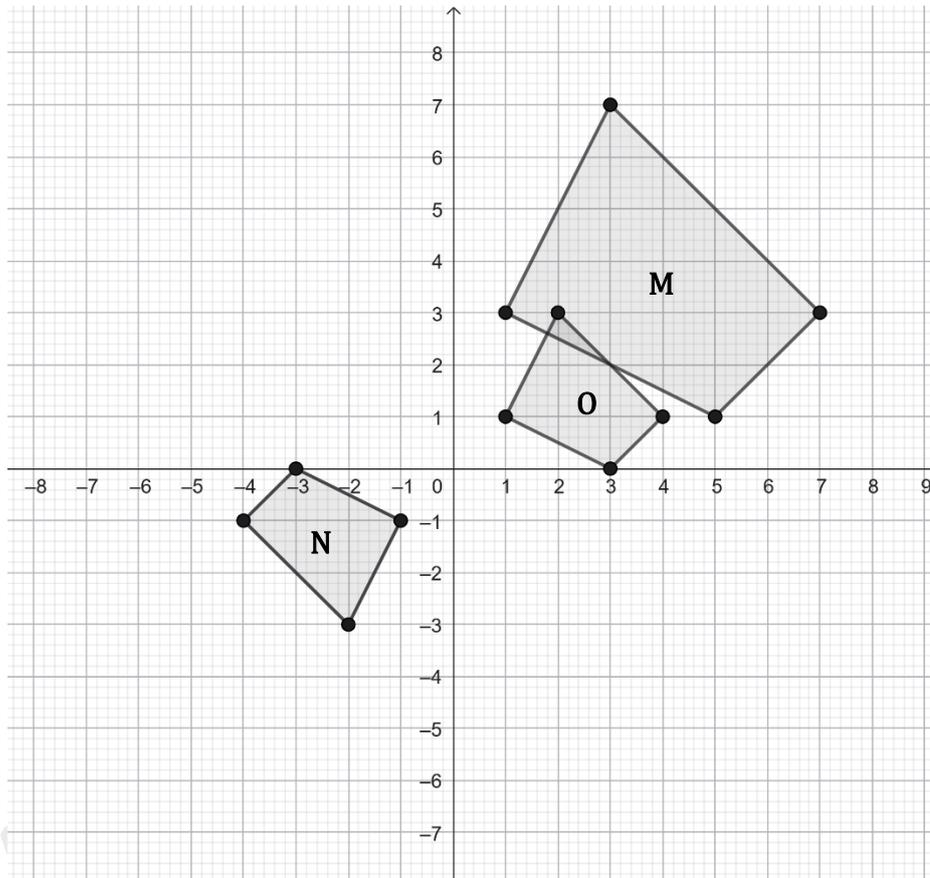
Title: Graph Showing the Functions $f(x) = x^2 + 3$ and $g(x) = 2x + 2$.



Total: 12 marks

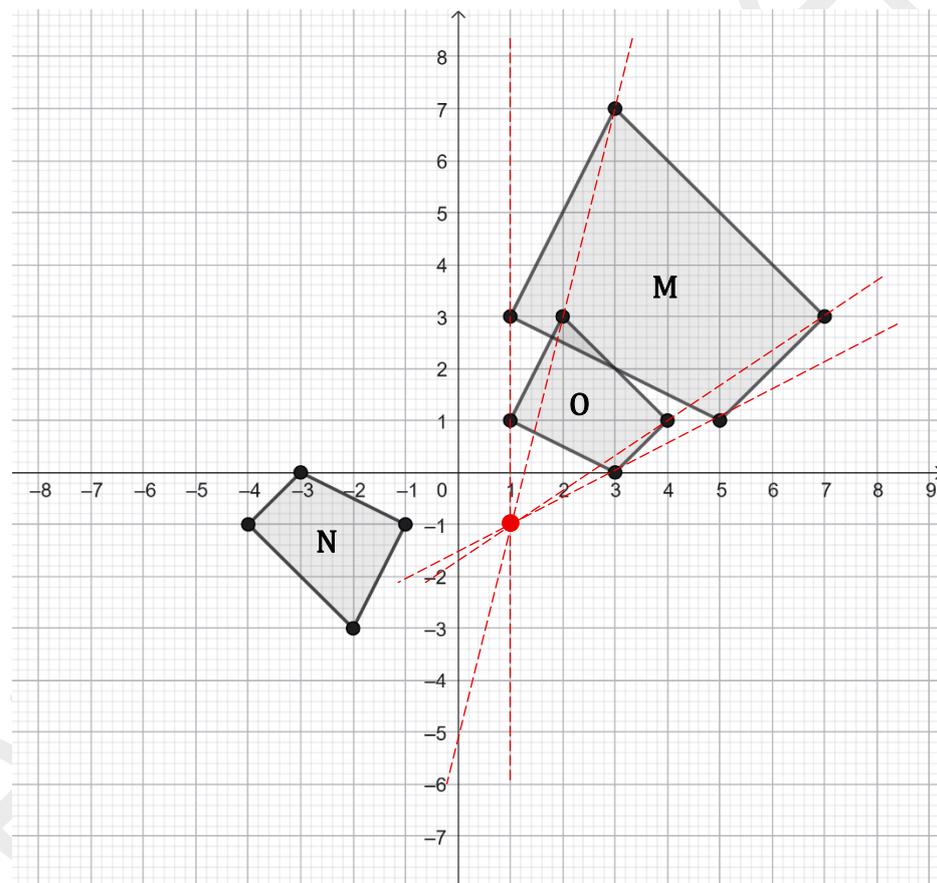
GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below shows quadrilaterals O , M and N . Quadrilaterals M and N are the images of Quadrilateral O after it has undergone 2 different transformations.



