CSEC Mathematics

January 2020 – Paper 2

Solutions

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of the following:

(i) $4\frac{1}{5} \times \frac{1}{3} - 1\frac{1}{4}$ [2]

$4\frac{1}{5} \times \frac{1}{3} - 1\frac{1}{4}$

$= \left(\frac{21}{5} \times \frac{1}{3}\right) - \frac{5}{4}$

$= \frac{7}{5} - \frac{5}{4}$

$= \frac{28-25}{20}$

$= \frac{3}{20}$

(ii) $\frac{4.1-1.25^2}{0.005}$ [2]

$\frac{4.1-1.25^2}{0.005}$

$= \frac{4.1-1.5625}{0.005}$

$= \frac{2.5375}{0.005}$

$= 507.5$

(b) A stadium currently has a seating capacity of 15 400 seats.

(i) Calculate the number of people in the stadium when 75% of the seats are occupied. [1]

Number of people in the stadium $= 75\%$ of 15 400

$= \frac{75}{100} \times \frac{15\,400}{1}$

$= 11\,550$ persons

$\therefore$ The number of people in the stadium when 75% of the seats are occupied11 550 persons.

(ii) The stadium is to be renovated with a new seating capacity of 20 790 seats. After the renovation, what will be the percentage increase in the number of seats? [2]

Increase in number of seats $= 20\,790 - 15\,400$

$= 5390$

Percentage Increase $= \frac{\textit{Increase in number of seats}}{\textit{Original number of seats}} \times 100$

$= \frac{5390}{15\,400} \times 100$

$= 35\%$

$\therefore$ The percentage increase in the number of seats is 35%.

(c) A neon light flashes five times every 10 seconds. Show that the light flashes 43 200 times in one day. [2]

1 day = 24 hours

1 hour = 60 minutes

24 hours = 24×60

$= 1440$ minutes

1 minute = 60 seconds

1440 minutes = 1440×60

$= 86\,400$ seconds

The light flashed five times in each 10-second interval.

In 86 400 seconds, the number of 10-second intervals $= \frac{86\,400}{10}$

$= 8640$

Since there are 5 flashes in every 10 seconds,

Then, the number of times the light flashes in one day $= 8640 \times 5$

$= 43\,200$

Total: 9 marks

2. (a) Factorize the following expressions completely.

(i) $5h^2 - 12hg$ [1]

$5h^2 - 12hg = h(5h - 12g)$

(ii) $2x^2 - x - 6$ [2]

$2x^2 - x - 6$

$= 2x^2 - 4x + 3x - 6$

$= 2x(x - 2) + 3(x - 2)$

$= (2x + 3)(x - 2)$

(b) Solve the equation

$r + 3 = 3(r - 5)$ [2]

$r + 3 = 3(r - 5)$

$r + 3 = 3r - 15$

$3 + 15 = 3r - r$

$18 = 2r$

$\frac{18}{2} = r$

$9 = r$

$\therefore r = 9$

(c) Make k the subject of the formula

$$2A = \pi k^2 + 3t \qquad [2]$$

$2A = \pi k^2 + 3t$

$\pi k^2 = 2A - 3t$

$k^2 = \frac{2A-3t}{\pi}$

$k = \sqrt{\frac{2A-3t}{\pi}}$

(d) A farmer plants two crops, potatoes and corn, on a ten-hectare piece of land.

The number of hectares of corn planted, c, must be at least twice the number of hectares of potatoes, p.

Write TWO inequalities to represent the scenario above. [2]

The area of the entire plot of land = 10 hectares

The number of hectares of corn = c

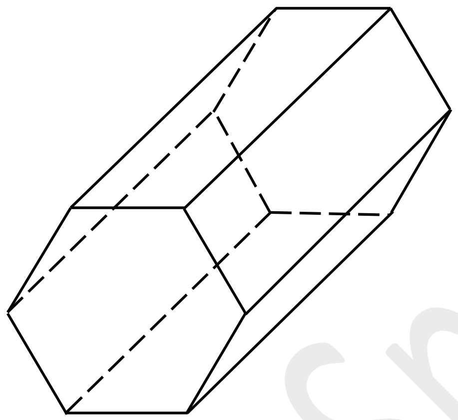
The number of hectares of potatoes = p

Given that the total area planted cannot exceed the area of the plot, then the inequality is: $c + p \leq 10$

Given that the number of hectares of corn is at least twice the number of hectares of potatoes, the inequality is: $c \geq 2p$

Total: 9 marks

3. (a) The diagram below shows a hexagonal prism.



Complete the following statement.

