

CSEC Mathematics
June 2016 – Paper 2
Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate

(i) $\frac{3\frac{3}{8} - 2\frac{1}{4}}{1\frac{1}{2}}$ giving your answer as a fraction in its lowest terms. [2]

$$\begin{aligned} \text{Numerator} &= 3\frac{3}{8} - 2\frac{1}{4} \\ &= \frac{27}{8} - \frac{9}{4} \\ &= \frac{27-18}{8} \\ &= \frac{9}{8} \end{aligned}$$

$$\begin{aligned} \text{Denominator} &= 1\frac{1}{2} \\ &= \frac{3}{2} \end{aligned}$$

Now,

$$\begin{aligned} \text{Numerator} \div \text{Denominator} &= \frac{9}{8} \div \frac{3}{2} \\ &= \frac{9}{8} \times \frac{2}{3} \\ &= \frac{3}{4} \end{aligned}$$

$$\therefore \frac{3\frac{3}{8} - 2\frac{1}{4}}{1\frac{1}{2}} = \frac{3}{4}$$

- (ii) $(2.86 + 0.75) + 0.481^2$ giving your answer correct to 2 decimal places. [2]

$$\begin{aligned} (2.86 + 0.75) + 0.481^2 &= 3.61 + 0.231361 \\ &= 3.841361 \\ &= 3.84 \quad (\text{to 2 decimal places}) \end{aligned}$$

- (b) Paul bought and sold a computer. He wrote his business activity as follows:

Cost price of computer = \$1064

Marked price of computer = \$1399

Discount on marked price = 5%

(if paid by cash)

Calculate

- (i) the selling price (paid cash) [2]

$$\begin{aligned} \text{Discount} &= 5\% \text{ of cash payment} \\ &= 0.05 \times 1399 \\ &= \$69.95 \end{aligned}$$

$$\begin{aligned} \text{Selling Price} &= \text{Marked price} - \text{Discount} \\ &= \$1399 - \$69.95 \\ &= \$1329.05 \end{aligned}$$

∴ The selling price is \$1329.05.

- (ii) the profit or loss as a percentage of the cost price. [2]

$$\begin{aligned} \text{Profit} &= \$1329.05 - \$1064.00 \\ &= \$265.05 \end{aligned}$$

$$\begin{aligned} \text{Percentage profit} &= \frac{\text{Profit}}{\text{Cost price}} \times 100 \\ &= \frac{\$265.05}{\$1064} \times 100 \\ &= 24.9\% \quad (\text{to 3 significant figures}) \end{aligned}$$

∴ The profit as a percentage of the cost price is 24.9%.

- (c) Orange juice is sold in cartons of three different sizes.

Carton Size	Selling Price
350 ml	\$4.20
450 ml	\$5.35
500 ml	\$5.80

Which size of orange juice is the most cost-effective buy?

Justify your answer. [3]

Carton Size of 350 ml = \$4.20

$$1 \text{ ml} = \frac{\$4.20}{350}$$
$$= \$0.012$$

Carton Size of 450 ml = \$5.35

$$1 \text{ ml} = \frac{\$5.35}{450}$$
$$= \$0.0119$$

Carton Size of 500 ml = \$5.80

$$1 \text{ ml} = \frac{\$5.80}{500}$$
$$= \$0.0116$$

The most cost-effective carton would be the 500 ml , as it costs least per ml.

Total: 11 marks

2. (a) Factorize completely:

(i) $4a^2 - 16$ [2]

$$\begin{aligned} &4a^2 - 16 \\ &= 4(a^2 - 4) \\ &= 4(a + 2)(a - 2) \end{aligned}$$

(ii) $3y^2 + 2y - 8$ [2]

$$\begin{aligned} &3y^2 + 2y - 8 \\ &= 3y^2 + 6y - 4y - 8 \\ &= 3y(y + 2) - 4(y + 2) \\ &= (3y - 4)(y + 2) \end{aligned}$$

