CSEC Add Maths

Paper 2

June 2018

Solutions

SECTION I

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) (i) Given that $f(x) = x^2 - 4$ for $x \geq 0$, find the inverse function, $f^{-1}(x)$, stating its domain. [4]

$f(x) = x^2 - 4$

Let $y = f(x)$.

$y = x^2 - 4$

Interchange the variables x and y.

$x = y^2 - 4$

Making y the subject of the formula.

$x + 4 = y^2$

$\sqrt{x+4} = y$

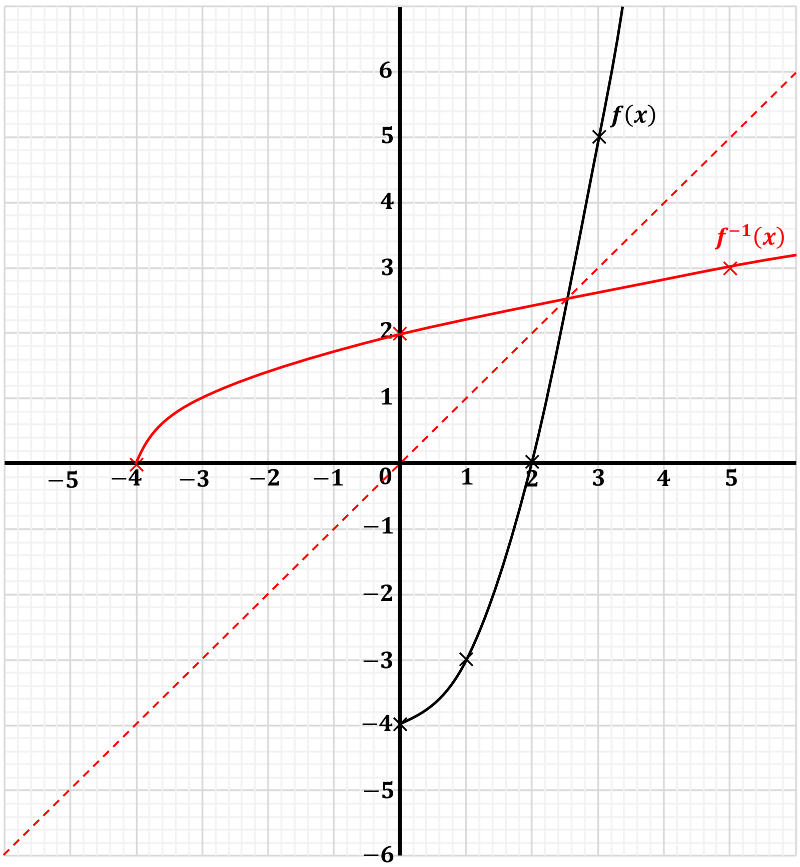
Now, for the domain of $f^{-1}(x)$,

$x + 4 \geq 0$

$x \geq -4$

$\therefore f^{-1}(x) = \sqrt{x+4}$ for $x \geq -4$

(ii) On the grid provided below, sketch $f^{-1}(x)$. [2]



Consider the coordinates:

$f(x)$	$\rightarrow$	$f^{-1}(x)$
$(0, -4)$	$\rightarrow$	$(-4, 0)$
$(2, 0)$	$\rightarrow$	$(0, 2)$
$(3, 5)$	$\rightarrow$	$(5, 3)$

(iii) State the relationship between $f(x)$ and $f^{-1}(x)$. [2]

The function $f(x)$ and the inverse function $f^{-1}(x)$ are reflections in the line $y = x$.

(b) Derive the polynomial, $P(x)$, of degree 3 which has roots equal to 1, 2 and -4. [3]

Since $P(x)$ has roots equal to 1, 2 and -4, then by the Factor Theorem, the corresponding factors are $(x-1)$, $(x-2)$ and $(x+4)$.

So, we have,

$$P(x) = (x-1)(x-2)(x+4)$$
$$= (x^2 - 3x + 2)(x+4)$$
$$= x^3 - 3x^2 + 2x + 4x^2 - 12x + 8$$
$$= x^3 + x^2 - 10x + 8$$

$\therefore$ The polynomial $P(x) = x^3 + x^2 - 10x + 8$.

(c) An equation relating V and t is given by $V = ka^t$ where k and a are constants.

(i) Use logarithms to derive an equation of the form $y = mx + c$ that can be used to find the values of k and a. [2]

$$V = ka^t$$
$$\log V = \log ka^t$$
$$\log V = \log k + \log a^t$$
$$\log V = \log k + t \log a$$
$$\log V = (\log a)t + \log k$$

which is in the form $y = mx + c$,

where $y = \log V$, $m = \log a$, $x = t$ and $c = \log k$.

(ii) If a graph of y versus x from the equation in part (c)(i) is plotted, a straight line is obtained. State an expression for the gradient of the graph. [1]

$\log V = (\log a)t + \log k$

which is in the form $y = mx + c$,

where $y = \log V$, $m = \log a$, $x = t$ and $c = \log k$.

$\therefore$ An expression for the gradient of the graph is $m = \log a$.

