CSEC Mathematics

June 2013 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

    (i) $\dfrac{1\frac{4}{5} - \frac{1}{3}}{2\frac{2}{5}}$ [2]

$$\text{Numerator} = 1\frac{4}{5} - \frac{1}{3}$$

$$= \frac{9}{5} - \frac{1}{3}$$

$$= \frac{27-5}{15}$$

$$= \frac{22}{15}$$

$$\text{Denominator} = 2\frac{2}{5}$$

$$= \frac{12}{5}$$

Now,

$$\text{Numerator} \div \text{Denominator} = \frac{22}{15} \div \frac{12}{5}$$

$$= \frac{22}{15} \times \frac{5}{12}$$

$$= \frac{110}{180}$$

$$= \frac{11}{18}$$

$$\therefore \frac{1\frac{4}{5} - \frac{1}{3}}{2\frac{2}{5}} = \frac{11}{18}$$

(ii) $\sqrt{1.5625} + (0.32)^2$ [2]

Using a calculator,

$$\sqrt{1.5625} + (0.32)^2 = 1.25 + 0.1024$$

$$= 1.3524$$

(b) Smiley Orange Juice is sold in cartons of two different sizes at the prices

shown in the table below.

Carton Size	Cost
350 ml	$4.20
450 ml	$5.13

Which size carton of orange juice is the BETTER buy?

Justify your answer. [3]

(c) Faye borrowed $9 600 at 8% per annum compound interest.

(i)   Calculate the interest on the loan for the first year. [1]

At the end of the first year, she repaid $4 368.

(ii)   How much did she still owe at the beginning of the second year? [2]

(iii)   Calculate the interest on the remaining balance for the second

year. [1]

Total: 11 marks

2. (a) Factorize completely:

    (i) $2x^3 - 8x$ [2]

    (ii) $3x^2 - 5x - 2$ [2]

(b) (i) Make C the subject of the formula $F = \frac{9}{5}C + 32$.   [2]

   (ii) Given that $F = 113$, calculate the value of C.   [1]

(c) 500 tickets were sold for a concert. Of these x tickets were sold at \$6 each, and the remainder at \$10 each.

    (i)   Write an expression, in terms of x, for

         (a) the number of tickets sold at \$10 each   [1]

         (b) the TOTAL amount of money collected for the sale of the 500

             tickets.   [1]

    (ii)   The sum of \$4108 was collected for the sale of the 500 tickets.

          Calculate the number of tickets sold at \$6 each.   [3]

Total: 12 marks

3. (a) A survey of the 30 students in Form 5 showed that some students used

cameras (C) or mobile phones (M) to take photographs.

20 students used mobile phones

$4x$ students used ONLY cameras

x students used BOTH mobile phones and cameras

2 students did not use either cameras or phones.
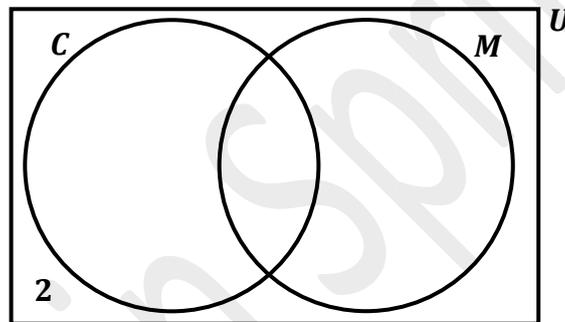
(i) Copy the Venn diagram below and complete it to show, in terms of x, the number of students in each region. [3]

(ii) Write an expression, in terms of x, which represents the TOTAL number of students in the survey. [1]

(iii) Determine the number of students in Form 5 who used ONLY cameras. [2]

(b) In the diagram below, **not drawn to scale**, AEC and ADB are straight lines.
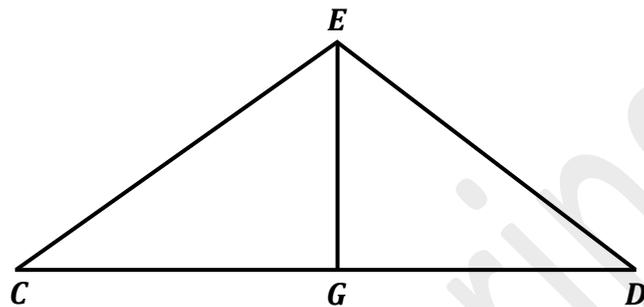
$\angle ABC = \angle ADE = 90°$