(b) Solve the simultaneous equations:

$$2x + y = 3$$

$$5x - 2y = 12 \quad [5]$$

$$2x + y = 3 \quad \rightarrow \text{Equation 1}$$

$$5x - 2y = 12 \quad \rightarrow \text{Equation 2}$$

Multiply Equation 1 by 2 to get:

$$4x + 2y = 6 \quad \rightarrow \text{Equation 3}$$

Equation 3 + Equation 2 gives:

$$9x = 18$$

$$x = \frac{18}{9}$$

$$x = 2$$

Substituting $x = 2$ into Equation 1 gives:

$$2(2) + y = 3$$

$$4 + y = 3$$

$$y = 3 - 4$$

$$y = -1$$

$$\therefore x = 2 \text{ and } y = -1$$

- (c) The table below shows corresponding values of the variables x and y , where y varies directly as x .

x	6	10	t
y	3	u	9

Calculate the values of t and u .

Since y is directly proportional to x , then

$$y \propto x$$

$$y = kx$$

Substituting $x = 6$ and $y = 3$ into $y = kx$ gives:

$$3 = k(6)$$

$$k = \frac{3}{6}$$

$$k = \frac{1}{2}$$

So, we have $y = \frac{1}{2}x$.

When $x = 10$ and $y = u$,

$$u = \frac{1}{2}(10)$$

$$u = 5$$

When $x = t$ and $y = 9$

$$9 = \frac{1}{2}t$$

$$t = 9 \times 2$$

$$t = 18$$

$\therefore u = 5$ and $t = 18$

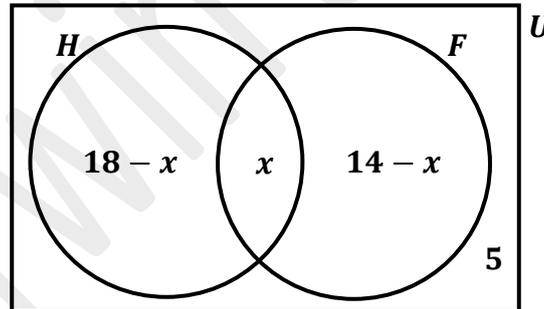
Total: 12 marks

3. (a) The Venn diagram below shows the number of students who study History and French in a class of 30 students.

$U = \{\text{students in the class}\}$

$H = \{\text{students who study History}\}$

$F = \{\text{students who study French}\}$



- (i) Write an expression, in x , in its simplest form, for the TOTAL number of students in the class. [2]

$$\begin{aligned} \text{Total number of students in class} &= 18 - x + x + 14 - x + 5 \\ &= 37 - x \end{aligned}$$

(ii) State whether the following relationships are true or false:

- $H \cup F = U$ **False**
- $H \cap F' = \emptyset$ **False**

$$(H \cup F)' = 5 \text{ which is not equal to } 0$$

$$\therefore H \cup F \neq U$$

Hence, the statement given is false.

Now,

$$\begin{aligned} H \cap F' &= 18 - 7 \\ &= 11 \end{aligned}$$

$$\text{So, } H \cap F' \neq \emptyset.$$

Hence, the statement given is false.

(iii) Determine the number of students who study BOTH History and French. [2]

There are a total of 30 students in the class.

So, we have,

$$37 - x = 30$$

$$x = 37 - 30$$

$$x = 7$$

So, $n(H \cap F) = 7$.

\therefore The number of students who study both History and French is 7.

(b)(i) Using a ruler, a pencil and a pair of compasses, construct triangle PQR

with $PQ = 5 \text{ cm}$, $\angle PQR = 60^\circ$ and $\angle QPR = 90^\circ$. [4]

(ii) Measure and state [2]