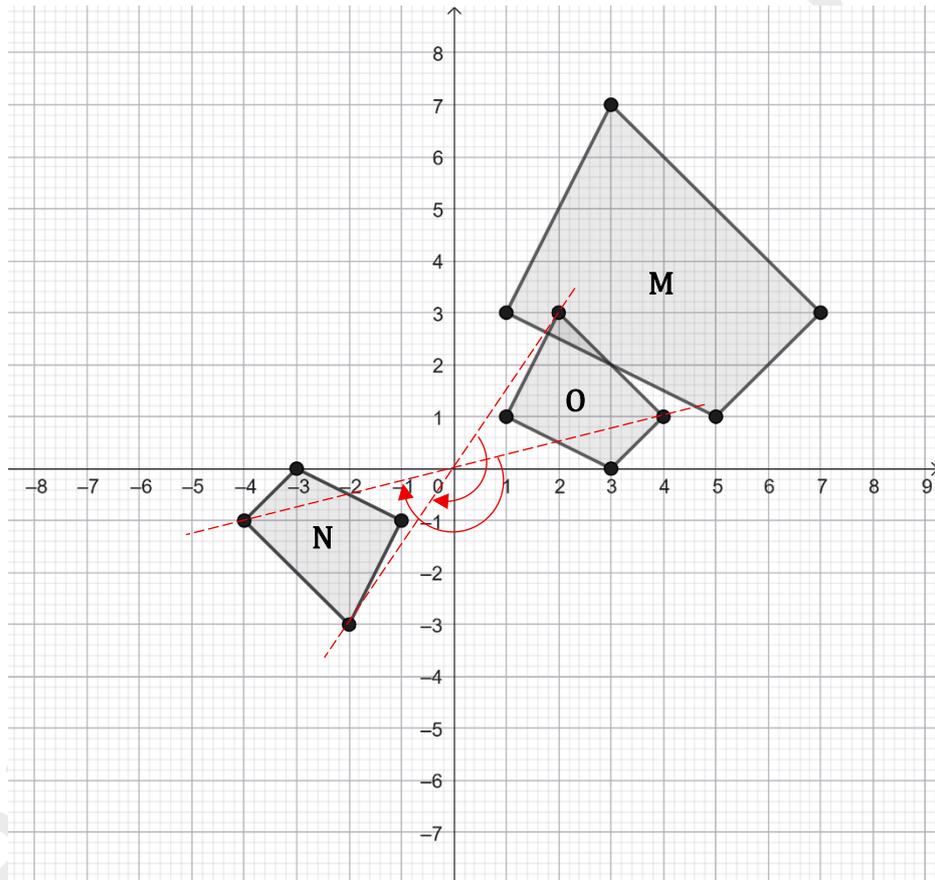
- (i) Describe fully the **single** transformation that maps Quadrilateral O onto Quadrilateral M . [3]

The single transformation that maps Quadrilateral O onto Quadrilateral M is an enlargement of scale factor 2 and where the centre of enlargement is $(1, -1)$.