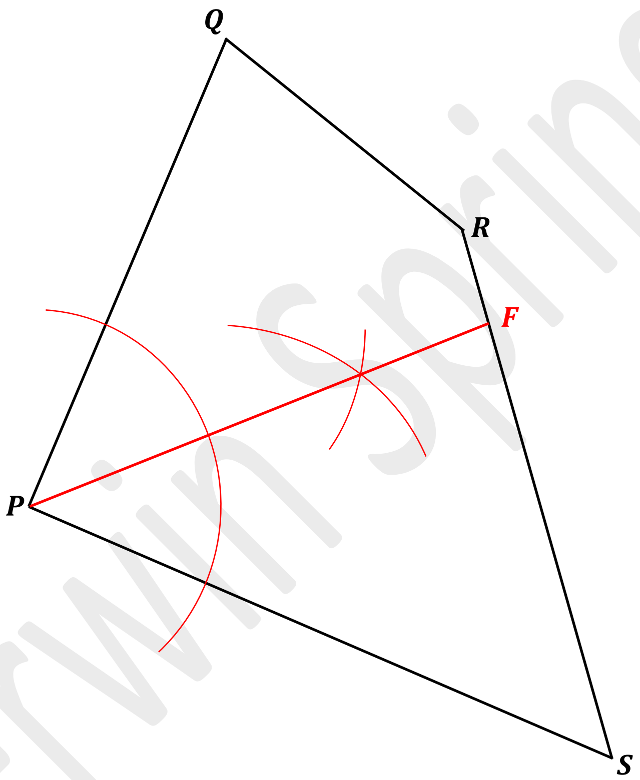
The prism has

………………………… 8 ……………………… faces,

………………………… 18 ……………………… edges and

………………………… 12 ……………………… vertices. [3]

(b) A sports club owns a field $PQRS$, in the shape of a quadrilateral. A scale diagram of this field is shown below. (1 centimetre represents 10 metres.)



In the following parts, show all your construction lines.

The field is to be divided with a fence from P to the side RS, so that different sports can be played at the same time.

Each point on the fence is the same distance from PQ as from PS.

(i) Using a straight edge and compasses only, construct the line representing the fence. [1]

See diagram above.

The line PF represents the fence.

(ii) Write down the length of this fence, in metres. [1]

By measurement, the line $BF = 7.4\ cm$.

$1\ cm$ represents 10 metres.

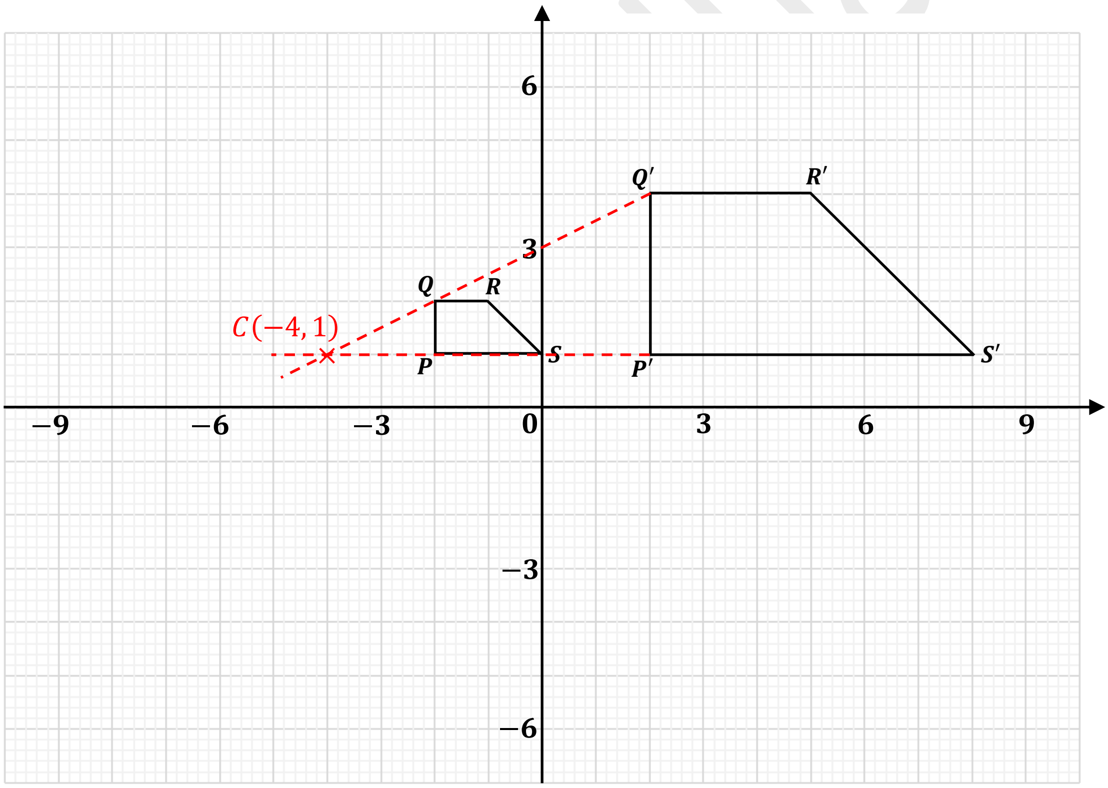
Now,

Length of fence $= 7.4 \times 10$

$= 74$ metres

$\therefore$ The length of this fence is 74 metres.

(c) A quadrilateral $PQRS$ and its image $P'Q'R'S'$ are shown on the grid below.



(i) Write down the mathematical name for the quadrilateral $PQRS$. [1]

The mathematical name for the quadrilateral $PQRS$ is a trapezium.

(ii) $PQRS$ is mapped onto $P'Q'R'S'$ by an enlargement with scale factor, k, about centre, $C(a, b)$. Using the diagram above, determine the values of a, b and k. [3]

When joining the image points to their corresponding object points, the centre of enlargement is found to be $C(-4, 1)$.

Since $C(a, b)$, then $a = -4$ and $b = 1$.

Now,

The scale factor, $k = \frac{P'Q'}{PQ}$

$= \frac{3}{1}$

$= 3$

$\therefore$ The values are $a = -4, b = 1$ and $k = 3$.