- the length of PR

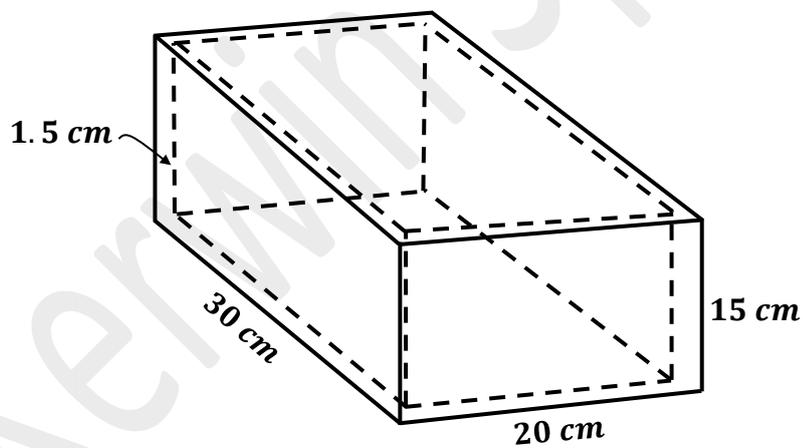
$PR =$

- the measure of $\angle PRQ$

$\angle PRQ =$

Total: 12 marks

4. The diagram below, **not drawn to scale**, shows a silver box with no lid. The sides and bottom of the box are 1.5 cm thick.



- (a) Calculate the volume of the box using the external measurements. [2]

$$\begin{aligned}
 \text{External volume of the box} &= l \times w \times h \\
 &= 30 \times 20 \times 15 \\
 &= 9000 \text{ cm}^3
 \end{aligned}$$

∴ The volume of the box using the external measurements is 9000 cm^3 .

(b) Complete EACH of the following statements.

(i) The internal length of the box is $30 \text{ cm} - 2 \times 1.5 \text{ cm} = \dots\dots\dots 27 \text{ cm}$ [1]

$$\begin{aligned} \text{Internal length} &= 30 - 2(1.5) \\ &= 30 - 3 \\ &= 27 \text{ cm} \end{aligned}$$

(ii) The internal width of the box is $\dots\dots\dots 17 \text{ cm}$ [2]

$$\begin{aligned} \text{Internal width} &= 20 - 2(1.5) \\ &= 20 - 3 \\ &= 17 \text{ cm} \end{aligned}$$

(iii) The internal depth of the box is $\dots\dots\dots 13.5 \text{ cm}$ [1]

$$\begin{aligned} \text{Internal depth} &= 15 - 1(1.5) \\ &= 15 - 1.5 \\ &= 13.5 \text{ cm} \end{aligned}$$

(c) Calculate the internal volume of the box. [2]

$$\begin{aligned}
 \text{Internal volume of the box} &= l \times w \times h \\
 &= 27 \times 17 \times 1305 \\
 &= 6196.5 \text{ cm}^3
 \end{aligned}$$

\therefore The internal volume of the box is 6196.5 cm^3 .

- (d) The box is made of silver which has a mass of 10.5 g for each cm^3 . Calculate the mass of the silver box, giving your answer in kg . [3]

Volume of silver = External volume of the box – Internal volume of the box

$$\text{Volume of silver} = 9000 - 6196.5$$

$$\text{Volume of silver} = 2803.5 \text{ cm}^3$$

Density of silver is 10.5 gcm^{-3} .

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$10.5 = \frac{\text{Mass}}{2803.5}$$

$$\text{Mass} = 2803.5 \times 10.5$$

$$\text{Mass} = 29436.75 \text{ g}$$

We need to convert this value to *kg*.

