

CSEC Mathematics
June 2015 – Paper 2
Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $2\frac{2}{5} - 1\frac{1}{3} + 3\frac{1}{2}$ [1]

$$\begin{aligned}
 2\frac{2}{5} - 1\frac{1}{3} + 3\frac{1}{2} &= \frac{12}{5} - \frac{4}{3} + \frac{7}{2} \\
 &= \frac{72}{30} - \frac{40}{30} + \frac{105}{30} \\
 &= \frac{72-40+105}{30} \\
 &= \frac{137}{30} \\
 &= 4\frac{17}{30}
 \end{aligned}$$

(ii) $(4.14 \div 5.75) + (1.62)^2$ [2]

$$(4.14 \div 5.75) + (1.62)^2 = 3.3444$$

(iii) $2 \times 3.142 \times 1.25$ [1]

$$2 \times 3.142 \times 1.25 = 7.855$$

(iv) $\sqrt{2.89} \times \tan 45^\circ$ [2]

$$\sqrt{2.89} \times \tan 45^\circ = 1.7$$

- (b) The table below shows a shopping bill prepared for Mrs. Rowe. The prices of some items are missing.

Shopping Bill		
Item	Unit Cost Price	Total Cost Price
3 kg sugar	X	\$10.80
4 kg rice	Y	Z
2 kg flour	\$1.60	\$3.20

- (i) Calculate the value of X , the cost of 1 kg of sugar. [1]

$$3 \text{ kg of sugar cost} = \$10.80$$

$$\therefore 1 \text{ kg of sugar costs} = \frac{\$10.80}{3}$$

$$= \$3.60$$

Hence, $X = \$3.60$.

- (ii) If the cost price of 1 kg of rice is 80 cents MORE than for 1 kg of flour, calculate the values of Y and Z . [2]

From the table, 1 kg of flour costs \$1.60.

The cost price of 1 kg of rice is 80 cents more than for 1 kg of flour.

So, we have,

$$\text{The cost of 1 kg of rice} = \$1.60 + \$0.80$$

$$= \$2.40$$

Hence, $Y = \$2.40$.

Now,

$$\begin{aligned} \text{The cost of 4 kg of rice} &= \$2.40 \times 4 \\ &= \$9.60 \end{aligned}$$

Hence, $Z = \$9.60$.

- (iii) A tax of 10% of the total cost price of the three items is to be added to Mrs. Rowe's bill.

What is Mrs. Rowe's TOTAL bill including the tax? [3]

$$\begin{aligned} \text{The sum of Mrs. Rowe's bill before tax} &= \$10.80 + \$9.60 + \$3.20 \\ &= \$23.60 \end{aligned}$$

Tax = 10% of \$23.60

$$\text{Tax} = \frac{10}{100} \times \$23.60$$

Tax = \$2.36

Now,

$$\begin{aligned} \text{Mrs. Rowe's total bill} &= \$23.60 + \$2.36 \\ &= \$25.96 \end{aligned}$$

\therefore Mrs. Rowe's total bill including tax is \$25.96.

2. (a) Given that $a = 4$, $b = 2$ and $c = -1$, find the value of:

(i) $a - b + c$ [1]

Substituting $a = 4$, $b = 2$ and $c = -1$ gives,

$$\begin{aligned} a - b + c &= (4) - (2) + (-1) \\ &= 4 - 2 - 1 \\ &= 1 \end{aligned}$$

(ii) $2a^b$ [1]

Substituting $a = 4$, $b = 2$ and $c = -1$ gives,

$$\begin{aligned} 2a^b &= 2(4)^2 \\ &= 2 \times 16 \\ &= 32 \end{aligned}$$

(b) A bottle contains **500 ml** of orange juice. Write an expression for EACH of the following.

The amount of juice left in the bottle after pouring out

(i) p ml [1]

The amount of juice left in the bottle

= The initial amount of juice – The amount of juice poured out

$$= 500 - p$$

∴ The expression is $(500 - p)$ ml.

(ii) q glasses each containing r ml. [1]

The amount of juice in q glasses with r ml each = qr ml

The amount of juice left in the bottle

= The initial amount of juice – The amount of juice poured out

$$= 500 - qr$$

∴ The expression is $(500 - qr)$ ml.