Total: 14 marks

2. (a) (i) Given that $g(x) = -x^2 + x - 3$, express $g(x)$ in the form $a(x+h)^2 + k$ where a, h and k are constants. [3]

$$-x^2 + x - 3$$

$$= -(x^2 - x) - 3$$

$$= -\left(x^2 - x + \tfrac{1}{4}\right) - 3 - (-1)\left(\tfrac{1}{4}\right)$$

$$= -\left(x - \tfrac{1}{2}\right)^2 - 3 + \tfrac{1}{4}$$

$$= -\left(x - \tfrac{1}{2}\right)^2 - \tfrac{11}{4}$$ which is in the form $a(x+h)^2 + k$,

where $a = -1, h = -\frac{1}{2}$ and $k = -\frac{11}{4}$.

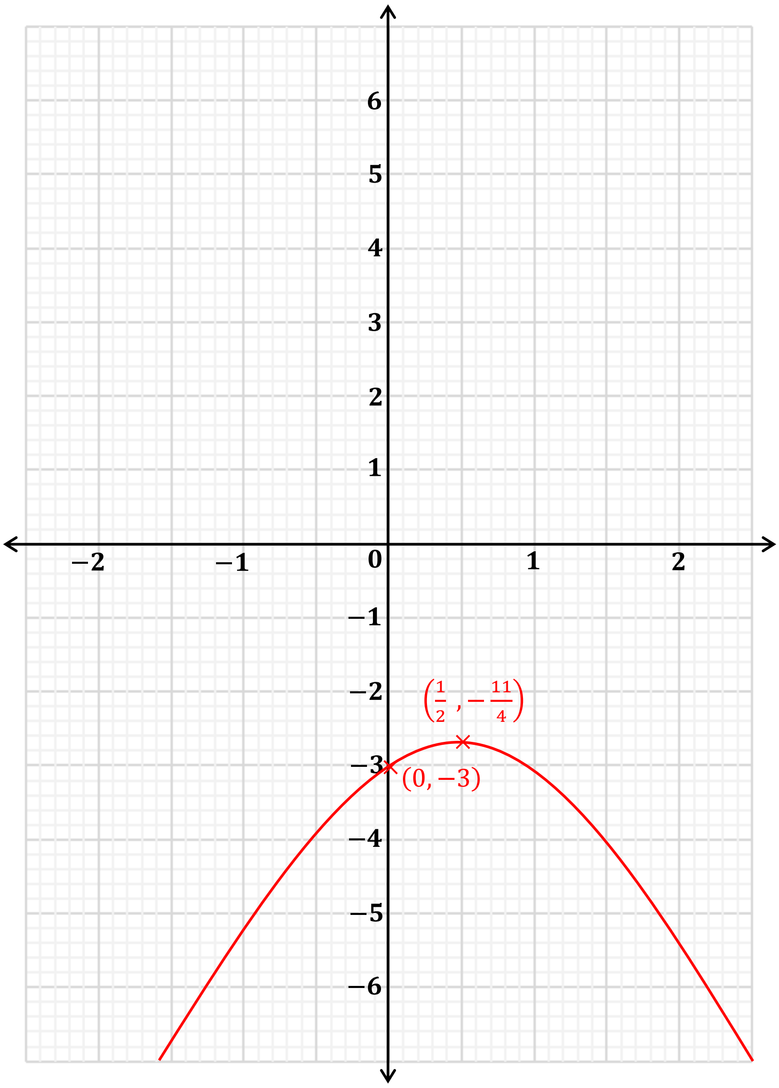
$$\therefore\ g(x) = -\left(x - \tfrac{1}{2}\right)^2 - \tfrac{11}{4}$$

(ii) On the grid provided below, sketch the graph of $g(x)$, showing the maximum point and the y-intercept. [3]

$$g(x) = -\left(x - \tfrac{1}{2}\right)^2 - \tfrac{11}{4}$$

The maximum point is of the form $(-h, k)$ which is $\left(\frac{1}{2}, -\frac{11}{4}\right)$

$g(x) = -x^2 + x - 3$ where the y-intercept is $c = -3$.



(b) In a geometric progression, the 3rd term is 25 and the sum of the 1st and 2nd terms is 150. Determine the sum of the first four terms, given that $r > 0$. [4]

For a geometric progression, the nth term is

$T_n = ar^{n-1}$

Let the 1st term be $T_1 = a$.

Let the 2nd term be $T_2 = ar$.

The sum of the 1st and 2nd terms is 150.

So, we have,

$a + ar = 150$

The 3rd term, T_3, is 25.

$ar^2 = 25$

We have two equations that we can solve simultaneously for a and r.

$a + ar = 150 \quad \rightarrow$ Equation 1

$ar^2 = 25 \quad \rightarrow$ Equation 2

Rearranging Equation 2 gives:

$a = \frac{25}{r^2} \quad \rightarrow$ Equation 3

Substituting Equation 3 into Equation 1 gives:

$$\frac{25}{r^2} + \left(\frac{25}{r^2}\right) r = 150$$

$$\frac{25}{r^2} + \frac{25}{r} = 150$$

$(\times r^2)$

$$25 + 25r = 150r^2$$

$(\div 25)$

$$1 + r = 6r^2$$

$$6r^2 - r - 1 = 0$$

$$(3r + 1)(2r - 1) = 0$$

Either $3r + 1 = 0$ or $2r - 1 = 0$

$r = -\frac{1}{3}$ $r = \frac{1}{2}$

Since $r > 0$, then $r = \frac{1}{2}$.