$$1000 \text{ g} = 1 \text{ kg}$$

$$1 \text{ g} = \frac{1}{1000} \text{ kg}$$

$$29436.75 \text{ g} = 29436.75 \times \frac{1}{1000}$$

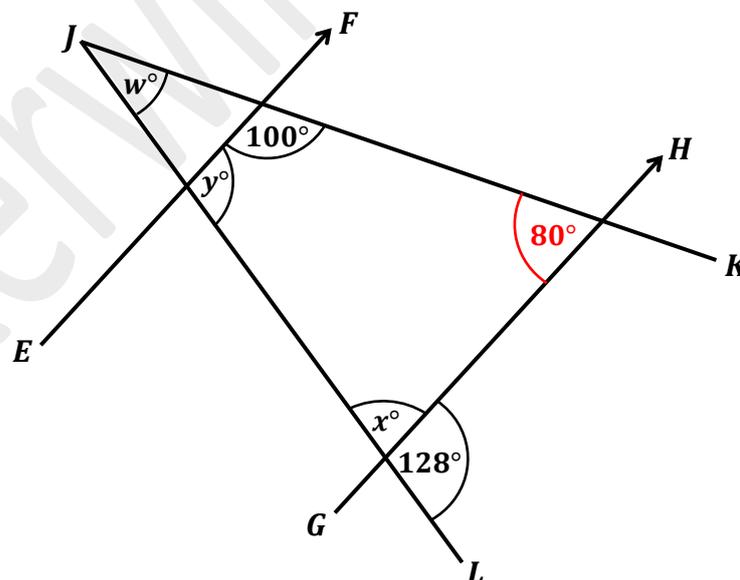
$$= 29.43675$$

$$= 29.4 \text{ kg} \quad (\text{to 3 significant figures})$$

\therefore The mass of the silver box is 29.4 kg .

Total: 11 marks

5. (a) The diagram below, **not drawn to scale**, shows two straight lines, *JK* and *JL*, intersecting a pair of parallel lines, *EF* and *GH*.



Determine, giving reasons for EACH of your answers, the value of

(i) x

Angles in a straight line sum to 180° .

$$x = 180^\circ - 128^\circ$$

$$= 52^\circ$$

(ii) y [2]

Angle y and 128° are corresponding angles.

Corresponding angles are equal to each other.

$$\therefore y = 128^\circ$$

(iii) w [2]

Angles in a quadrilateral add up to 360° .

$$\angle JHG = 360^\circ - (100^\circ + y^\circ + x^\circ)$$

$$\angle JHG = 360^\circ - (100^\circ + 128^\circ + 52^\circ)$$

$$\angle JHG = 360^\circ - 280^\circ$$

$$= 80^\circ$$

The sum of angles in a triangle is equal to 180° .

$$w^\circ + \angle JHG + x^\circ = 180^\circ$$

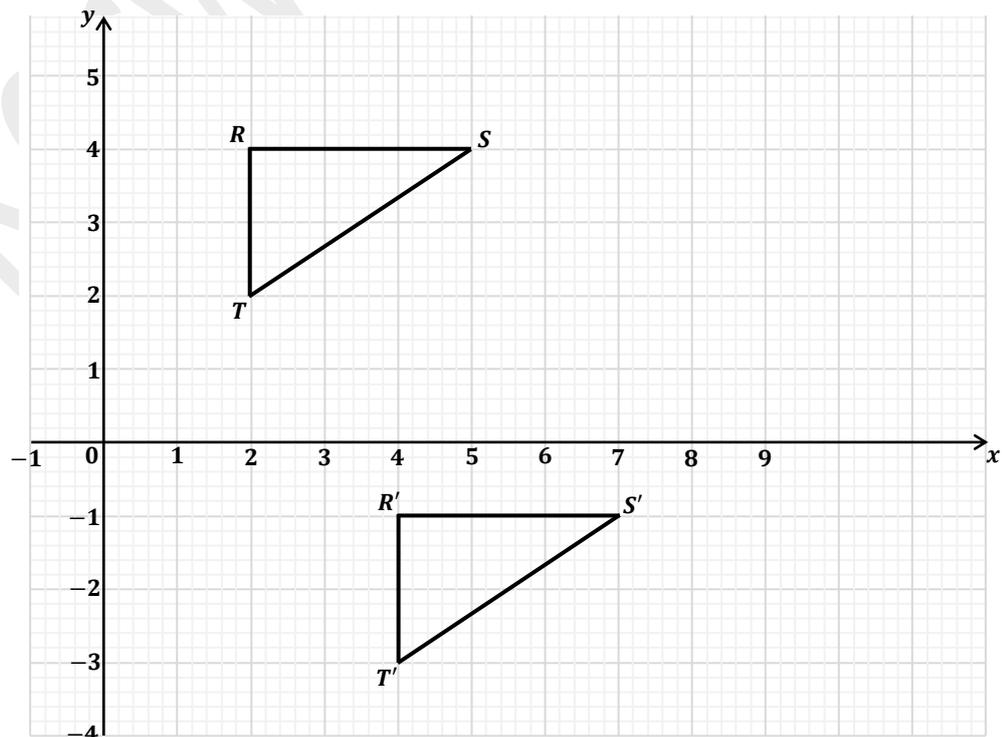
$$w^\circ + 80^\circ + 52^\circ = 180^\circ$$

$$w^\circ + 132^\circ = 180^\circ$$

$$w^\circ = 180^\circ - 132^\circ$$

$$w^\circ = 48^\circ$$

(b) The diagram below shows triangle RST and its image $R'S'T'$ after a transformation.