(c) Write as a single fraction, as simply as possible

$$\frac{2k}{3} + \frac{2-k}{5} \quad [5]$$

$$= \frac{2k}{3} + \frac{2-k}{5}$$

$$= \frac{5(2k)+3(2-k)}{15}$$

$$= \frac{10k+6-3k}{15}$$

$$= \frac{7k+6}{15}$$

(d) Four mangoes and two pears cost \$24.00, while two mangoes and three pears cost \$16.00.

- (i) Write a pair of simultaneous equations in x and y to represent the information given above. [2]

Let the cost of one mango be represented by x and the cost of one pear be represented by y .

The information provided can be represented by the pair of simultaneous equations:

$$4x + 2y = 24$$

$$2x + 3y = 16$$

- (ii) State clearly what x and y represent. [1]

x represents the cost of 1 mango

y represents the cost of 1 pear

(e) Factorize completely:

- (i) $a^3 - 12a$ [1]

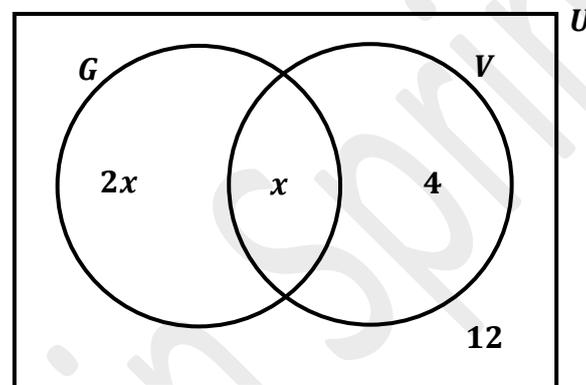
$$a^3 - 12a$$
$$= a(a^2 - 12)$$

(ii) $2x^2 - 5x + 3$ [2]

$$2x^2 - 5x + 3$$
$$= 2x^2 - 2x - 3x + 3$$
$$= 2x(x - 1) - 3(x - 1)$$
$$= (2x - 3)(x - 1)$$

Total: 12 marks

3. (a) The Venn diagram below shows the number of students who play the guitar (G) or the violin (V) in a class of 40 students.



- (i) How many students play neither the guitar nor the violin? [1]

Number of students who play neither guitar nor violin = 12 students

- (ii) Write an expression, in terms of x , which represents the TOTAL number of students in the class. [1]

$$\begin{aligned} \text{Total number of students in the class} &= 2x + x + 4 + 12 \\ &= 3x + 16 \end{aligned}$$

- (iii) Write an equation which may be used to determine the total number of students in the class. [1]

We are given that there are 40 students in the class.

Total number of students in the class as an equation is: $3x + 16 = 40$

- (iv) How many students play the guitar? [2]

$$3x + 16 = 40$$

$$3x = 40 - 16$$

$$3x = 24$$

$$x = \frac{24}{3}$$

$$x = 8$$

$$\begin{aligned} \text{Number of students who play the guitar} &= 2x + x \\ &= 3x \end{aligned}$$

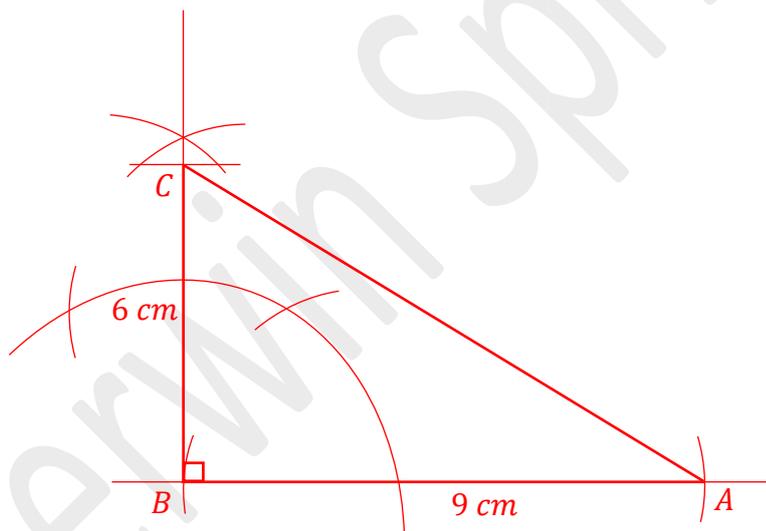
Substituting $x = 8$ gives:

$$\begin{aligned} \text{Number of students who play the guitar} &= 3(8) \\ &= 24 \end{aligned}$$

\therefore 24 students played the guitar.