Total: 9 marks

4. (a) The function f is defined as

$$f(x) = \frac{2x+7}{5}.$$

(i) Find the value of $f(4) + f(-4)$. [2]

$$f(4) + f(-4) = \frac{2(4)+7}{5} + \frac{2(-4)+7}{5}$$

$$= \frac{8+7}{5} + \frac{-8+7}{5}$$

$$= \frac{15}{5} + \left(-\frac{1}{5}\right)$$

$$= 3 - \frac{1}{5}$$

$$= 2\frac{4}{5} \quad \text{or} \quad \frac{14}{5}$$

(ii) (a) Calculate the value of x for which $f(x) = 9$. [2]

$$f(x) = \frac{2x+7}{5}$$

$$f(x) = 9$$

So, we have,

$$\frac{2x+7}{5} = 9$$

$$2x + 7 = 45$$

$$2x = 45 - 7$$

$$2x = 38$$

$$x = \frac{38}{2}$$

$$x = 19$$

(b) Hence or otherwise, determine the value of $f^{-1}(9)$. [1]

$$f(x) = \frac{2x+7}{5}$$

Let $y = f(x)$.

$$y = \frac{2x+7}{5}$$

Interchange the variables x and y.

$$x = \frac{2y+7}{5}$$

Make y the subject of the formula.

$$5x = 2y + 7$$

$$5x - 7 = 2y$$

$$\frac{5x-7}{2} = y$$

So, we have, $f^{-1}(x) = \frac{5x-7}{2}$

Now,

$$f^{-1}(9) = \frac{5(9)-7}{2}$$

$$= \frac{45-7}{2}$$

$$= \frac{38}{2}$$

$$= 19$$

Alternatively, using the result from part (a),

$$\frac{2x+7}{5} = 9$$

$$2x + 7 = 45$$

$$2x = 45 - 7$$

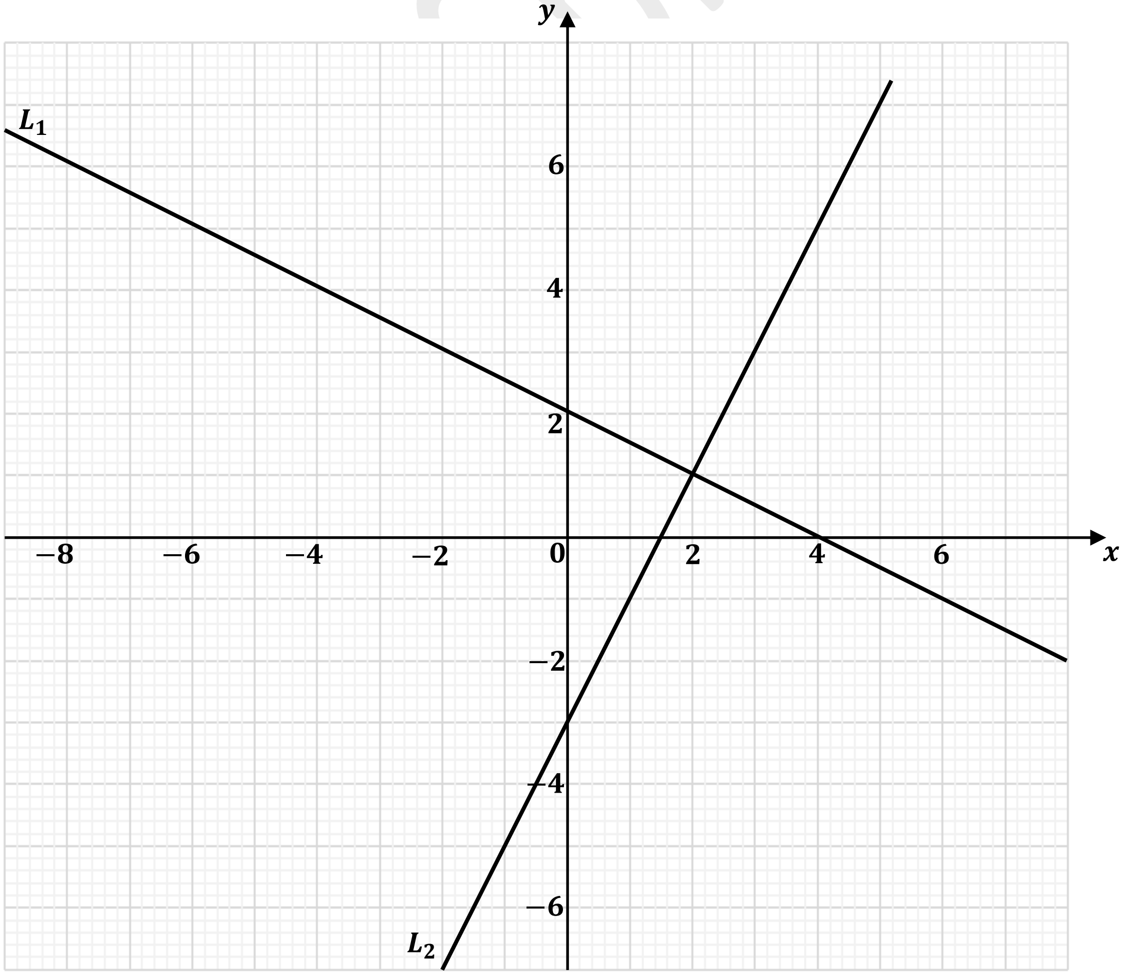
$$2x = 38$$

$$x = \frac{38}{2}$$

$$x = 19$$

$$\therefore f^{-1}(9) = 19$$

(b) The graph below shows two straight lines, L_1 and L_2. L_1 intercepts the x and y axes at $(4, 0)$ and $(0, 2)$ respectively. L_2 intercepts the x and y axes at $(1.5, 0)$ and $(0, -3)$ respectively.