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(i) Describe FULLY the transformation which maps RST onto $R'S'T'$. [3]

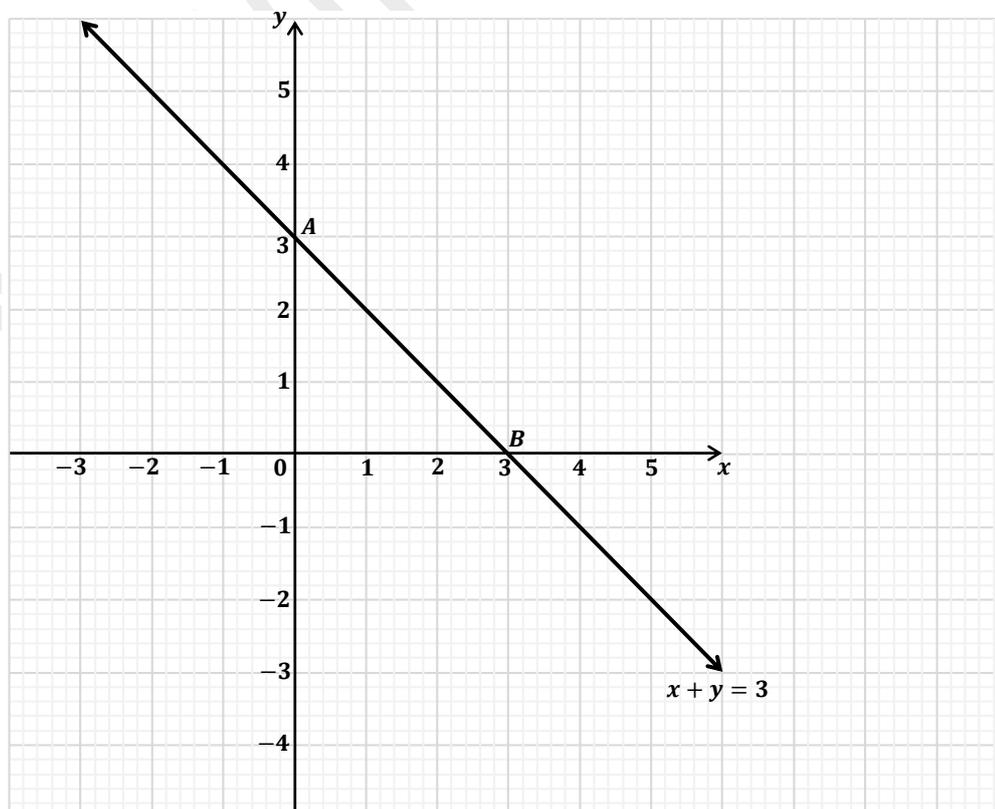
(ii) Triangle RST is reflected in the line $x = 6$. On the graph above, draw triangle $R''S''T''$, the image of ΔRST , after the reflection.

Write down the coordinates of R'' .

$R'' = \dots\dots\dots$ [3]

Total: 12 marks

6. (a) The diagram below shows the graph of the straight line $x + y = 3$.



Determine the equation of the line which is

- (i) parallel to the line $x + y = 3$ and passes through the origin [2]

Consider the line $x + y = 3$.

Rearranging gives:

$$y = -x + 3 \quad \text{which is in the form } y = mx + c,$$

where $m = -1$ and $c = 3$.

Parallel lines have equal gradients.

So, the gradient of the line parallel to the line $x + y = 3$ is $m = -1$.

Now, this parallel line also passes through the origin.

This means that the y -intercept = 0.

So, the equation of the parallel line is:

$$y = -x + 0$$

$$y = -x \quad \text{which is in the form } y = mx + c,$$

where $m = -1$ and $c = 0$.