- (b) (i) Using a ruler, a pencil and a pair of compasses, construct triangle ABC with $AB = 9\text{ cm}$, angle $ABC = 90^\circ$ and $BC = 6\text{ cm}$. [4]

The construction of triangle ABC is shown below:

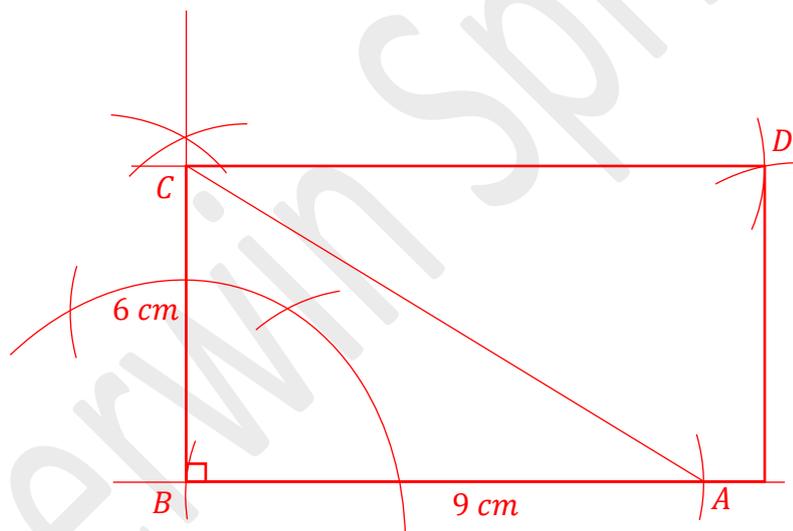


- (ii) Measure and state the size of angle BAC . [1]

With the use of a protractor, the measure of $\angle BAC = 34^\circ$.

(iii) On the diagram, show the point D such that $ABCD$ is a parallelogram. [2]

The construction of parallelogram $ABCD$ is shown below:



Total: 12 marks

4. A graph sheet is provided for this question.

The table below is designed to show values of x and y for the function

$y = x^2 - 2x - 3$ for integer values of x from -2 to 4 .

x	-2	-1	0	1	2	3	4
y	5		-3	-4	-3		5

(a) Complete the table for the function $y = x^2 - 2x - 3$. [2]

(b) On the graph on page 13, plot the graph of $y = x^2 - 2x - 3$ using a scale of 2 cm to represent 1 unit on the x -axis and 1 cm to represent 1 unit on the y -axis. [4]

(c) On the graph on page 13, draw a smooth curve passing through the points on your graph. [1]