Substituting $r = \frac{1}{2}$ into Equation 3 gives:

$$a = \frac{25}{\left(\frac{1}{2}\right)^2}$$

$$= \frac{25}{\left(\frac{1}{4}\right)}$$

$$= 100$$

The sum of n terms is:

$$S_n = \frac{a(1-r^n)}{1-r}$$

Substituting $a = 100$, $r = \frac{1}{2}$ and $n = 4$ gives:

$$S_4 = \frac{100\left[1-\left(\frac{1}{2}\right)^4\right]}{1-\left(\frac{1}{2}\right)}$$

$$= \frac{100\left(1-\frac{1}{16}\right)}{\frac{1}{2}}$$

$$= 2(100)\left(\frac{15}{16}\right)$$

$$= 187\frac{1}{2}$$

$\therefore$ The sum of the first four terms is $S_4 = 187\frac{1}{2}$.

(c) If α and β are the roots of the equation $2x^2 - 5x + 3 = 0$, determine the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. [4]

$2x^2 - 5x + 3 = 0$

which is in the form $ax^2 + bx + c = 0$,

where $a = 2, b = -5$ and $c = 3$.

$$\alpha + \beta = -\frac{b}{a}$$

$$= -\frac{(-5)}{2}$$

$$= \frac{5}{2}$$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{3}{2}$$

Consider,

$$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{5}{2}\right)^2 - 2\left(\frac{3}{2}\right)$$

$$= \frac{25}{4} - 3$$

$$= \frac{13}{4}$$

Now,

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

$$=\frac{\left(\frac{13}{4}\right)}{\left(\frac{3}{2}\right)^2}$$

$$=\frac{13}{4}\div\left(\frac{3}{2}\right)^2$$

$$=\frac{13}{4}\div\frac{9}{4}$$

$$=\frac{13}{4}\times\frac{4}{9}$$

$$=\frac{13}{9}$$

$\therefore$ The value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{13}{9}$.

Total: 14 marks

SECTION II

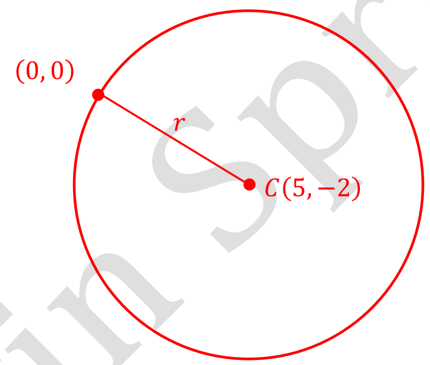
Answer BOTH questions.

ALL working must be clearly shown.

3. (a) Determine the equation of the circle that has centre $(5, -2)$ and passes through the origin. [3]

Points are $C(5, -2)$ and $(0, 0)$.

A sketch is shown below:



Radius of circle, $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(5 - 0)^2 + (-2 - 0)^2}$$

$$= \sqrt{(5)^2 + (-2)^2}$$

$$= \sqrt{25 + 4}$$

$$= \sqrt{29} \text{ units}$$

The standard form for the equation of the circle is:

$(x - a)^2 + (y - b)^2 = r^2$

Substituting $C(5, -2)$ and $r = \sqrt{29}$ units gives:

$(x-5)^2 + (y+2)^2 = \left(\sqrt{29}\right)^2$

$(x-5)^2 + (y+2)^2 = 29$

$\therefore$ The equation of the circle is: $(x-5)^2 + (y+2)^2 = 29$

(b) Determine whether the following pair of lines is parallel. [2]

$x + y = 4$

$3x - 2y = -3$

$x + y = 4$ $\rightarrow$ Equation 1

$3x - 2y = -3$ $\rightarrow$ Equation 2

Rearranging Equation 1 gives:

$y = -x + 4$ which is in the form $y = mx + c$,

where $m = -1$ and $c = 4$.

Rearranging Equation 2 gives:

$-2y = -3x - 3$

$y = \frac{3}{2}x + \frac{3}{2}$ which is in the form $y = mx + c$,

where $m = \frac{3}{2}$ and $c = \frac{3}{2}$.

Since $-1 \neq \frac{3}{2}$, the gradients are not the same.

$\therefore$ The lines are not parallel.

(c) The position vectors of two points, A and B, relative to a fixed origin, O, are given by $\overrightarrow{OA} = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ and $\overrightarrow{OB} = 3\hat{\boldsymbol{\imath}} - 5\hat{\boldsymbol{\jmath}}$, where $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ represent the unit vectors in the x and y directions respectively. Calculate

(i) the magnitude of $\overrightarrow{AB}$ [4]

$\overrightarrow{OA} = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ and $\overrightarrow{OB} = 3\hat{\boldsymbol{\imath}} - 5\hat{\boldsymbol{\jmath}}$

$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

Using the triangle law,

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3-2 \\ -5-1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -6 \end{pmatrix}$

Now,

Magnitude of $\overrightarrow{AB} = \sqrt{(1)^2 + (-6)^2}$

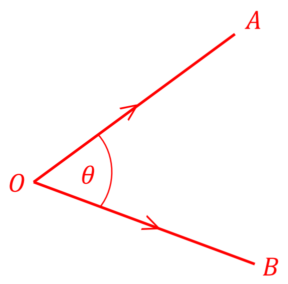
$= \sqrt{1+36}$

$= \sqrt{37}$ units

$\therefore$ The magnitude of $\overrightarrow{AB}$ is $\sqrt{37}$ units.