- (ii) perpendicular to the line $x + y = 3$ and passes through the midpoint of AB . [2]

- (b) The function $y = x^2 - 2x - 3$ is defined in the domain $-2 \leq x \leq 4$. The table below shows the corresponding values of y for five selected values of x .

x	-2	-1	0	1	2	3	4
y	5	0	-3		-3		5

- (i) Complete the table by calculating and inserting the missing values of y . [2]
- (ii) On the same axes used in Part (a) on page 16, draw the graph of $y = x^2 - 2x - 3$. [2]
- (iii) Using information from your graph, complete EACH of the following statements.
- The minimum value of $y = x^2 - 2x - 3$ occurs when $x = \dots\dots\dots$ [1]

- The values of x for which $x^2 - 2x - 3 = -x + 3$ are

$x = \dots\dots\dots$ and $x = \dots\dots\dots$

[2]

Total: 11 marks

7. Twenty bags of sugar were weighed. The weights, to the nearest kg, are as follows:

3	38	17	33	28
12	43	38	31	30
11	8	23	18	26
50	22	35	39	5

(a) Complete the frequency table for the data shown above.

[4]

Weight (kg)	Tally	Number of Bags
1 – 10		
11 – 20		
21 – 30		
31 – 40		

41 – 50		
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(b) For the class interval 21 – 30, state:

- (i) the upper class boundary [1]
- (ii) the class width [1]
- (iii) the class midpoint [1]

(c) On the grid on page 19, using a scale of **2 cm to represent 10 kg on the x-axis**, and **1 cm to represent 1 bag on the y-axis**, draw a histogram to represent the data contained in your frequency table above. [4]

Total: 11 marks

8. The diagram below show the first three figures in a sequence. Each figure is made up of knots and strings. Each knot connects exactly 3 strings.



Figure 1

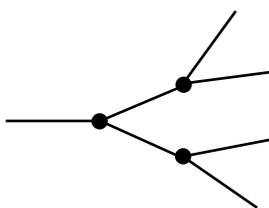


Figure 2

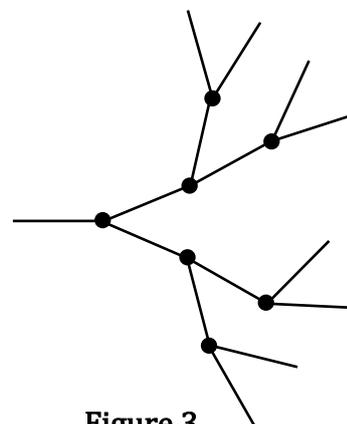


Figure 3

(a) Draw Figure 4 of the sequence. [2]

Figure 4

(b) The table below shows the number of knots and strings in each figure used to draw Figures 1, 2 and 3. Complete the table by inserting the missing values in the rows numbered (i), (ii) and (iii).

	Figure Number (N)	Number of Knots (K)	Number of Strings (S)	
	1	1	3	
	2	3	7	
	3	7	15	
(i)	4	[2]
(ii)	255	[3]

(iii) Best Online Lessons in the Caribbean
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Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The functions $f(x)$ and $g(x)$ are defined as $f(x) = 2x - 7$ and $g(x) = x^2 + 1$, respectively.

(i) Write an expression, in terms of x , for EACH of the following:

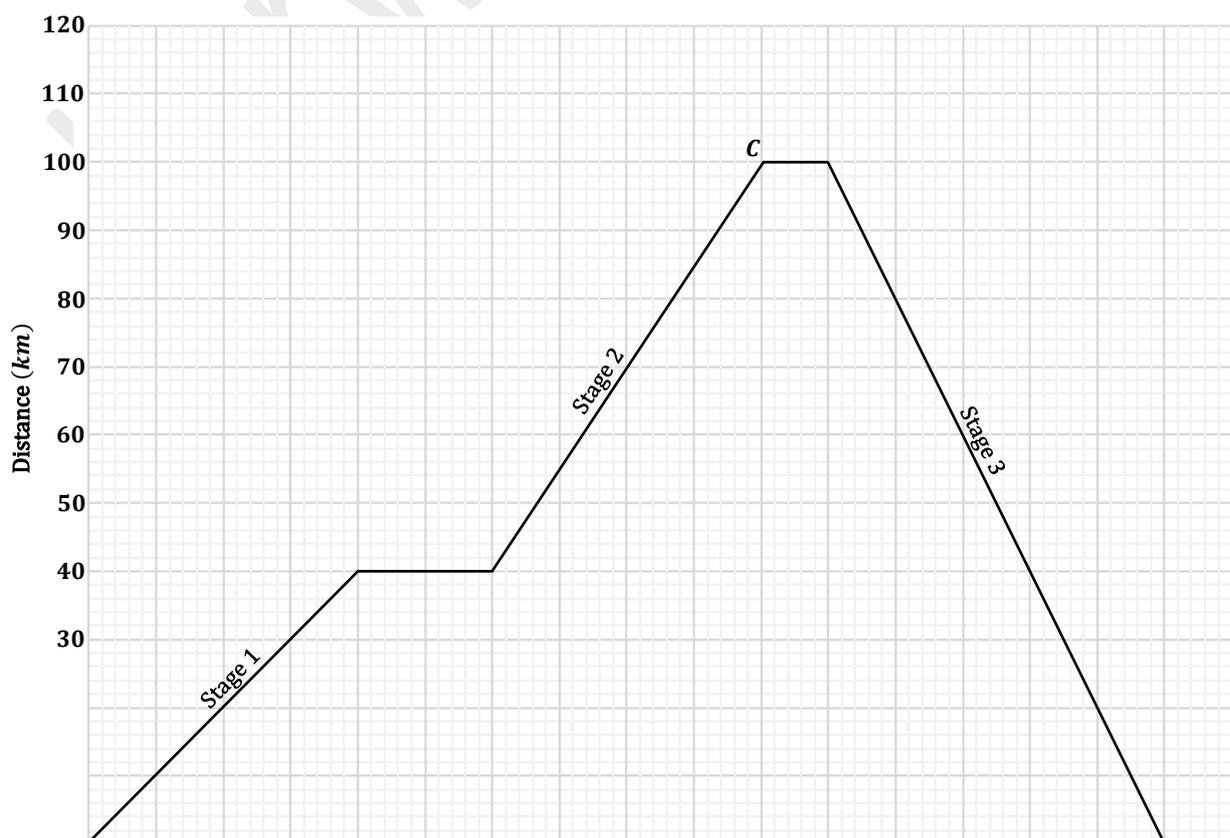
- $f^{-1}(x)$
- $g^{-1}(x)$
- $fg(x)$
- $(fg)^{-1}(x)$

[4]

(ii) Show that $(fg)^{-1}(5) = g^{-1}f^{-1}(5)$.

[3]

(c) The distance-time graph below shows the three-stage journey of a car travelling from Town A to Town C and back to Town A.



20

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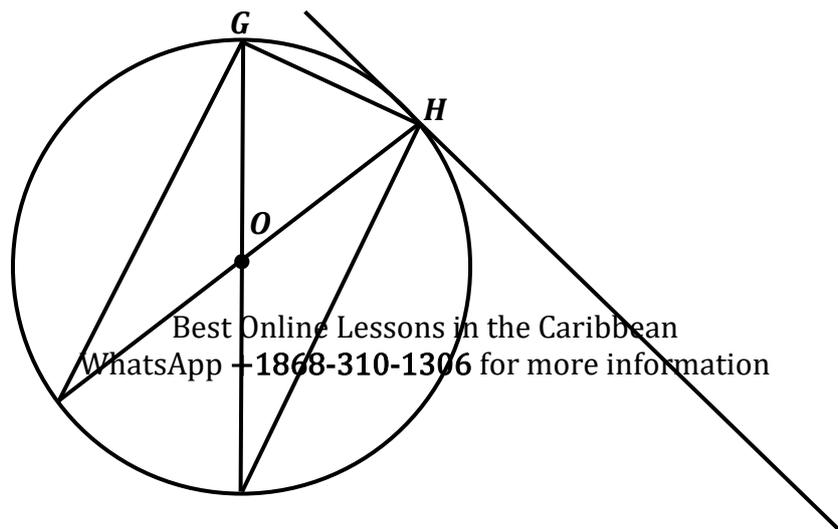
06:00	07:00	08:00	09:00	10:00
Time of day				