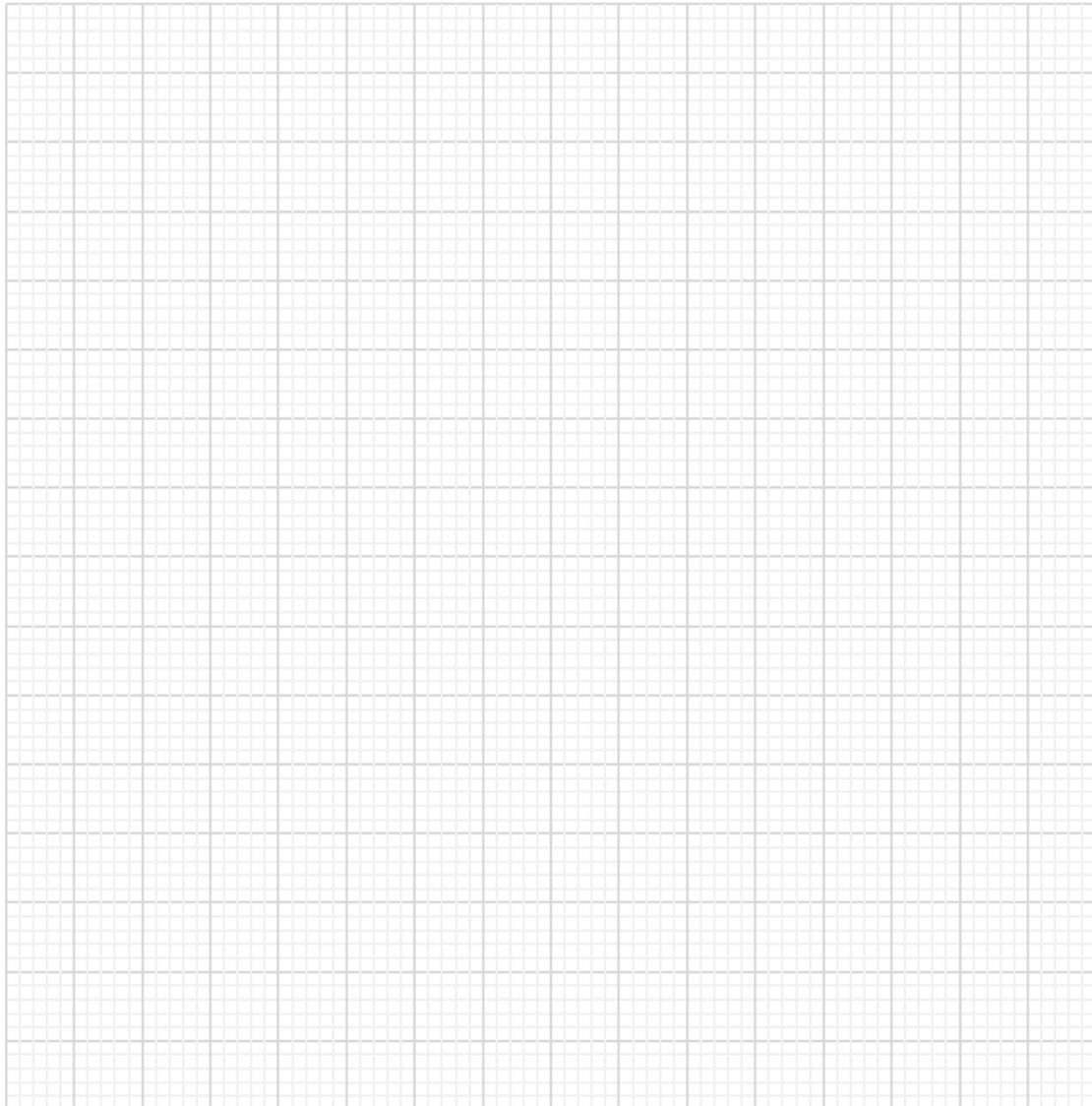
(d) Complete the following sentences using information from your graph.

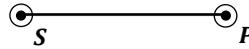
(i) The values of x for which $x^2 - 2x - 3 = 0$ are and [1]

(ii) The **minimum** value of $x^2 - 2x - 3$ is [1]

(iii) The equation of the line of symmetry of the graph of
 $y = x^2 - 2x - 3$ is [1]

Total: 10 marks





- (i) Determine, in centimetres, the distance from Q to R on the map.

$QR = \dots\dots\dots$ cm [1]

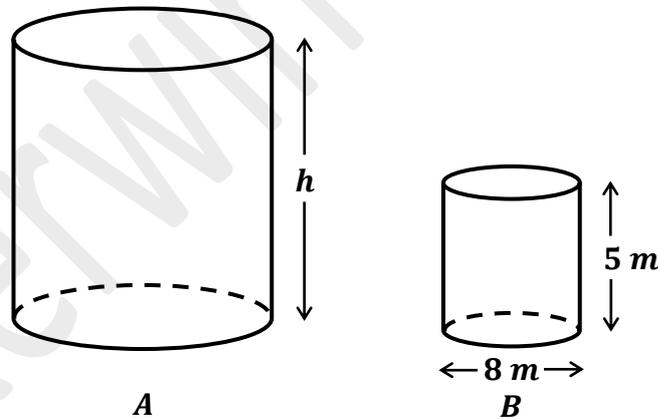
- (ii) Determine, by counting, the area in square centimetres of the plane $PQRS$ on the map. [2]