$AC = 10\ m$, $AB = 8\ m$ and $DB = 3.2\ m$
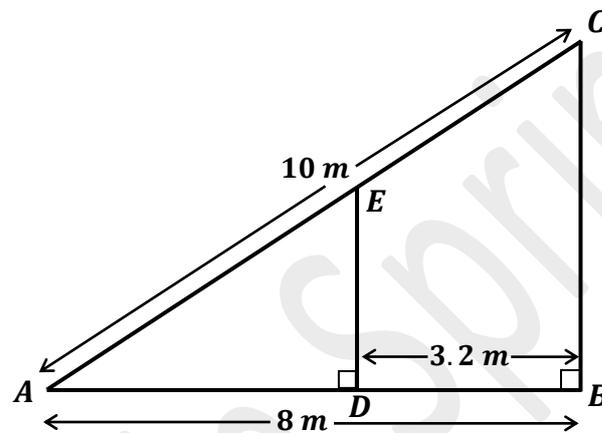
(i)   Calculate the length of BC.   [2]

(ii)   Explain why triangles ABC and ADE are similar.   [1]

(iii)   Determine the length of DE.   [3]

Total: 12 marks

4. (a) The diagram below shows an isosceles triangle CDE. G is the midpoint of CD.

(i) Measure and state, in centimetres, the length of DE. [1]

(ii) Measure and state, in degrees, the size of $\angle ECD$. [1]

(iii) Determine the perimeter of the triangle CDE. [2]

(iv) Calculate the area of the triangle CDE. [1]

(b) $A(-1, 4)$ and $B(3, 2)$ are the end points of a line segment AB. Determine

(i) the gradient of AB [2]

(ii) the coordinates of the midpoint of AB [2]

(iii) the equation of the perpendicular bisector of AB. [3]

Total: 12 marks

5. (a) The incomplete table below shows one pair of values for A and R where A is

directly proportional to the square of R.

A	36		196
R	3	5	

   (i)   Express A in terms of R and a constant k.   [1]

   (ii)   Calculate the value of the constant k.   [2]

   (iii) Copy and complete the table.   [2]

(b) Given that $f(x) = \frac{2x+1}{3}$ and $g(x) = 4x + 5$, determine the values of:

   (i) $fg(2)$ [3]

   (ii) $f^{-1}(3)$ [3]

Total: 11 marks

6. (a) A car, travelling along a straight road at a constant speed of 54 km/h, takes

20 seconds to travel the distance between two sign posts.

Calculate

(i)   the speed of the car in m/s [2]

(ii)   the distance, in metres, between the two sign posts   [2]

(b) **An answer sheet is provided for this question**.

The graph below shows triangle LMN and its image $L'M'N'$ after undergoing

a single transformation.
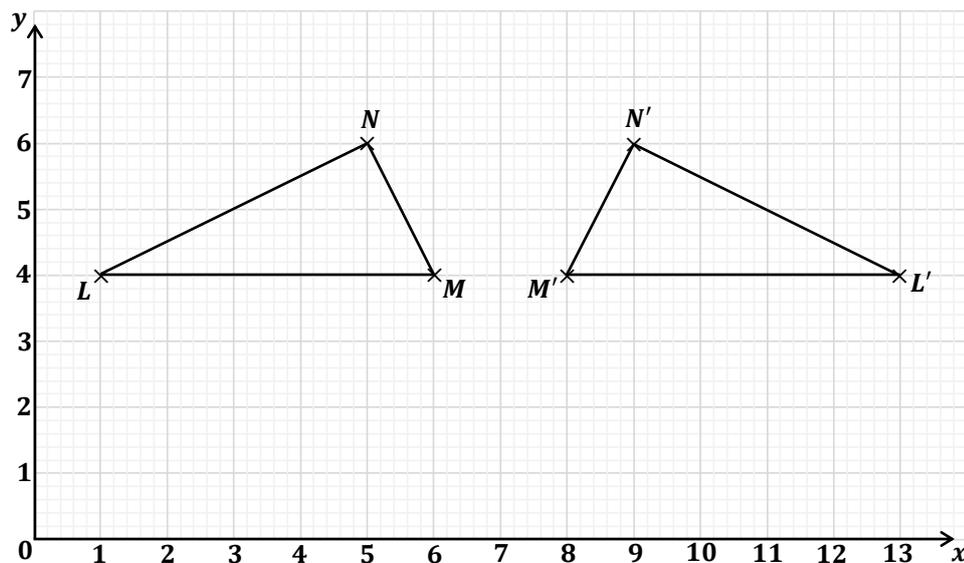
(i)   Describe **fully** the transformation that maps ΔLMN onto $\Delta L'M'N'$. [2]

(ii)   **On the answer sheet provided**, draw triangle $L''M''N''$, the image of

triangle LMN, after a translation by the vector $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$. [2]

(iii)   Name and describe a combination of TWO transformations which

may be used to map $\Delta L''M''N''$ onto $\Delta L'M'N'$. [3]

Total: 11 marks

7.   The table below shows the amount, to the nearest dollar, spent by a group of 40

students at the school canteen during a period of one week.