(i) Determine the equation of the line L_1. [3]

Points on L_1 are $(4, 0)$ and $(0, 2)$.

Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-0}{0-4}$

$= -\frac{1}{2}$

From graph the y-intercept is $c = 2$.

Substituting $m = -\frac{1}{2}$ and $c = 2$ into $y = mx + c$ gives:

$y = -\frac{1}{2}x + 2$

$\therefore$ The equation of the line L_1 is: $y = -\frac{1}{2}x + 2$

(ii) What is the gradient of the line L_2, given that L_1 and L_2 are perpendicular? [1]

Since L_1 and L_2 are perpendicular, then the product of the gradients is -1.

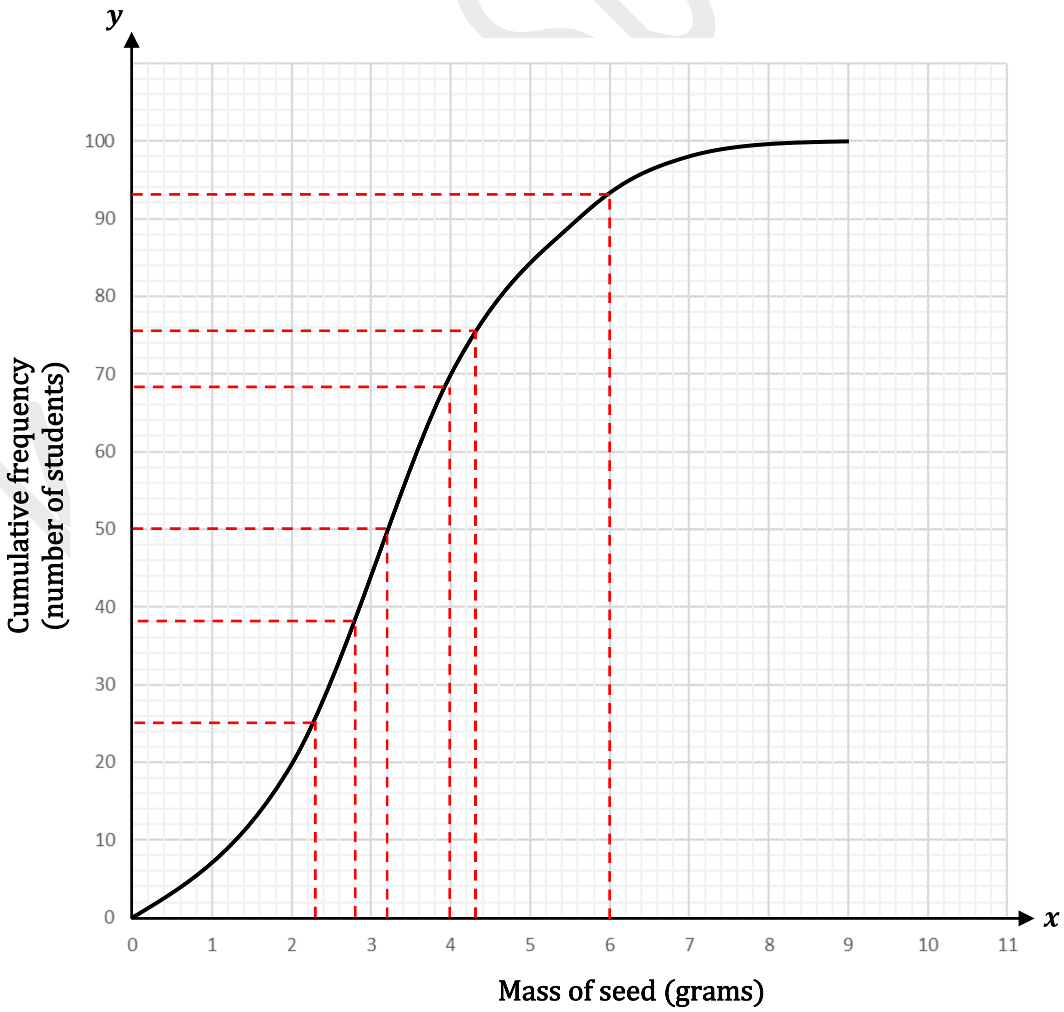
Gradient of $L_2 = -1 \div \left(-\frac{1}{2}\right)$

$= 2$

∴ The gradient of the line L_2 is 2.

Total: 9 marks

5. A group of 100 students estimated the mass, m (grams), of a seed. The cumulative frequency curve below shows the results.



(a) Using the cumulative frequency curve, estimate the

(i) median [1]

The median occurs at the $= \frac{n+1}{2}$

$= \frac{100+1}{2}$

$= 50.5^{th}$ value

From the graph, the median is 3.2 grams.

(ii) upper quartile [1]

The upper quartile occurs at the $= \frac{3}{4}(n+1)$

$= \frac{3}{4}(100+1)$

$= \frac{3}{4}(101)$

$= 75.75^{th}$ value

From the graph, the upper quartile is 4.3 grams.