- (iii) Calculate the ACTUAL distance, in kilometres, between Q and R . [2]

- (iv) Calculate the ACTUAL area, in square metres, of the plane $PQRS$. [2]

Total: 12 marks

6. (a) The diagram below, **not drawn to scale**, shows two cylindrical water tanks, *A* and *B*. Tank *B* has base diameter 8 m and height 5 m . Both tanks are filled with water.

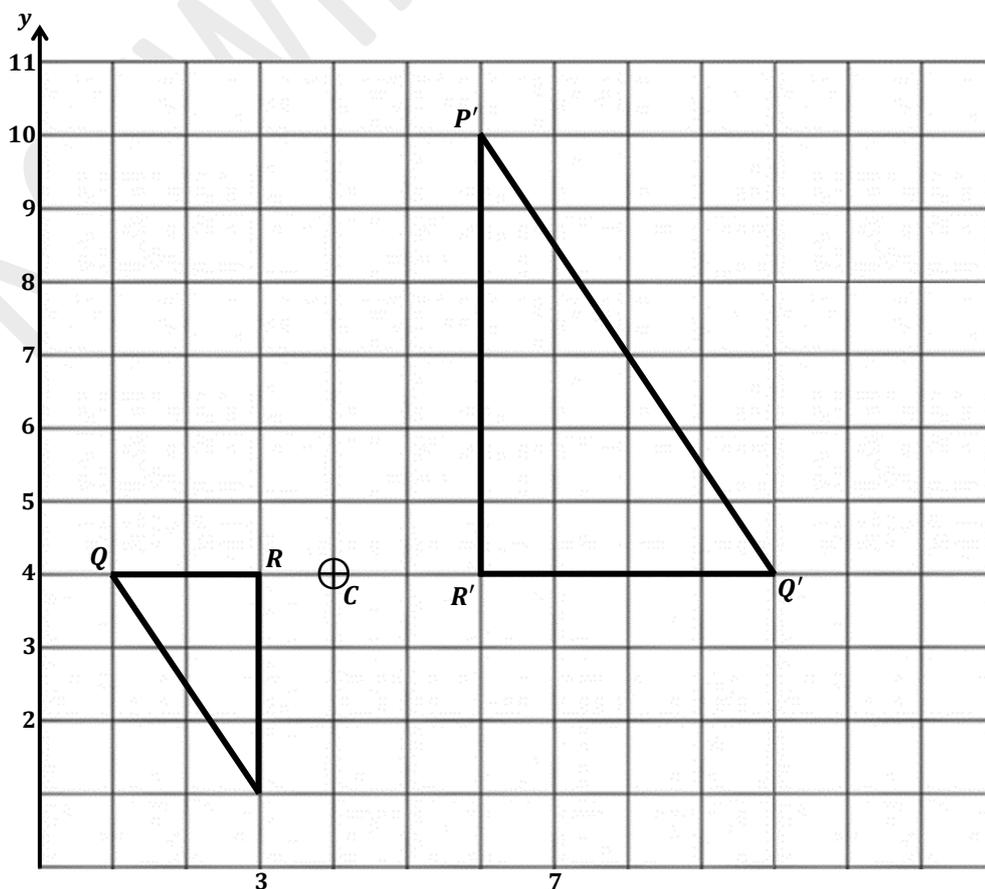


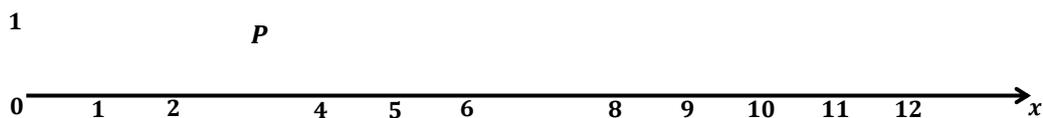
Take $\pi = 3.14$.