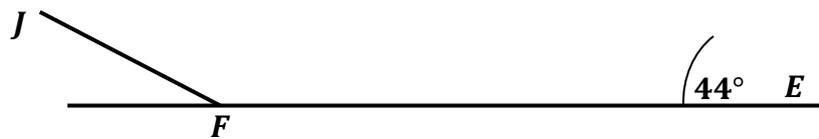
- (i) State the time of day at which the car arrived at Town C. [1]
- (ii) Calculate the TOTAL time, in minutes, for which the car stopped during the journey. [2]
- (iii) Determine the constant speed of the car during Stage 2 of the journey. [2]
- (iv) Calculate the average speed of the car for the time during which it was moving. [3]

Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle, centre O . EH and EF are tangent to the circle. FOG and JOH are straight lines. The measure of $\angle FEH = 44^\circ$.

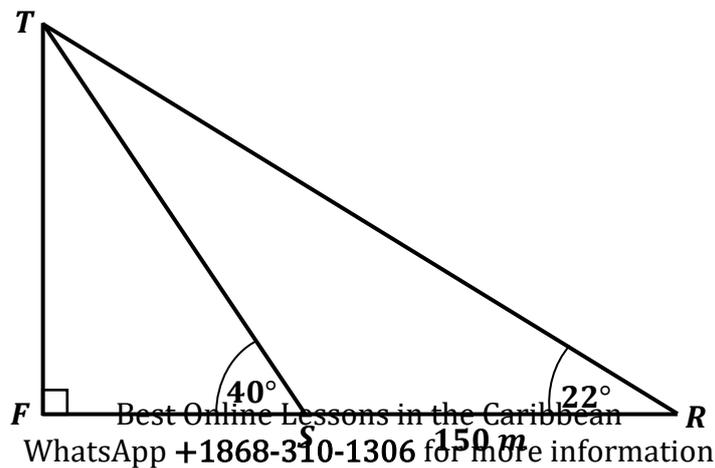




Calculate, **giving reasons for your answer**, the measure of:

- (i) $\angle EHF$ [2]
- (ii) $\angle FGH$ [2]
- (iii) $\angle JHE$ [2]
- (iv) $\angle JGH$ [2]

- (b) The diagram below, **not drawn to scale**, shows two ships, R and S at anchor on a lake of calm water. FT is a vertical tower. FSR is a straight line and $RS = 150\text{ m}$. The angles of elevation of T , the top of a tower, from R and S , are 22° and 40° respectively. F is the foot of the tower.



Calculate, giving your answer to 1 decimal place where appropriate

- (i) the measure of $\angle RTS$ [1]
- (ii) the length of ST [3]
- (iii) the height of the tower, FT [3]

Total: 15 marks

VECTORS AND MATRICES

11. (a) The position vectors of points A , B and C , relative to the origin O , are

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}; \overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \end{pmatrix} \text{ and } \overrightarrow{OC} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} \text{ respectively.}$$

- (i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ the vectors
- $\overrightarrow{AB} = \dots\dots\dots$
 - $\overrightarrow{AC} = \dots\dots\dots$ [3]

- (ii) Hence, determine whether A, B and C are collinear, **giving the reasons for your answer.** [3]

(b) Determine the value of x for which the matrix $\begin{pmatrix} 3 & x \\ 2 & 4 \end{pmatrix}$ is singular. [2]

(c) N and P are 2×2 matrices such that $N = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 5 \\ 2 & 1 \end{pmatrix}$.

(i) Determine NP . [1]

(ii) Given that $PN = \begin{pmatrix} 19 & 11 \\ 11 & 4 \end{pmatrix}$, determine whether matrix multiplication is commutative. [1]

(iii) Determine N^{-1} , the inverse of N . [2]

(iv) Hence, calculate the values of x and y for which

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}. \quad [3]$$

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.