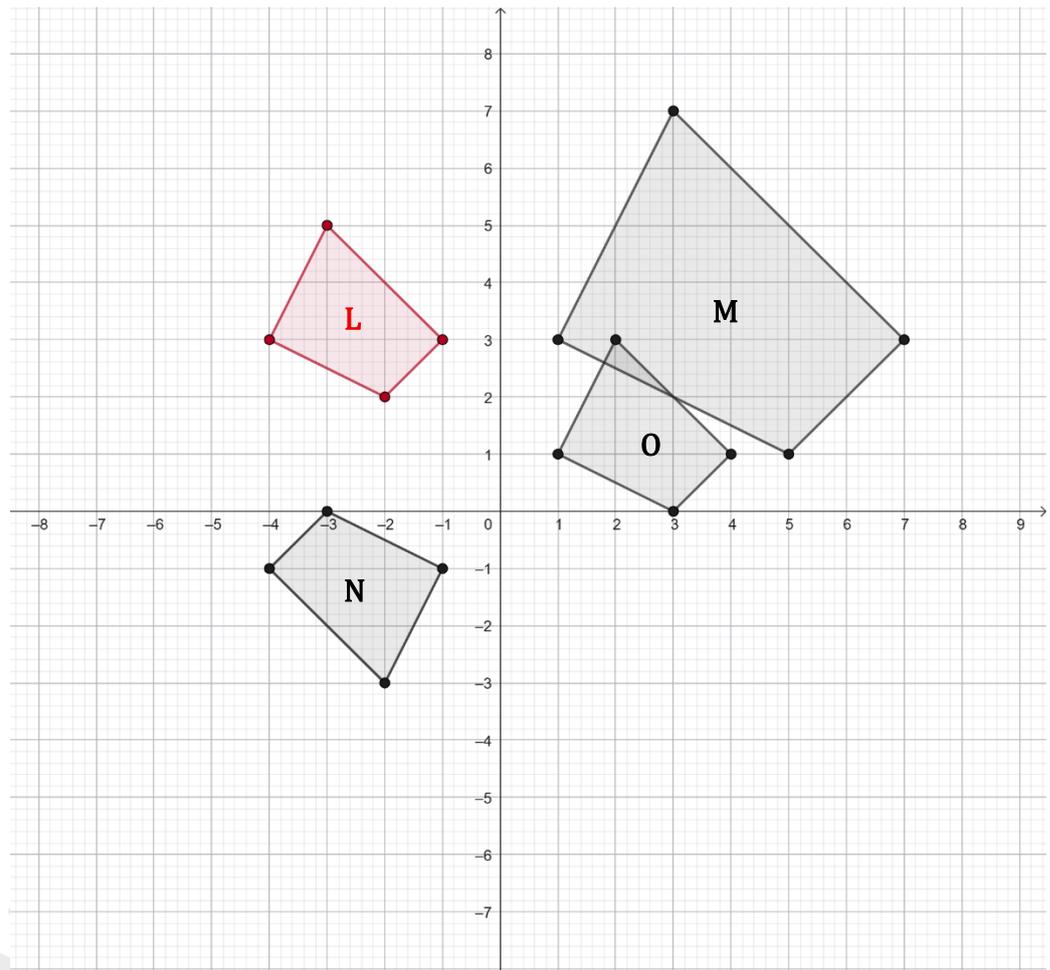
- (ii) Describe fully the **single** transformation that maps Quadrilateral O onto Quadrilateral N . [3]

The single transformation that maps Quadrilateral O onto Quadrilateral N is a clockwise/anticlockwise rotation of 180° about the origin $(0, 0)$.