- (i) Calculate the volume of water in Tank *B*. [2]

- (ii) If the area of the base of A is 314 m^2 , calculate the length of the radius of Tank A . [1]
- (iii) Tank A holds 8 times as much water as Tank B . Calculate the height, h , of Tank A . [2]

- (b) The diagram below shows triangle PQR and its image, triangle $P'Q'R'$, after an enlargement centred at the point C on the diagram.





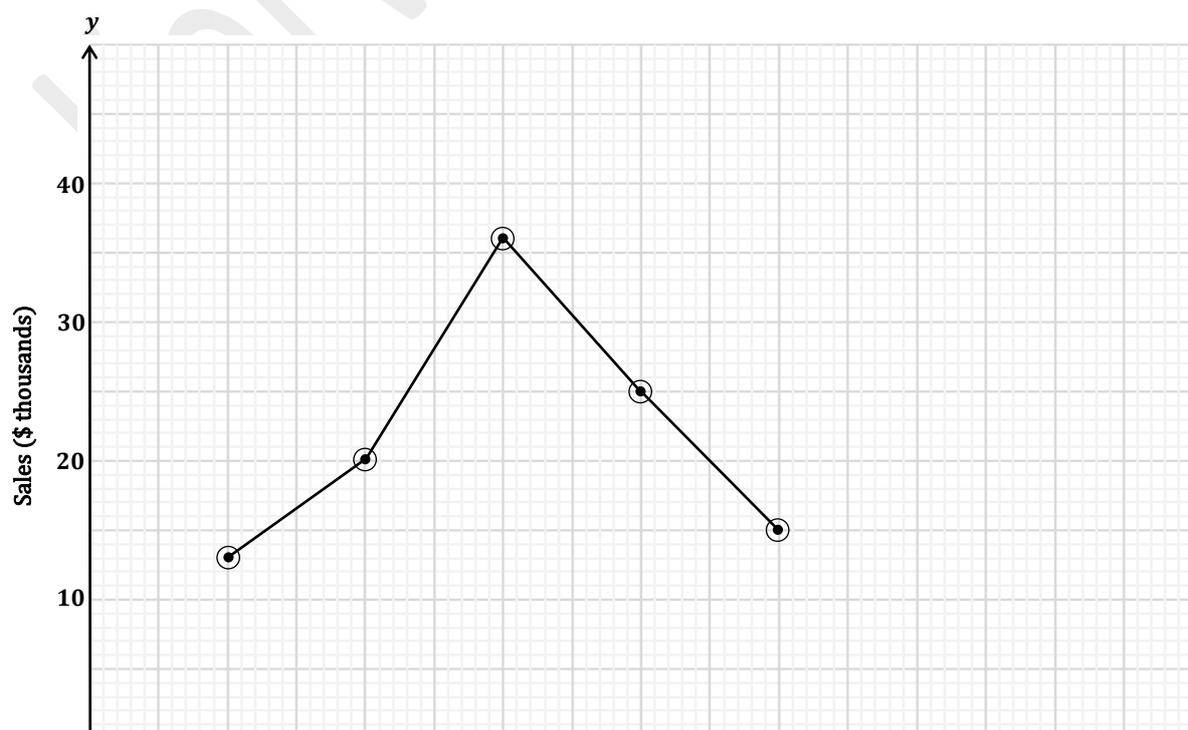
Use the information from the diagram to complete the statements below.

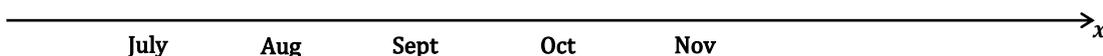
- (i) The size of the scale factor is [1]
- (ii) The scale factor is negative because.....

 [1]
- (iii) The length of PQ is $\sqrt{13}$ units, therefore,
 the length of $P'Q'$ is units. [1]
- (iv) The area of triangle PQR is square units. [1]
- (v) The area of $P'Q'R'$ is times the area of triangle
 PQR which is square units. [2]

Total: 11 marks

7. The line graph below shows the monthly sales, in thousands of dollars, at a car dealership for the period July to November 2014.