(iii) semi-interquartile range [2]

The lower quartile occurs at the $= \frac{1}{4}(n+1)$

$= \frac{1}{4}(100+1)$

$= 25.25^{th}$ value

From the graph, the lower quartile is 2.3 grams.

Now,

$$SIQR = \frac{IQR}{2}$$

$$= \frac{Q_3 - Q_1}{2}$$

$$= \frac{4.3 - 2.3}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

∴ The semi-interquartile range is 1.

(iv) number of students whose estimate is 2.8 grams or less [1]

From the graph, the cumulative frequency that corresponds to a mass of 2.8 grams is 38 students.

∴ The number of students whose estimate is 2.8 grams or less is 38 students.

(b) (i) Use the cumulative frequency curve on **page 18** to complete the frequency table below. [2]

Mass of Seed, m (grams)	**Frequency**
$0 < m \leq 2$	20
$2 < m \leq 4$	48

$4 < m \leq 6$	25
$6 < m \leq 8$	6
$8 < m \leq 10$	1

From the graph, the cumulative frequency that corresponds to $m \leq 4$ is 68 students.

So, the frequency for $2 < m \leq 4$ is $68 - 20 = 48$ students.

From the graph, the cumulative frequency that corresponds to $m \leq 6$ is 93 students.

So, the frequency for $4 < m \leq 6$ is $93 - 68 = 25$ students.

(ii) A student is chosen at random. Find the probability that the student estimated the mass to be greater than 6 grams. [2]

The number of students who estimated the mass to be greater than 6 grams is $6 + 1 = 7$ students.

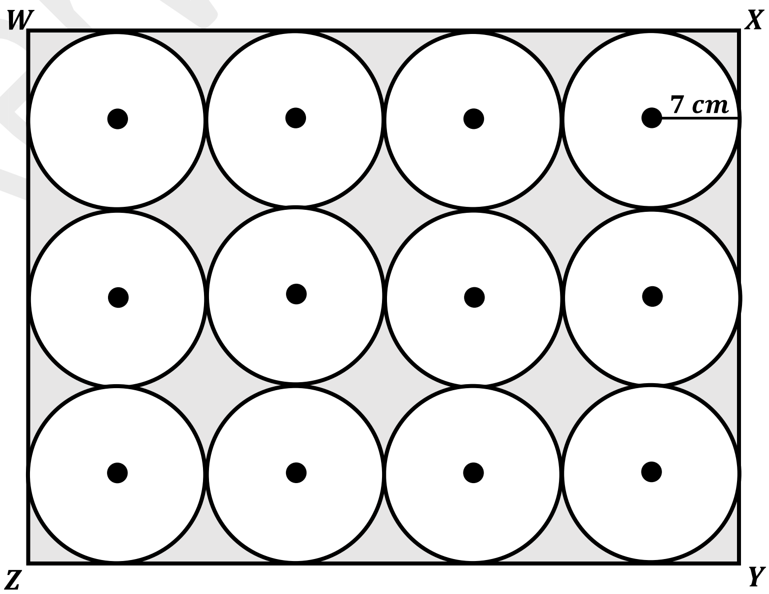
$$\text{Probability} = \frac{\textit{Number of desired outcomes}}{\textit{Total number of outcomes}}$$

$$= \frac{7}{100} \quad \text{or} \quad 0.07 \quad \text{or} \quad 7\%$$

Total: 9 marks

6. (a) The radius of EACH circle in the rectangle $WXYZ$ shown below is 7 cm.

The circles fit **exactly** into the rectangle.



(i) Show that the area of the rectangle is 2 352 cm^2. [2]

Since the circles fit exactly into the rectangle, the length of the rectangle is equal to 8 times the radius of the circle.

Length of rectangle $= 8 \times 7$

$= 56\ cm$

The width of the rectangle is equal to 6 times the radius of the circle.

Width of rectangle $= 6 \times 7$

$= 42\ cm$

Now ,

Area of rectangle $= l \times w$

$= 56 \times 42$

$= 2\ 352\ cm^2$

$\therefore$ The area of the rectangle is 2 352 cm^2.

Q.E.D.

(ii) Calculate the area of the shaded region.

(Take π to be $\frac{22}{7}$.) [3]

Area of 1 circle $= \pi r^2$

Area of 12 circles $= 12\pi r^2$

$= 12 \times \frac{22}{7} \times (7)^2$

$= 1848\ cm^2$

Area of shaded region = Area of rectangle − Area of 12 circles

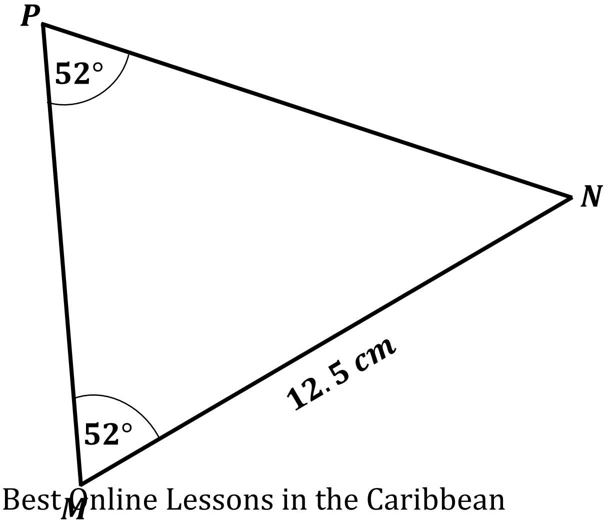
$= 2352 - 1848$

$= 504\ cm^2$

$\therefore$ The area of the shaded region is $504\ cm^2$.