Amount Spent ($)	Number of Students	Cumulative Frequency
$1 - 10$	3	3
$11 - 20$	7	10
$21 - 30$	9	19
$31 - 40$	11	
$41 - 50$	8	
$51 - 60$	2	

(a) Copy and complete the table to show the cumulative frequency. [2]

(b) Using a scale **of 1 cm to represent \$5 on the horizontal axis and 1 cm to**

represent 5 students on the vertical axis, draw the cumulative frequency

graph for the data. [5]

(Marks will be awarded for axes appropriately labelled, points correctly

plotted, and a smooth curve carefully drawn.)

(c) Use your graph to estimate

   (i)   the median amount of money spent   [2]

   (ii)   the probability that a student chosen at random spent less than $23

           during the week.   [2]

           Show on your graph, using broken lines, how these estimates were

           determined.

                                                              Total: 11 marks

8. **An answer sheet is provided for this question.**

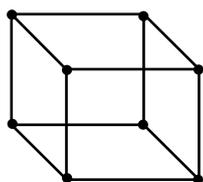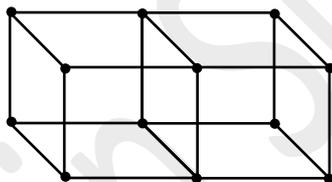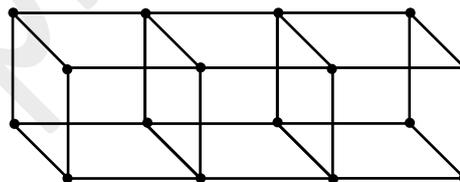   The drawings below show the first three diagrams in a sequence.
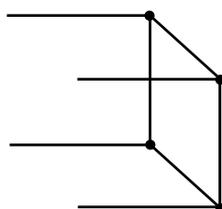

    Each diagram is made up of wires of equal length which are joined at the ends by

    balls of plasticine. Diagram 1 is made of 12 wires and 8 balls. Each new diagram in

    the sequence is formed by fitting the frame shown below to the right of the

    previous diagram.

    Thus, Diagram 2 has 8 more wires and 4 more balls than Diagram 1.

On the answer sheet provided:

(a) Draw a sketch of Diagram 4, the fourth diagram in the sequence.   [2]

(b) Complete the table by inserting the missing values at the rows marked

(i) and (ii).

	Name of Diagram (N)	No. of Wires (W)	No. of Balls (B)	
	1	12	8	
	2	20	12	
	3	28	16	
(i)	4	_____	_____	[2]
(ii)	20	_____	_____	[4]

(c) Write the rules which may be used to find the values of W and of B where N

is known.

(i) $W =$ _____   [1]

(ii) $B =$ _____   [1]

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) **An answer sheet is provided for this question.**

   Trish wishes to buy x oranges and y mangoes which she intends to carry in her bag. Her bag has space for only 6 fruits.

   (i)   Write an inequality to represent this information.   [1]

   To get a good bargain, she must buy AT LEAST 2 mangoes.

   (ii)   Write an inequality to represent this information.   [1]

   More information about the number of oranges and mangoes associated with the good bargain is represented by

   $y \leq 2x.$

(iii)   Write the information represented by this inequality as a sentence in

your own words.   [2]

(iv)   **On the answer sheet provided**, draw the lines associated with the two

inequalities obtained in (i) and (ii) above.   [3]

(v)   Shade on your graph the region which represents the solution set for

the three inequalities.   [1]

(b)(i) Write $3x^2 - 12x + 8$ in the form $a(x + h)^2 + k$, where a, h and k are

constants.   [3]

(ii) Sketch the graph of $y = 3x^2 - 12x + 8$, showing on your sketch

(a) the intercept on the y-axis

(b) the coordinates of the minimum point.   [4]