(a) Complete the table below to show the sales for EACH month. [2]

Month	July	August	September	October	November
Sales in \$ Thousands	13		36		

(b) (i) Between which TWO consecutive months was there the GREATEST increase in sales? [1]

..... and

(ii) Between which TWO consecutive months was there the SMALLEST increase in sales? [1]

..... and

(iii) What feature of the line graph enables you to infer that the increase in sales between two consecutive months was the greatest or the smallest? [2]

(c) Calculate the mean monthly sales for the period July to November 2014. [2]

(d) The TOTAL sales for the period July to December was \$130 000.

(i) Calculate the sales, in dollars, for the month of December. [1]

(ii) Complete the line graph to show the sales for December. [2]

Total: 11 marks

8. A sequence of figures is made up of equilateral triangles, called unit triangles with unit sides. The first three figures in the sequence are shown below.



Figure 1

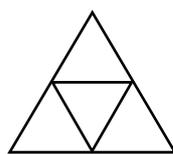


Figure 2

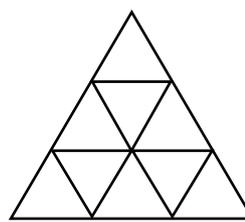


Figure 3

(a) Draw Figure 4 of the sequence. [2]

(b) Study the patterns of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures. Some rows have not been included in the table.

Complete the rows numbered (i), (ii), (iii) and (iv).

	Figure	Number of Unit Triangles	Number of Unit Sides	
	1	1	$\frac{(3 \times 1)(1+1)}{2} = 3$	
	2	4	$\frac{(3 \times 2)(2+1)}{2} = 9$	
	3	9	$\frac{(3 \times 3)(3+1)}{2} = 18$	
(i)	4			[2]
(ii)		144		[2]
(iii)	25			[2]
(iv)	n			[2]

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) A graph sheet is provided for this question.

A teacher marks an examination out of a maximum of 120 marks. The marks are then converted to percentages.

(i) Calculate the percentage for a student who scores

- 60 marks
- 120 marks

[2]

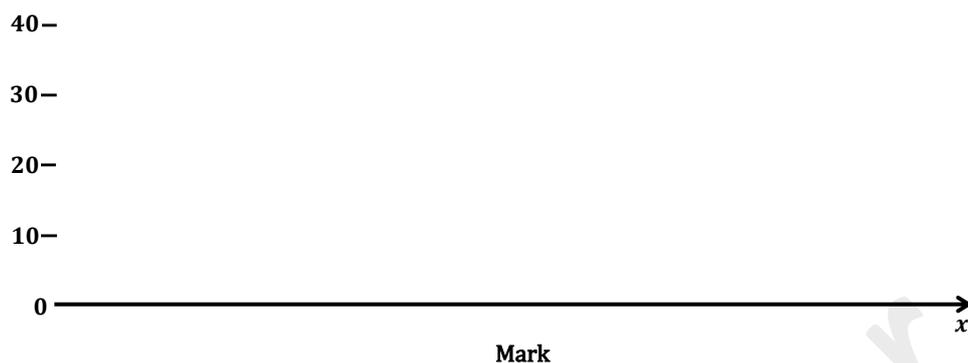
(ii) On the graph sheet on page 27, plot a graph to show the information in (i). [2]

(iii) A candidate is awarded 95 marks on the examination. Use the graph drawn at (ii) to determine the candidate's percentage.

Draw lines on your graph to show how the percentage was obtained.

Percentage: [1]





- (iv) A candidate is awarded a Grade A if her percentage is 85% or more. Use the graph drawn at (ii) to determine the minimum mark the candidate needs to be awarded a Grade A.

Draw lines on your graph to show how the percentage was obtained.

Minimum mark: [2]

- (b) The functions $f(x)$ and $g(x)$ are defined as

$$f(x) = 3x + 2 \qquad g(x) = \frac{x^2 - 1}{3}$$

- (i) Evaluate $g(5)$. [2]
- (ii) Write an expression in terms of x for $f^{-1}(x)$. [2]
- (iii) Write an expression for $gf(x)$, in the form $(x + a)(x + b)$, where

a and $b \in \mathbb{R}$.