(b) The diagram below, **not drawn to scale**, shows triangle MNP in which

angle MPN = angle $PMN = 52°$ and $MN = 12.5\ cm$.



(i) State the type of triangle shown above. [1]

Since the triangle has two equal angles, then it is an isosceles triangle.

(ii) Determine the value of angle PNM. [1]

All angles in a triangle add up to 180°.

$\angle PNM = 180° - 2(52°)$

$= 180° - 104°$

$= 76°$

(iii) Calculate the area of the triangle MNP. [2]

Area of the triangle $MNP = \frac{1}{2}ab \sin C$

$= \frac{1}{2} \times 12.5 \times 12.5 \times \sin 76°$

$= 75.8\ cm^2$ (to 3 significant figures)

Total: 9 marks

7. A sequence of figures is made up of stars, using dots and sticks of different length.

The first three figures in the sequence are shown below.

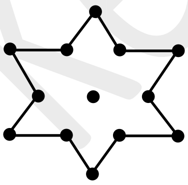


Figure 1

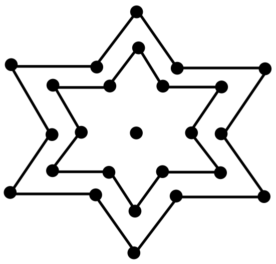


Figure 2

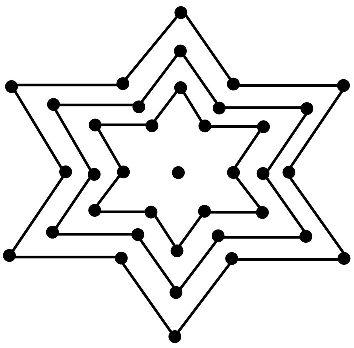


Figure 3

Study the pattern of numbers in each row of the table below. Each row relates to a figure in the sequence of figures started above. Some rows have not been included in the table.

(a) Complete Rows (i), (ii) and (iii).

	Figure	Number of Sticks (S)	Number of Dots (D)	
	1	12	13	
	2	24	25	
	3	36	37	
	4	48	49	
(i)	5	60	61	[2]
	⋮	⋮	⋮	
(ii)	13	156	157	[2]
	⋮	⋮	⋮	
(iii)	n	$12n$	$12n + 1$	[2]

For the nth figure,

Number of sticks, $S = 12n$

Number of dots, $D = 12n + 1$

(i) When $n = 5$,

$S = 12(5)$

$= 60$ sticks

When $n = 5$,

$D = 12(5) + 1$

$= 60 + 1$

$= 61$

(ii) When $S = 156$,

$12n = 156$

$n = \frac{156}{12}$

$n = 13$

When $n = 13$,

$D = 12(13) + 1$

$= 156 + 1$

$= 157$ dots

(b) The sum of the number of dots in two consecutive figures are recorded. This information for the first three pairs of consecutive figures are shown in the table below.

Figure 1 and Figure 2	**Figure 2 and Figure 3**	**Figure 3 and Figure 4**
$13 + 25 = 38$	$25 + 37 = 62$	$37 + 49 = 86$

Determine the TOTAL number of dots in

(i) Figure 7 and Figure 8 [2]

(ii) Figure n and Figure $(n+1)$ [2]

Total: 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) Solve the pair of simultaneous equations: [4]

$$y^2 + 2y + 11 = x$$

$$x = 5 - 3y$$

$y^2 + 2y + 11 = x$ → Equation 1

$x = 5 - 3y$ → Equation 2

Equating both equations gives:

$$y^2 + 2y + 11 = 5 - 3y$$

$$y^2 + 2y + 3y + 11 - 5 = 0$$

$$y^2 + 5y + 6 = 0$$

$$y^2 + 3y + 2y + 6 = 0$$

$$y(y + 3) + 2(y + 3) = 0$$

$$(y + 2)(y + 3) = 0$$

Either $y + 2 = 0$ or $y + 3 = 0$

$y = -2$ $y = -3$

Now, we need to find the corresponding values of x.

When $y = -2$,

$x = 5 - 3(-2)$

$= 5 + 6$

$= 11$

When $y = -3$,

$x = 5 - 3(-3)$

$= 5 + 9$

$= 14$

$\therefore$ The solutions are $x = 14, y = -3$ and $x = 11, y = -2$.

(b) The function f is defined as follows:

$$f(x) = 4x^2 - 8x - 2$$

(i) Express $f(x)$ in the form $a(x+h)^2 + k$, where a, h and k are constants. [3]

$h = \frac{b}{2a}$

$= \frac{(-8)}{2(4)}$

$= \frac{-8}{8}$

$= -1$

$k = \frac{4ac - b^2}{4a}$

$= \frac{4(4)(-2) - (-8)^2}{4(4)}$

$= \frac{-32 - 64}{16}$

$= -6$

$\therefore 4x^2 - 8x - 2 = 4(x-1)^2 - 6$ which is in the form $a(x+h)^2 + k$, where $a = 4, h = -1$ and $k = -6$.

(ii) State the minimum value of $f(x)$. [1]

The minimum value of $f(x)$ is the value of k which is equal to -6.