(ii) the angle $A\hat{O}B$, giving your answer to the nearest whole number. [3]

Consider the sketch below:



$\overrightarrow{OA} = 2\hat{\imath} + \hat{\jmath}$ and $\overrightarrow{OB} = 3\hat{\imath} - 5\hat{\jmath}$

$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 3 \\ -5 \end{pmatrix}$

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

$$= (2)(3) + (1)(-5)$$

$$= 6 - 5$$

$$= 1$$

$$|\overrightarrow{OA}| = \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{4 + 1}$$

$$= \sqrt{5}$$

$$|\overrightarrow{OB}| = \sqrt{(3)^2 + (-5)^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34}$$

Now,

$$\cos\theta = \frac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$$

$$\cos\theta = \frac{1}{\sqrt{5}\sqrt{34}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}\sqrt{34}}\right)$$

$$\theta = 86^\circ \qquad \text{(to the nearest whole number)}$$

$\therefore$ The angle $A\hat{O}B = 86^\circ$.

Total: 12 marks

4. (a) A wire in the form of a circle with radius 4 cm is reshaped in the form of a sector of a circle with radius 10 cm. Determine, in radians, the angle of the sector, giving your answer in terms of π. [4]

Circumference of circle $= 2\pi r$

$$= 2\pi(4)$$

$$= 8\pi\ cm$$

Perimeter of sector $= r + r + r\theta$

$$= 10 + 10 + 10\theta$$

$$= 20 + 10\theta$$

Now,

Circumference of circle = Perimeter of sector

$$8\pi = 20 + 10\theta$$

$$8\pi - 20 = 10\theta$$

$$\frac{8\pi - 20}{10} = \theta$$

$$\theta = \frac{2(4\pi - 10)}{2(5)}$$

$$\theta = \frac{4\pi - 10}{5} \text{ radians}$$

$\therefore$ The angle of the sector is $\theta = \frac{4\pi - 10}{5}$ radians.

(b) Solve the equation $\sin^2\theta + 3\cos 2\theta = 2$ for $0 \leq \theta \leq \pi$. Give your answer(s) to 1 decimal place. [4]

$$\sin^2\theta + 3\cos 2\theta = 2$$

$$\sin^2\theta + 3(1 - 2\sin^2\theta) = 2$$

$$\sin^2\theta + 3 - 6\sin^2\theta = 2$$

$$3 - 5\sin^2\theta = 2$$

$$-5\sin^2\theta = 2 - 3$$

$$-5\sin^2\theta = -1$$

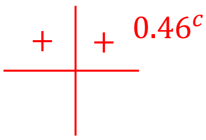
$$\sin^2\theta = \frac{-1}{-5}$$

$$\sin^2\theta = \frac{1}{5}$$

$$\sin\theta = \pm\frac{1}{\sqrt{5}}$$

Consider,

$$\sin\theta = \frac{1}{\sqrt{5}}$$



$$\theta = 0.46^c,\ \pi - 0.46^c$$

$$\theta = 0.46^c,\ 2.68^c$$

$$\theta = 0.5^c,\ 2.7^c \qquad \text{(to 1 decimal place)}$$

Consider,

$$\sin\theta = -\frac{1}{\sqrt{5}}$$

No solutions since sine is not negative for the interval $0 \leq \theta \leq \pi$.

$\therefore \theta = 0.5^c$ and $\theta = 2.7^c$ for $0 \leq \theta \leq \pi$.

(c) Prove the identity $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} \equiv \frac{2\tan x}{\cos x}$. [4]

Taking L.H.S:

$$\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$= \frac{1+\sin x-1+\sin x}{1-\sin^2 x}$$

$$= \frac{2\sin x}{1-\sin^2 x}$$

$$= \frac{2\sin x}{\cos^2 x}$$

$$= \frac{2\sin x}{\cos x} \times \frac{1}{\cos x}$$

$$= \frac{2\tan x}{1} \times \frac{1}{\cos x}$$

$$= \frac{2\tan x}{\cos x}$$

= R.H.S.

$$\therefore \frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2\tan x}{\cos x}$$

Q.E.D.

Total: 12 marks

SECTION III

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) Given that $y = x^3 + 2x^2 - 1$, determine

(i) the coordinates of the stationary points [5]

$y = x^3 + 2x^2 - 1$

$\frac{dy}{dx} = 3x^2 + 4x$

At stationary points, $\frac{dy}{dx} = 0$.

When $\frac{dy}{dx} = 0$,

$3x^2 + 4x = 0$

$x(3x + 4) = 0$

Either $x = 0$ or $3x + 4 = 0$

$x = -\frac{4}{3}$

When $x = 0$,

$y = (0)^3 + 2(0)^2 - 1$

$= -1$

When $x = -\frac{4}{3}$

$y = \left(-\frac{4}{3}\right)^3 + 2\left(-\frac{4}{3}\right)^2 - 1$

$= \frac{5}{27}$

$\therefore$ The coordinates of the stationary points are $(0, -1)$ and $\left(-\frac{4}{3}, \frac{5}{27}\right)$.