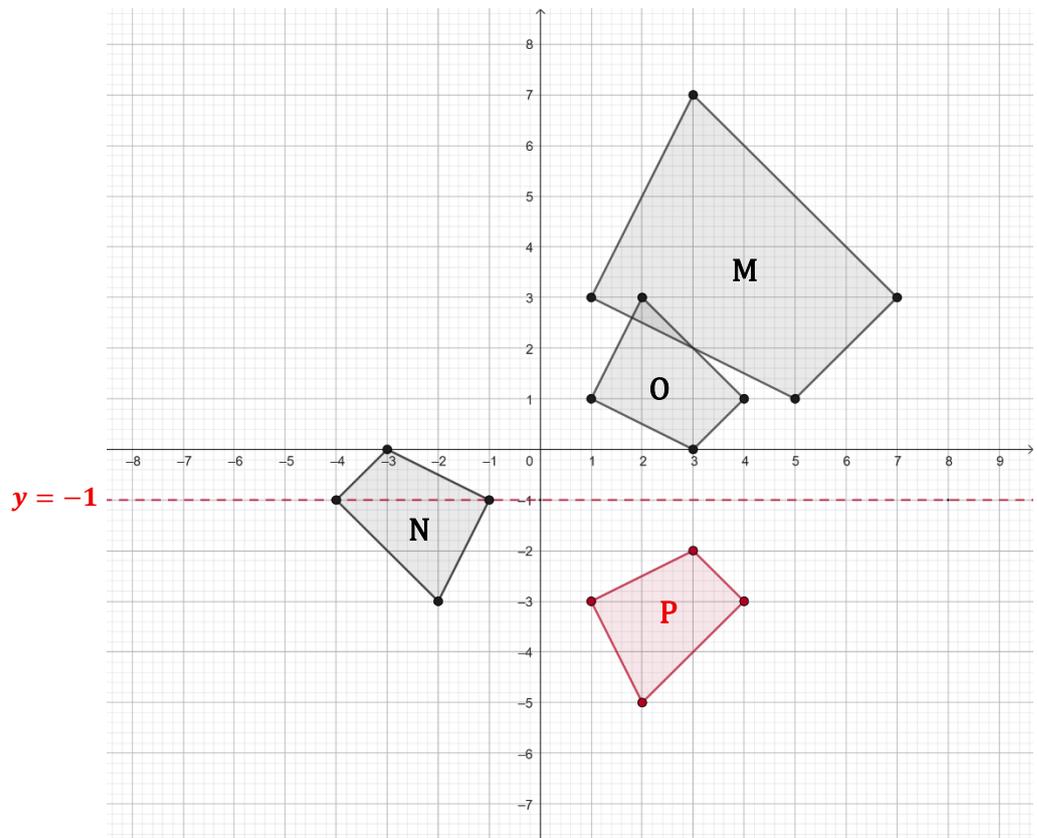


(iii) On the diagram on page 24, draw the image of Quadrilateral O after it undergoes the following transformations.

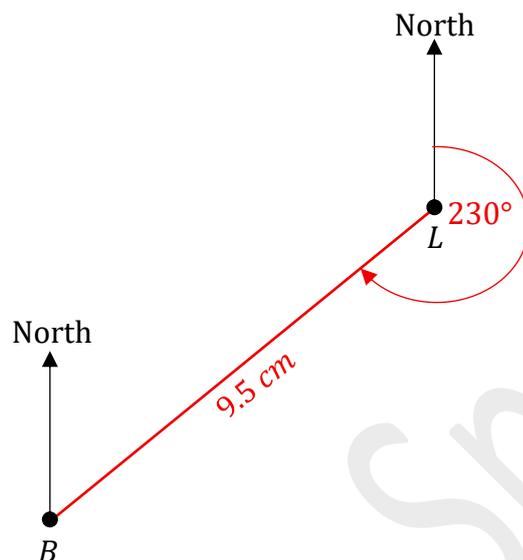
- a. Translation by the vector $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$. Label this image L . [1]



b. Reflection in the line $y = -1$. Label this image P .



(b) A buoy (B) and a lighthouse (L) are 95 km apart. The bearing of B from L is 230° . Using a scale of 1 cm : 10 km and the space provided below, complete the diagram to show the buoy (B) relative to the lighthouse (L). Indicate the given bearing on your drawing. [3]



Scale given is 1 cm : 10 km

$$1 \text{ cm} = 10 \text{ km}$$

$$10 \text{ km} = 1 \text{ cm}$$

$$1 \text{ km} = \frac{1}{10} \text{ cm}$$

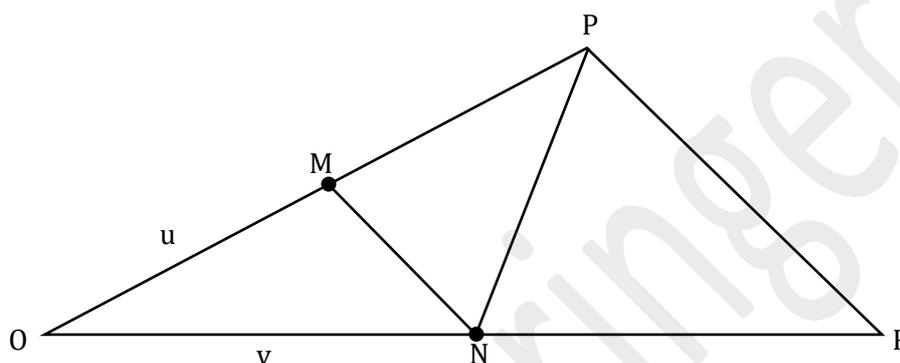
$$95 \text{ km} = \frac{1}{10} \times 95 \text{ cm}$$