(iii) Determine the equation of the axis of symmetry. [1]

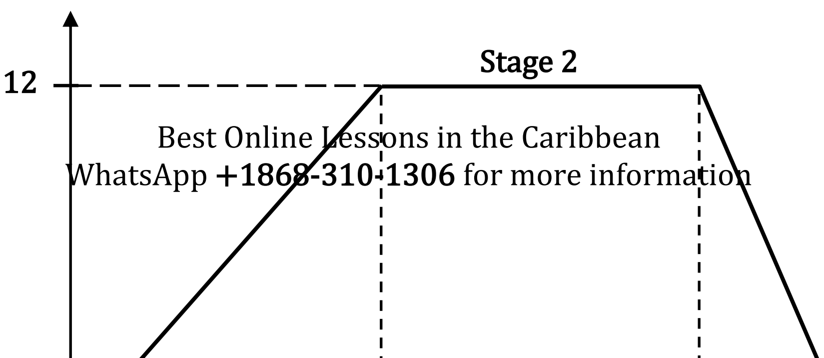
$x = -h$

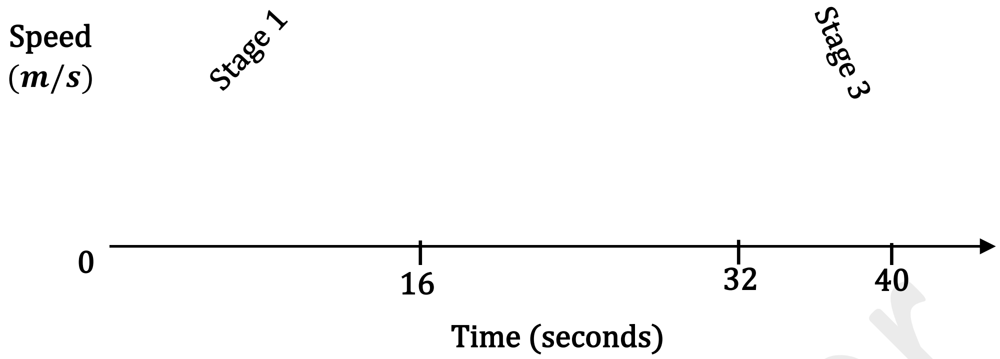
$x = -(-1)$

$x = 1$

$\therefore$ The equation of the axis of symmetry is $x = 1$.

(c) The speed-time graph below, **not drawn to scale**, shows the three-stage journey of a car over a period of 40 seconds.





Determine the acceleration of the car for EACH of the following stages of the journey. [3]

Stage 2

Stage 2 is represented by a horizontal line.

The gradient of a horizontal line is 0.

……………………………………………………………………………………………………

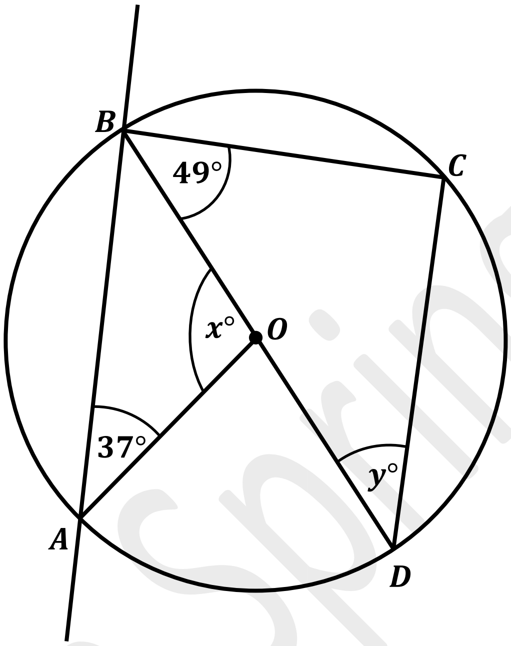
Stage 3

……………………………………………………………………………………………………

Total: 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) The circle shown below has centre O and the points A, B, C and D lying on the circumference. A straight line passes through the points A and B. Angle $CBD = 49°$ and angle $OAB = 37°$.



(i) Write down the mathematical names of the straight lines BC and OA. [2]

BC ....

OA ....

(ii) Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

(a) x [2]

Reason

……

……

……

……

(b) y [2]

Reason

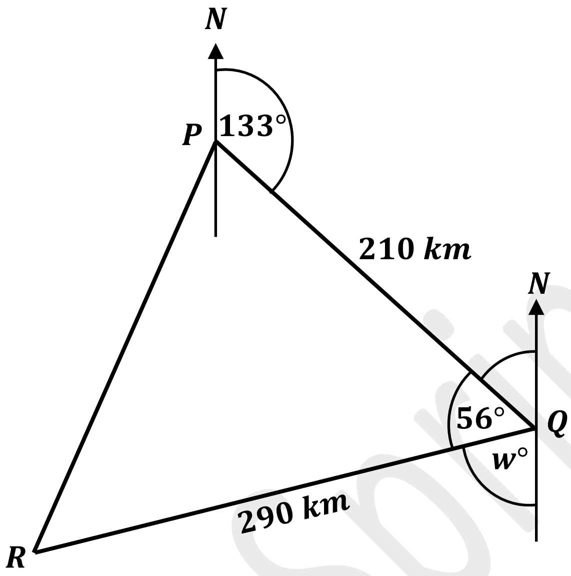
……

……

……

……

(b) The diagram below, **not drawn to scale**, shows the route of a ship cruising from Palmcity (P) to Quayton (Q) and then to Rivertown (R). The bearing of Q from P is 133° and the angle PQR is 56°.