(ii) the nature of EACH stationary point [5]

$$\frac{dy}{dx} = 3x^2 + 4x$$

$$\frac{d^2y}{dx^2} = 6x + 4$$

When $x = 0$,

$$\frac{d^2y}{dx^2} = 6(0) + 4$$

$$= 0 + 4$$

$$= 4 \quad (> 0)$$

Since $\frac{d^2y}{dx^2} > 0$, then $(0, -1)$ is a minimum point.

When $x = -\frac{4}{3}$,

$$\frac{d^2y}{dx^2} = 6\left(-\frac{4}{3}\right) + 4$$

$$= -8 + 4$$

$$= -4 \quad (< 0)$$

Since $\frac{d^2y}{dx^2} < 0$, then $\left(-\frac{4}{3}, \frac{5}{27}\right)$ is a maximum point.

(b) Differentiate $y = 2x\sqrt{(4-8x)}$ with respect to x, simplifying your answer. [4]

$y = 2x\sqrt{(4-8x)}$

$y = 2x(4-8x)^{\frac{1}{2}}$

which is in the form $y = uv$.

Let $u = 2x$, $v = (4-8x)^{\frac{1}{2}}$

$\frac{du}{dx} = 2$, $\frac{dv}{dx} = \frac{1}{2}(-8)(4-8x)^{\frac{1}{2}-1}$

$\frac{dv}{dx} = -4(4-8x)^{-\frac{1}{2}}$

Using the product rule,

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x)(-4)(4-8x)^{-\frac{1}{2}} + (4-8x)^{\frac{1}{2}}(2)$

$= -8x(4-8x)^{-\frac{1}{2}} + 2(4-8x)^{\frac{1}{2}}$

$= (4-8x)^{-\frac{1}{2}}[-8x + 2(4-8x)^{1}]$

$= (4-8x)^{-\frac{1}{2}}(-8x + 8 - 16x)$

$= (4-8x)^{-\frac{1}{2}}(8-24x)$

$= \frac{8(1-3x)}{(4-8x)^{\frac{1}{2}}}$

$\therefore \frac{dy}{dx} = \frac{8(1-3x)}{\sqrt{(4-8x)}}$

Total: 14 marks

6. (a) Show, using integration, that the finite area of the curve $y = \sin x$ in the first quadrant bounded by the line $x = \frac{4\pi}{9}$ is smaller than the finite region of $y = \cos x$ in the same quadrant and bounded by the same line. [6]

$y = \sin x$

The limits are $x = 0$ and $x = \frac{4\pi}{9}$.

$$\text{Area of region} = \int_a^b y \ dx$$

$$= \int_0^{\frac{4\pi}{9}} \sin x \ dx$$

$$= [-\cos x]_0^{\frac{4\pi}{9}}$$

$$= -\cos\left(\frac{4\pi}{9}\right) - [-\cos(0)]$$

$$= -0.174 + 1$$

$$= 0.826 \text{ units}^2 \qquad \text{(to 3 decimal places)}$$

Now, $y = \cos x$.

The limits are $x = 0$ and $x = \frac{4\pi}{9}$.

$$\text{Area of region} = \int_a^b y \ dx$$

$$= \int_0^{\frac{4\pi}{9}} \cos x \ dx$$

$$= [\sin x]_0^{\frac{4\pi}{9}}$$

$$= \sin\left(\frac{4\pi}{9}\right) - \sin(0)$$

$$= 0.985 \text{ units}^2 \qquad \text{(to 3 decimal places)}$$

Since $0.826 < 0.985$, then the area of the curve $y = \sin x$ in the first quadrant bounded by the line $x = \frac{4\pi}{9}$ is smaller than the finite region of $y = \cos x$ in the same quadrant and bounded by the same line.

Q.E.D.

(b) The finite region in the first quadrant bounded by the curve $y = x^2 + x + 3$, the x-axis and the line $x = 4$ is rotated completely about the x-axis.

Determine the volume of the solid of revolution formed. [4]

$y = x^2 + x + 3$

$y^2 = (x^2 + x + 3)^2$

The limits are $x = 0$ and $x = 4$.

Volume of the solid $= \pi \int_a^b y^2 \; dx$

$$= \pi \int_0^4 (x^2 + x + 3)^2 \; dx$$

$$= \pi \int_0^4 (x^4 + x^3 + 3x^2 + x^3 + x^2 + 3x + 3x^2 + 3x + 3)\, dx$$

$$= \pi \int_0^4 (x^4 + 2x^3 + 7x^2 + 6x + 9)\, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^4}{4} + \frac{7x^3}{3} + \frac{6x^2}{2} + 9x\right]_0^4$$

$$= \pi \left[\frac{x^5}{5} + \frac{x^4}{2} + \frac{7x^3}{3} + 3x^2 + 9x\right]_0^4$$

$$= \pi \left[\frac{(4)^5}{5} + \frac{(4)^4}{2} + \frac{7(4)^3}{3} + 3(4)^2 + 9(4)\right] - 0$$

$$= \pi \left(\frac{1024}{5} + 128 + \frac{448}{3} + 48 + 36\right)$$

$$= \frac{8492\pi}{15} \text{ units}^3$$

$\therefore$ The volume of the solid of revolution formed is $\frac{8492\pi}{15}$ units^3.

(c) A curve which has a gradient of $\frac{dy}{dx} = 3x - 1$ passes through the point $A(4, 1)$.