$$= 9.5 \text{ cm}$$

Total: 12 marks

VECTORS AND MATRICES

10. (a) The diagram below shows Triangle OPR . The point M is the midpoint of OP and the point N is the midpoint of OR . In the diagram, $\overrightarrow{OM} = \mathbf{u}$ and $\overrightarrow{ON} = \mathbf{v}$.



- (i) Find in terms of u and v , simplified expressions for

a. \overrightarrow{MN}

[1]

Using the triangle law,

$$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$$

$$= -\mathbf{u} + \mathbf{v}$$

$$= \mathbf{v} - \mathbf{u}$$

b. \overrightarrow{NP}

Since M is the midpoint of OP and $\overrightarrow{OM} = u$ then $\overrightarrow{MP} = u$.

Therefore, $\overrightarrow{OP} = 2u$.

Using the triangle law,

$$\begin{aligned}\overrightarrow{NP} &= \overrightarrow{NO} + \overrightarrow{OP} \\ &= -v + 2u\end{aligned}$$

(ii) Show that MN is parallel to PR . [2]

Since N is the midpoint of OR and $\overrightarrow{ON} = v$ then $\overrightarrow{NR} = v$.

Therefore, $\overrightarrow{OR} = 2v$.

Using the triangle law,

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PO} + \overrightarrow{OR} \\ &= -2u + 2v \\ &= 2v - 2u \\ &= 2(v - u)\end{aligned}$$

$$\overrightarrow{MN} = v - u$$

Since $\overrightarrow{PR} = 2\overrightarrow{MN}$, then \overrightarrow{PR} is a scalar multiple of \overrightarrow{MN} and hence \overrightarrow{MN} is parallel to \overrightarrow{PR} .

(b) The matrix P , written in terms of a where a is a real constant, is as follows.

$$P = \begin{pmatrix} 3a & 4 \\ 6 & 2a \end{pmatrix}$$

(i) Calculate the determinant of P , in terms of a . [1]

$$\begin{aligned} \det(P) &= ad - bc \\ &= (3a)(2a) - (4)(6) \\ &= 6a^2 - 24 \end{aligned}$$

(ii) Determine the values of a for which Matrix P is singular. [2]

Since Matrix P is singular, then $\det(P) = 0$.

$$6a^2 - 24 = 0$$

$$6a^2 = 24$$

$$a^2 = \frac{24}{6}$$

$$a^2 = 4$$

$$a = \pm\sqrt{4}$$

$$a = \pm 2$$

$$a = -2 \text{ and } +2$$

\therefore The values of a for which Matrix P is singular are $a = -2$ and $a = 2$.

- (iii) Determine P^{-1} , the inverse of P , for which P is non-singular.

$$\det(P) = 6a^2 - 24$$

$$\begin{aligned} \text{adj}(P) &= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \\ &= \begin{pmatrix} 2a & -4 \\ -6 & 3a \end{pmatrix} \end{aligned}$$

The inverse of P , for which P is non-singular is:

$$\begin{aligned} P^{-1} &= \frac{1}{\det(P)} \times \text{adj}(P) \\ &= \frac{1}{6a^2 - 24} \times \begin{pmatrix} 2a & -4 \\ -6 & 3a \end{pmatrix} \end{aligned}$$

$$\therefore \text{The inverse of } P \text{ is } P^{-1} = \frac{1}{6a^2 - 24} \begin{pmatrix} 2a & -4 \\ -6 & 3a \end{pmatrix}$$

(c) Write down the 2×2 matrix that represents a

- (i) Counterclockwise rotation of 90° about the origin. Label this matrix X .

[1]

$$X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

- (ii) Counterclockwise rotation of 90° about the origin followed by a reflection in the y -axis, given that $Q = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y -axis. Label this matrix T .

[2]

$$X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad Q = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$T = QX$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} (-1 \times 0) + (0 \times 1) & (-1 \times -1) + (0 \times 0) \\ (0 \times 0) + (1 \times 1) & (0 \times -1) + (1 \times 0) \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 0 & 1 + 0 \\ 0 + 1 & 0 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.