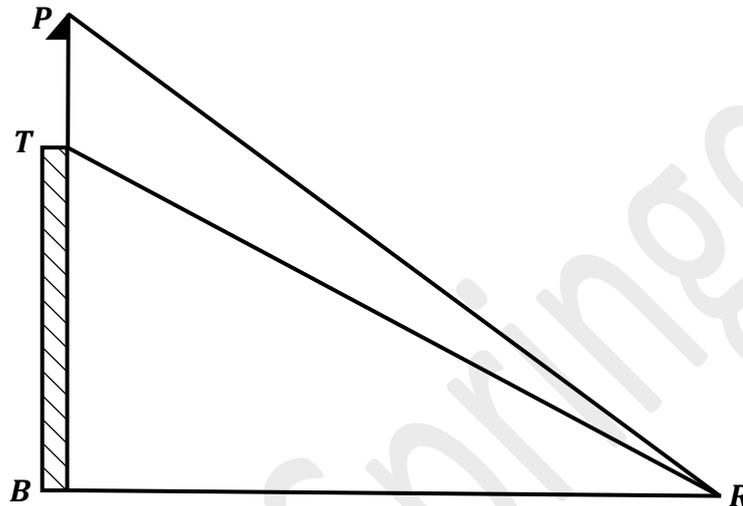
Total: 15 marks

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MEASUREMENT, GEOMETRY AND TRIGONOMETRY

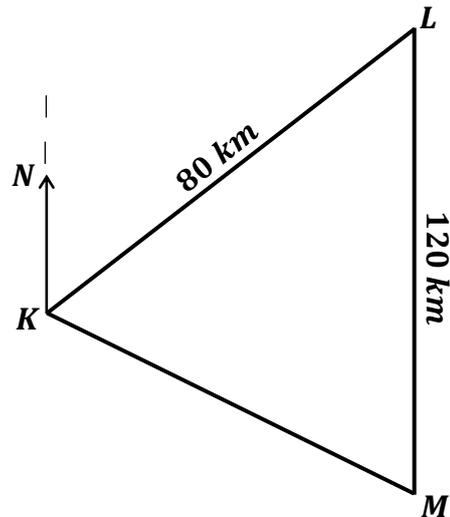
10. (a) The diagram below, **not drawn to scale**, shows a vertical tower, BT , with a

flagpole, TP , mounted on it. A point R is on the same horizontal ground as B , such that $RB = 60\text{ m}$, and the angles of elevation of T and P from R are 35° and 42° , respectively.



- (i) Label the diagram to show
- the distance 60 m
 - the angles of 35° and 42°
 - any right angle(s) [3]
- (ii) Calculate the length of the flagpole, giving your answer to the nearest metre. [4]

- (b) The diagram below, **not drawn to scale**, shows the relative positions of three fishing boats, K , L and M . L is on a bearing of 040° from K and M due south of L . $LM = 120\text{ km}$ and $KL = 80\text{ km}$.



- (i) On the diagram, show the bearing of 040° . [1]
- (ii) Calculate the measure of $\angle KLM$. [1]
- (iii) Calculate the length, to the nearest kilometre, of KM . [3]
- (iv) Calculate the measure of $\angle LKM$ to the nearest degree. [2]
- (v) Calculate the bearing of M from K . [1]

Total: 15 marks

VECTORS AND MATRICES

11. (a) (i) Calculate the matrix product AB where $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. [2]

(ii) Show that the matrix product of A and B is NOT commutative, that is,

$$AB \neq BA. \quad [2]$$

(iii) Find A^{-1} , the inverse of A . [2]

(iv) Given that $M = \begin{pmatrix} 2x & 2 \\ 9 & 3 \end{pmatrix}$, calculate the value(s) of x for which

$$|M| = 0. \quad [2]$$

(b) The position vectors of the points R, S and T , relative to an origin, O , are

$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ respectively.

(i) Calculate the value of $|\overrightarrow{OR}|$. [1]

(ii) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, the vectors \overrightarrow{RS} and \overrightarrow{ST} . [2]

$$\overrightarrow{RS} =$$

$$\overrightarrow{ST} =$$

(iii) Using the results of combining the vectors in (b)(ii) on **page 33**, justify that RS is parallel to ST and that RST is a straight line. [2]

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END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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