Find the equation of the curve. [4]

$\frac{dy}{dx} = 3x - 1$

$y = \int \frac{dy}{dx}\, dx$

$y = \int (3x - 1)\, dx$

$y = \frac{3x^2}{2} - x + c$

Substituting point $A(4, 1)$ gives:

$1 = \frac{3(4)^2}{2} - (4) + c$

$1 = 24 - 4 + c$

$1 = 20 + c$

$c = 1 - 20$

$c = -19$

$\therefore$ The equation of the curve is: $y = \frac{3x^2}{2} - x - 19$

Total: 14 marks

SECTION IV

Answer only ONE question.

ALL working must be clearly shown.

7. (a) The number of runs scored by a cricketer for 18 consecutive innings is illustrated in the following stem-and-leaf diagram.

0	|	2	3	6	7	
1	|	0	3	5	8	9
2	|	4	4	6	8	
3	|	1	4	5		
4	|	5	7			

Key 0|6 means 6

(i) Determine the median score. [2]

The raw data set is:

$\cancel{2}$ $\cancel{3}$ $\cancel{6}$ $\cancel{7}$ $\cancel{10}$ $\cancel{13}$ $\cancel{15}$ $\cancel{18}$ 19

24 $\cancel{24}$ $\cancel{26}$ $\cancel{28}$ $\cancel{31}$ $\cancel{34}$ $\cancel{35}$ $\cancel{45}$ $\cancel{47}$

$$\text{Median} = \frac{19+24}{2}$$

$$= \frac{43}{2}$$

$$= 21.5$$

$\therefore$ The median score is 21.5.

(ii) Calculate the interquartile range of the scores. [3]

The lower quartile occurs at the middle value between the 1st and the 9th value. This is the 5th value.

So, $Q_1 = 10$.

The upper quartile occurs at the middle value between the 10th and the 18th value. This is the 14th value.

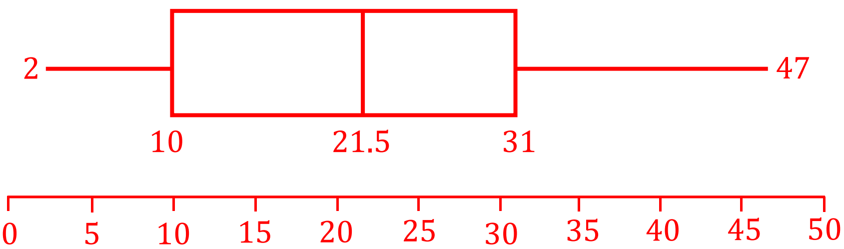
So, $Q_3 = 31$.

Now,

$$\text{Interquartile Range} = Q_3 - Q_1$$
$$= 31 - 10$$
$$= 21$$

(iii) In the space below, construct a box-and whisker plot to illustrate the data and comment on the shape of the distribution. [4]

The box-and-whisker plot is shown below:

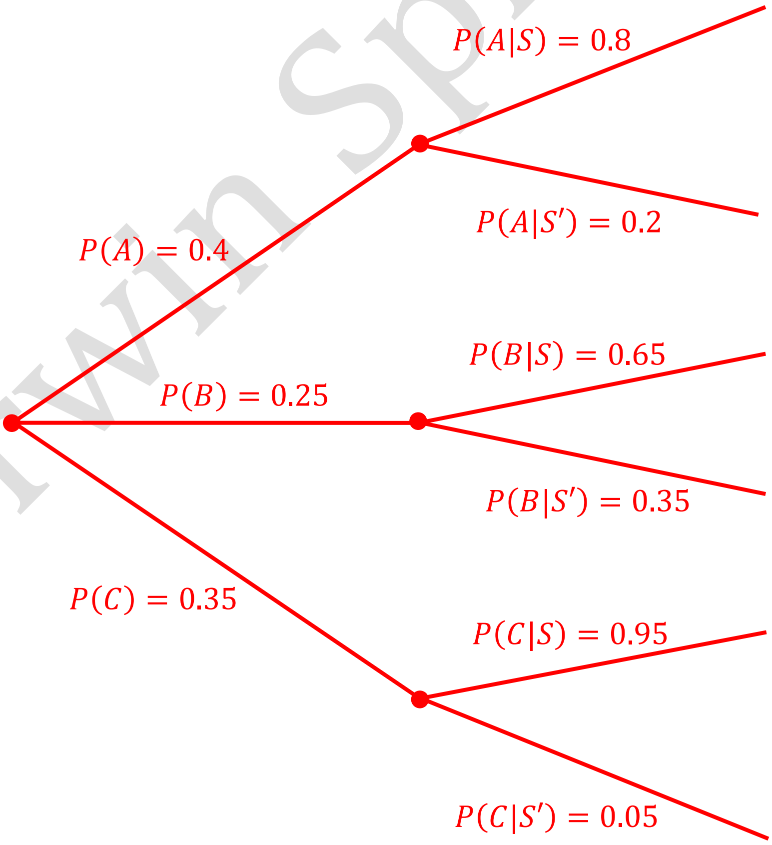


The median is located just around the middle of the box and which indicates that the data is almost symmetric. However, the fourth quartile which is the right whisker is noticeably longer than the other three quartiles. This indicates that there is more variability among the larger scores than among the smaller scores.

(b) Insecticides A, B or C are applied on lots Q, R and S. The same crop is planted on each lot and the lots are of the same size. The probability that a group of farmers will select A, B or C is 40%, 25% and 35% respectively. The probability that insecticide A is successful is 0.8, that B is successful is 0.65, and that C is successful is 0.95.