**Total: 15 marks**

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle with centre O. EBC is a

tangent to the circle. $\angle OBA = 40°$ and $\angle OBF = 35°$.
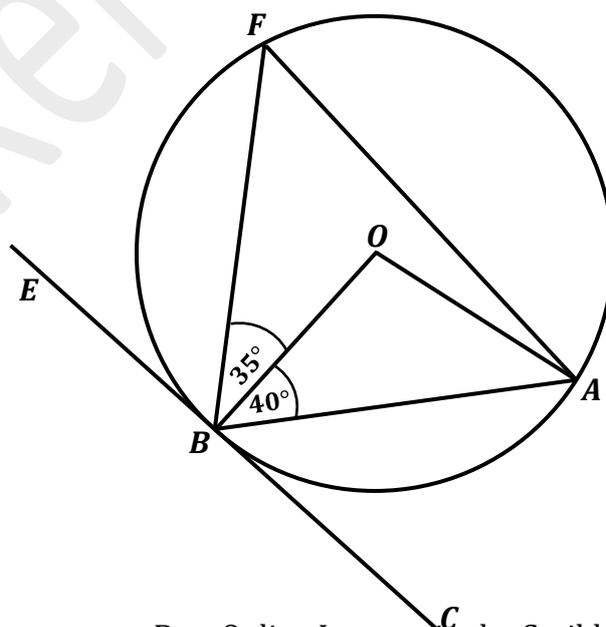
Calculate, **giving reasons for your answer**, the measure of

(i) $\angle EBF$ [1]

(ii) $\angle BOA$ [2]

(iii) $\angle AFB$ [2]

(iv) $\angle OAF$ [2]

(b) The diagram below, **not drawn to scale**, shows three points R, S and F on the horizontal ground. FT is a vertical tower of height 25 m. The angle of elevation of the top of the tower, T, from R is 27°. R is due east of F and S is due south of F. $SF = 43.3\ m$.
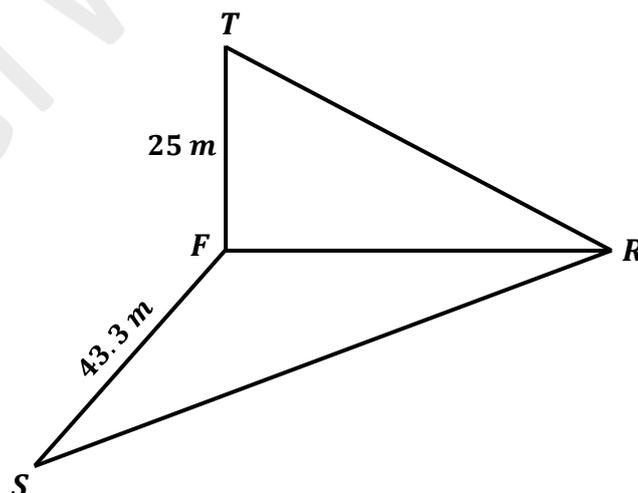
(i)   Sketch **separate** diagrams of the triangles RFT, TFS and SFR. Mark on

         EACH diagram the given measures of sides and angles.   [3]

(ii)   Show, by calculation, that $RF = 49.1$ m.   [2]

(iii)   Calculate the length of SR correct to 1 decimal place.   [1]

(iv)   Calculate the angle of elevation of the top of the tower, T, from S. [2]

                                                              Total: 15 marks

VECTORS AND MATRICES

11. (a) In the diagram below, **not drawn to scale**, P and Q are the midpoints of OA

    and AB respectively.

    $\overrightarrow{OA} = 2\boldsymbol{a}$ and $\overrightarrow{OB} = 2\boldsymbol{b}$.
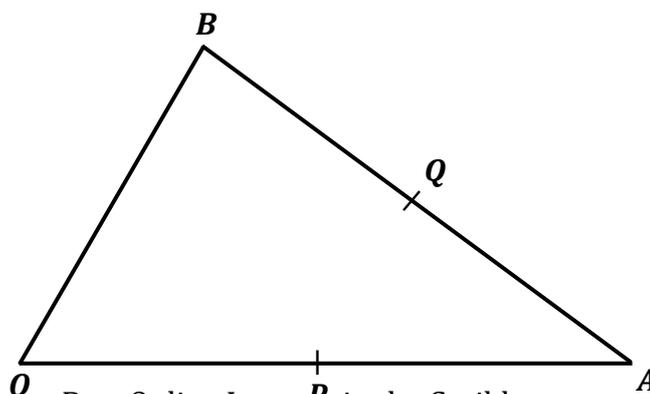
(i)   Express in terms of **a** and **b** the vectors

(a) $\overrightarrow{AB}$ [2]

(b) $\overrightarrow{PQ}$ [2]

(ii)   State TWO geometrical relationships that exist between OB and PQ.

Give reasons for your answers.   [2]

(b) Given that $M = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

(i)   Evaluate M^{-1}, the inverse of M.   [2]

(ii)   Show that $M^{-1}M = I$.   [2]

(iii)   Use a matrix method to solve for r, s, t and u in the equation

$$\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}.$$ [5]

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.