(i) Calculate the value of angle w. [2]

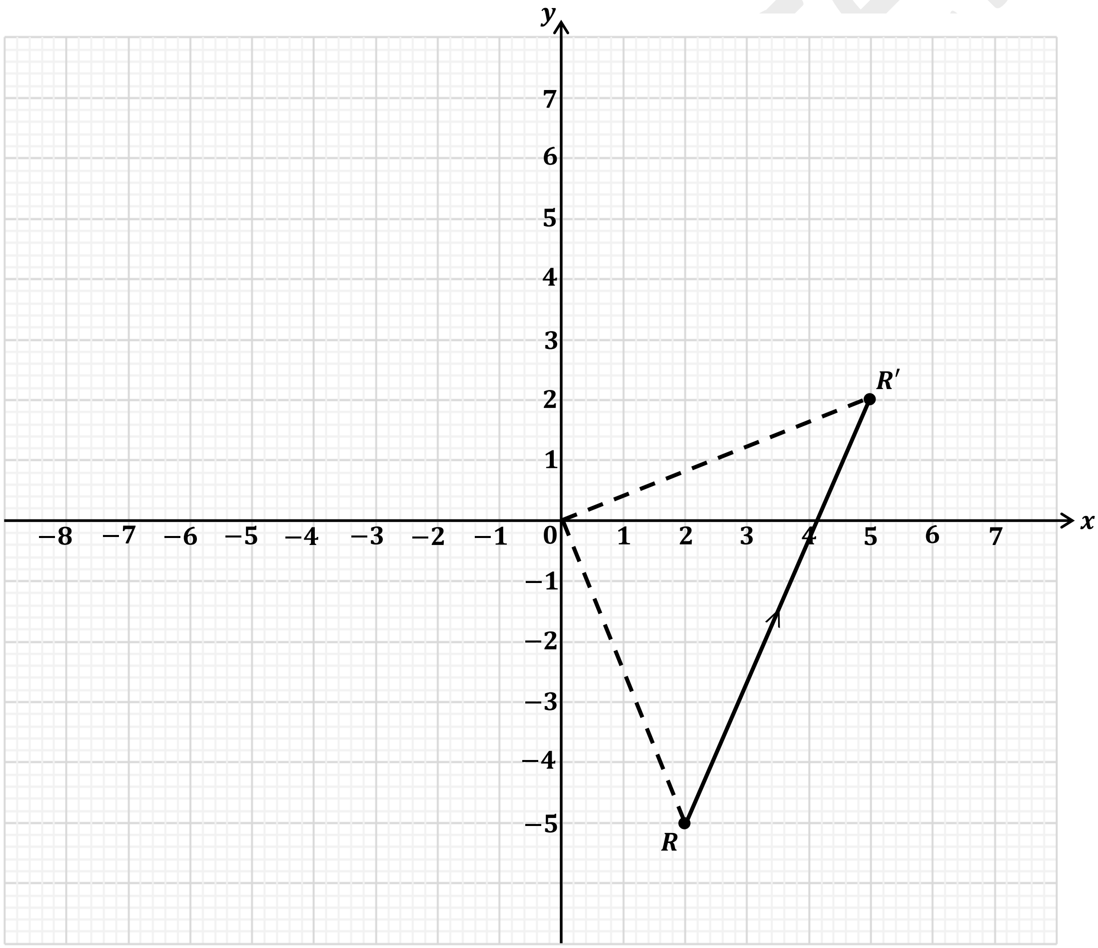
(ii) Determine the bearing of P from Q. [1]

(iii) Calculate the distance RP. [3]

Total: 12 marks

VECTORS AND MATRICES

10. (a) The transformation $M = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$ maps the point R onto R' as shown in the diagram below.

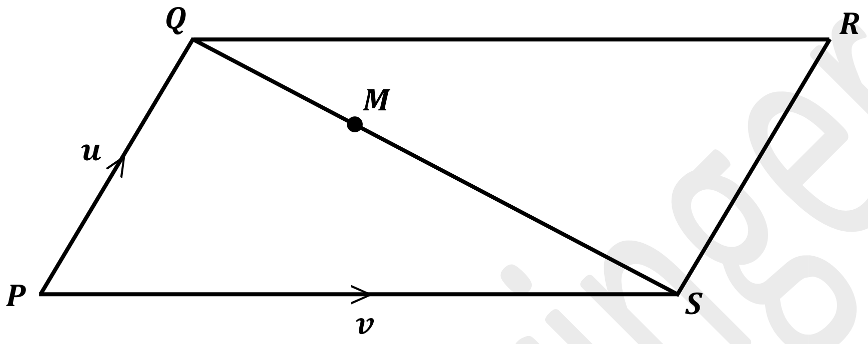


(i) Determine the values of p and q. [2]

(ii) Describe fully the transformation, M. [3]

(b) $PQRS$ is a parallelogram in which $\overrightarrow{PQ} = \boldsymbol{u}$ and $\overrightarrow{PQ} = \boldsymbol{v}$.

M is a point on QS such that $QM : MS = 1 : 2$.



(i) Write in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$ an expression for

(a) $\overrightarrow{QS}$ [1]

(b) $\overrightarrow{QM}$ [1]

(ii) Show that $\overrightarrow{MR} = \frac{1}{3}(\boldsymbol{u} + 2\boldsymbol{v})$. [2]

(iii) T is the mid-point of PQ. Prove that R, M and T are collinear. [3]

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.