(i) Illustrate this information on a tree diagram showing ALL the probabilities on ALL branches. [3]

The tree diagram is shown below:



(ii) An insecticide is selected at random, determine the probability that it is unsuccessful. [3]

$$\text{Probability} = P(A \cap S') + P(B \cap S') + P(C \cap S')$$

$$= P(A)P(A|S') + P(B)P(B|S') + P(C)P(C|S')$$

$$= (0.4)(0.2) + (0.25)(0.35) + (0.35)(0.05)$$

$$= 0.08 + 0.0875 + 0.0175$$

$$= 0.185$$

$\therefore$ The probability that it is unsuccessful is 0.185.

(c) A regular six-sided die is tossed 2 times.

(i) Calculate the probability of obtaining a 5 on the 2^{nd} toss, given that a 5 was obtained on the 1^{st} toss. [2]

Let A represent the event that a '5' is obtained on the second toss.

Let B represent the event that a '5' is obtained on the first toss.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{6}}{\frac{1}{6}}$$

$$= \frac{1}{6}$$

$\therefore$ The probability of obtaining a 5 on the 2^{nd} toss, given that a 5 was obtained on the 1^{st} toss is $\frac{1}{6}$.

(ii) Determine the probability that a 5 is obtained on both tosses. [2]

$$P(5\ on\ both\ tosses) = P(5\ on\ 1st\ toss) \times P(5\ on\ 2nd\ toss)$$

$$P(5\ on\ both\ tosses) = \frac{1}{6} \times \frac{1}{6}$$

$$P(5\ on\ both\ tosses) = \frac{1}{36}$$

$\therefore$ The probability that a 5 is obtained on both tosses is $\frac{1}{36}$.

(iii) Explain why the answers in (c)(i) and (c)(ii) are different. [1]

The answer in (c)(i) is based on conditional probability. It requires the probability of A only given that B has occurred before.

The answer in (c)(ii) was calculated based on two independent events, that is, the occurrence of the first event does not affect the occurrence of the second event.

Total: 20 marks

8. (a) A particle moves in a straight line so that its distance, s metres, after t seconds, measured from a fixed point, O, is given by the function

$s = t^3 - 2t^2 + t - 1.$

Determine

(i) its velocity when $t = 2$ [2]

$s = t^3 - 2t^2 + t - 1$

$v = \frac{ds}{dt}$

$= \frac{d}{dt}(t^3 - 2t^2 + t - 1)$

$= 3t^2 - 4t + 1$

When $t = 2$,

$v = 3(2)^2 - 4(2) + 1$

$= 3(4) - 8 + 1$

$= 12 - 8 + 1$

$= 5\ ms^{-1}$

$\therefore$ The velocity when $t = 2$ is $v = 5\ ms^{-1}$.

(ii) the values of t when the particle is at rest [4]

When the particle is at rest, $v = 0$.

$v = 3t^2 - 4t + 1.$

When $v = 0$,

$$3t^2 - 4t + 1 = 0$$

$$(3t - 1)(t - 1) = 0$$

Either $3t - 1 = 0$ or $t - 1 = 0$

$3t = 1$ $\quad$ $t = 1$

$t = \frac{1}{3}$

$\therefore$ The values of t when the particle is at rest are $t = \frac{1}{3}$ and $t = 1$.

(iii) the distance between the rest points [3]

The rest points occurs at $t = \frac{1}{3}$ and $t = 1$.

When $t = \frac{1}{3}$,

$$s = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) - 1$$

$$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3} - 1$$

$$= -\frac{23}{27}\ m$$

When $t = 1$,

$$s = (1)^3 - 2(1)^2 + (1) - 1$$

$$= 1 - 2 + 1 - 1$$

$$= -1\ m$$

Distance between rest points $= -\frac{23}{27} - (-1)$

$$= -\frac{23}{27} + 1$$

$$= -\frac{23}{27} + \frac{27}{27}$$

$$= \frac{4}{27}\ m$$

$\therefore$ The distance between rest points is $\frac{4}{27}\ m$.

(iv) the time at which the maximum velocity occurs [3]

$$v = 3t^2 - 4t + 1$$

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt}(3t^2 - 4t + 1)$$

$$= 6t - 4$$

At maximum velocity, $a = 0$.

When $a = 0$,

$$6t - 4 = 0$$

$$6t = 4$$

$$t = \frac{4}{6}$$

$$t = \frac{2}{3}$$

$\therefore$ The time at which the maximum velocity occurs is $t = \frac{2}{3}$.

(b) A bus starts from rest at Station A and travels a distance of 80 km in 60 minutes to Station B. Since the bus arrived at Station B early, it remained there for 20 minutes then started the journey to Station C. The time taken to travel from Station B to Station C was 90 minutes at an average speed of 80 kmh^{-1}.

(i) On the grid provided on page 27, draw a distance-time graph to illustrate the motion of the bus. [3]

From Station A, the bus travels a distance of 80 km in 60 minutes.

This is represented by a straight line with positive gradient from point $(0, 0)$ to point $(60, 80)$.

Next, the bus remains for 20 minutes at B.

$$\text{Time passed} = 60 + 20$$
$$= 80 \text{ minutes}$$

The distance covered remains at 80 km.

So, the point is $(80, 80)$.

Lastly, the time taken to travel from Station B to Station C was 90 minutes at an average speed of 80 kmh^{-1}.

$$90 \text{ minutes} = \frac{90}{60} \text{ hours}$$
$$= \frac{3}{2} \text{ hours}$$

Recall:

$$\text{Speed} = \frac{Distance}{Time}$$

$$80 = \frac{Distance}{\frac{3}{2}}$$

$$\text{Distance} = \frac{3}{2} \times 80$$

$$\text{Distance} = 120\ km$$

$$\text{Time passed} = 80 + 90$$

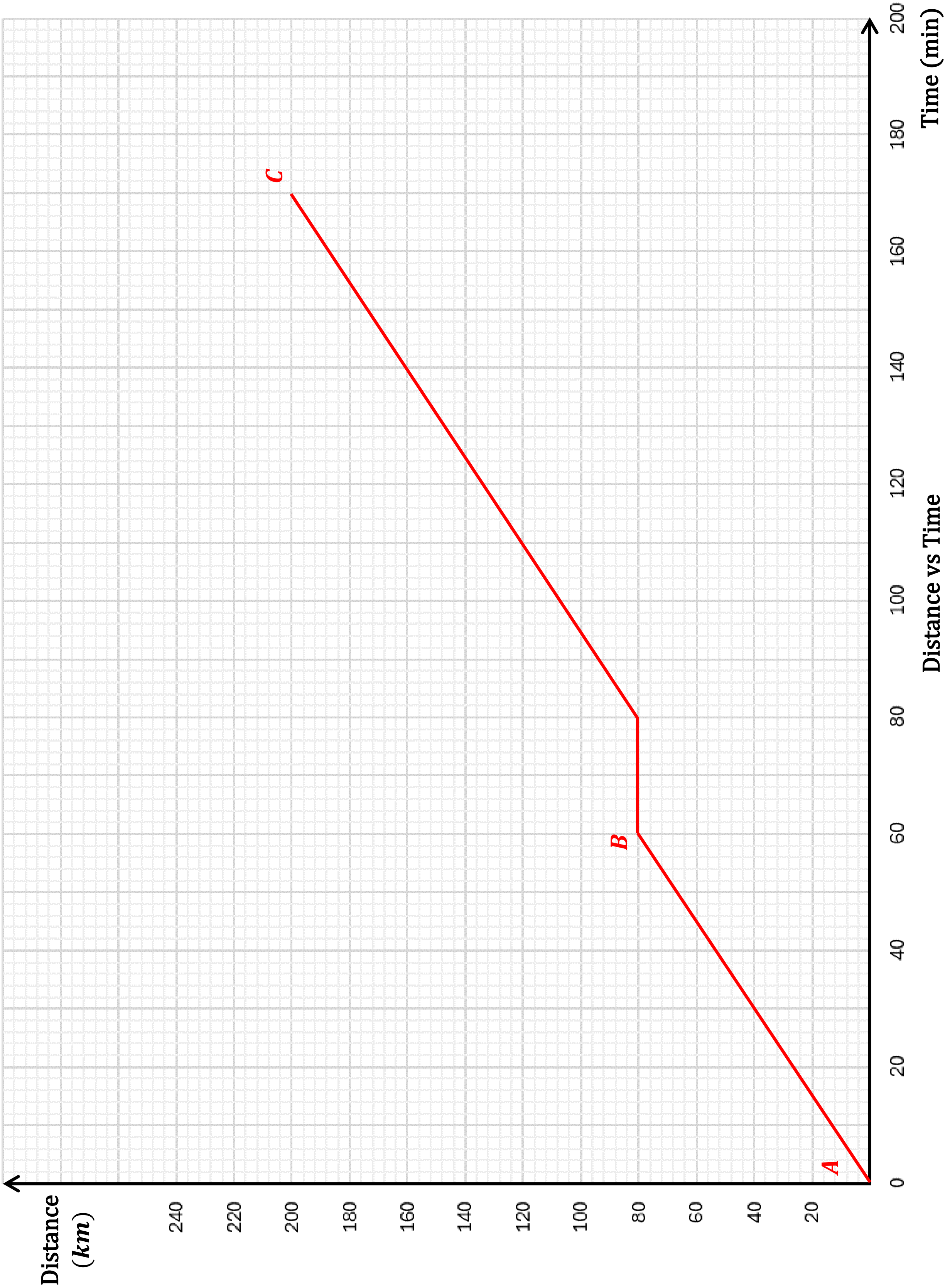
$$= 170 \text{ minutes}$$

$$\text{Distance covered} = 80 + 120$$

$$= 200\ km$$

So, the point is $(170, 200)$.

Title: Graph showing distance vs. time



(ii) Determine the distance from Station B to Station C. [2]

$\text{Distance} = 200 - 80$

$= 120\ m$

$\therefore$ The distance from Station B to Station C is $120\ m$.

(iii) Determine the average speed from Station A to Station B,

in kmh^{-1}. [3]

$\text{Distance} = 80\ km$

$\text{Time} = 60 \text{ minutes}$

$= 1 \text{ hour}$

$\text{Average speed} = \frac{Distance}{Time}$

$= \frac{80}{1}$

$= 80\ kmh^{-1}$

$\therefore$ The average speed from Station A to Station B is $80\ kmh^{-1}$.

Total: 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.