CSEC Add Maths

Paper 2

June 2019

Solutions

SECTION I

Answer BOTH questions.

ALL working must be clearly shown.

1. (a) The function f is such that $f(x) = 2x^3 + 7x^2 + 3x$.

(i) Determine **all** linear factors of $f(x)$. [3]

$$f(x) = 2x^3 + 7x^2 + 3x$$
$$= x(2x^2 + 7x + 3)$$
$$= x(2x + 1)(x + 3)$$

$\therefore$ The linear factors of $f(x)$ are $x, 2x + 1$ and $x + 3$.

(ii) Compute the roots of the function $f(x)$. [2]

$$f(x) = x(2x + 1)(x + 3)$$

When $f(x) = 0$,

$$0 = x(2x + 1)(x + 3)$$

Either $x = 0$ or $2x + 1 = 0$ or $x + 3 = 0$

$2x = -1$ $\quad$ $x = -3$

$x = -\frac{1}{2}$

$\therefore$ The roots of the function $f(x)$ are $x = 0, x = -\frac{1}{2}$ and $x = -3$.

(b) Two functions are such that $g(x) = x^2 - x$ and $h(x) = 2x - 3$.

(i) Determine $gh(x)$. [2]

$gh(x) = g[h(x)]$

$= g(2x - 3)$

$= (2x - 3)^2 - (2x - 3)$

$= 4x^2 - 12x + 9 - 2x + 3$

$= 4x^2 - 14x + 12$

(ii) Given that $hg(x) = 2x^2 - 2x - 3$, show that the values of x, for which $hg(x) = 0$, can be expressed as $\frac{1 \pm \sqrt{7}}{2}$. [3]

$hg(x) = 2x^2 - 2x - 3$

When $hg(x) = 0$,

$2x^2 - 2x - 3 = 0$ which is in the form $ax^2 + bx + c = 0$,

where $a = 2, b = -2$ and $c = -3$.

Using the quadratic formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-3)}}{2(2)}$

$= \frac{2 \pm \sqrt{4 + 24}}{4}$

$= \frac{2 \pm \sqrt{28}}{4}$

$= \frac{2 \pm \sqrt{4 \times 7}}{4}$

$= \frac{2 \pm \sqrt{4}\sqrt{7}}{4}$

$= \frac{2 \pm 2\sqrt{7}}{4}$

$= \frac{2(1 \pm \sqrt{7})}{2(2)}$

$= \frac{1 \pm \sqrt{7}}{2}$

$\therefore$ The values of x for which $hg(x) = 0$ can be expressed as $\frac{1 \pm \sqrt{7}}{2}$.

Q.E.D.

(c) Solve $3x \log 2 + \log 8^x = 2$. [4]

$3x \log 2 + \log 8^x = 2$

$3x \log 2 + \log 2^{3x} = 2$

$3x \log 2 + 3x \log 2 = 2$

$6x \log 2 = 2$

$6x = \frac{2}{\log 2}$

$x = \frac{2}{\log 2} \times \frac{1}{6}$

$x = \frac{2}{6 \log 2}$

$x = 1.11$ (to 3 significant figures)

Total: 14 marks

2. (a) (i) Express $f(x) = -2x^2 - 7x - 6$ in the form $a(x+h)^2 + k$. [3]

$$-2x^2 - 7x - 6$$

$$= -2\left(x^2 + \frac{7}{2}x\right) - 6$$

$$= -2\left(x^2 + \frac{7}{2}x + \frac{49}{16}\right) - 6 + 2\left(\frac{49}{16}\right)$$

$$= -2\left(x + \frac{7}{4}\right)^2 - 6 + \frac{49}{8}$$

$$= -2\left(x + \frac{7}{4}\right)^2 + \frac{1}{8}$$

which is in the form $a(x+h)^2 + k$,

where $a = -2, h = \frac{7}{4}$ and $k = \frac{1}{8}$.

(ii) State the maximum value of $f(x)$. [1]

$$f(x) = -2\left(x + \frac{7}{4}\right)^2 + \frac{1}{8}$$

∴ The maximum value of $f(x)$ is $\frac{1}{8}$.

(iii) State the value of x for which $f(x)$ is a maximum. [1]

Consider,

$$x + \frac{7}{4} = 0$$

$$x = -\frac{7}{4}$$

∴ The value of x for which $f(x)$ is a maximum is $x = -\frac{7}{4}$.

(iv) Use your answer in (a)(i) to determine **all** values of x when

$f(x) = 0.$ [3]

$$f(x) = -2\left(x + \frac{7}{4}\right)^2 + \frac{1}{8}$$

When $f(x) = 0$,

$$-2\left(x + \frac{7}{4}\right)^2 + \frac{1}{8} = 0$$

$$-2\left(x + \frac{7}{4}\right)^2 = -\frac{1}{8}$$

$$\left(x + \frac{7}{4}\right)^2 = -\frac{1}{8} \times -\frac{1}{2}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{1}{16}$$

$$x + \frac{7}{4} = \pm\sqrt{\frac{1}{16}}$$

$$x + \frac{7}{4} = \pm\frac{1}{4}$$

$$x = -\frac{7}{4} \pm \frac{1}{4}$$

Either $x = -\frac{7}{4} - \frac{1}{4}$ or $x = -\frac{7}{4} + \frac{1}{4}$

$= -\frac{8}{4}$ $\quad$ $= -\frac{6}{4}$

$= -2$ $\quad$ $= -\frac{3}{2}$

$\therefore x = -2$ or $x = -\frac{3}{2}$

(v) Sketch the function $f(x)$ and show your solution set to (a)(iv) when

$f(x) < 0$. [2]

When $f(x) = 0, x = -2$ and $x = -\frac{3}{2}$.

The points are $(-2, 0)$ and $\left(-\frac{3}{2}, 0\right)$.

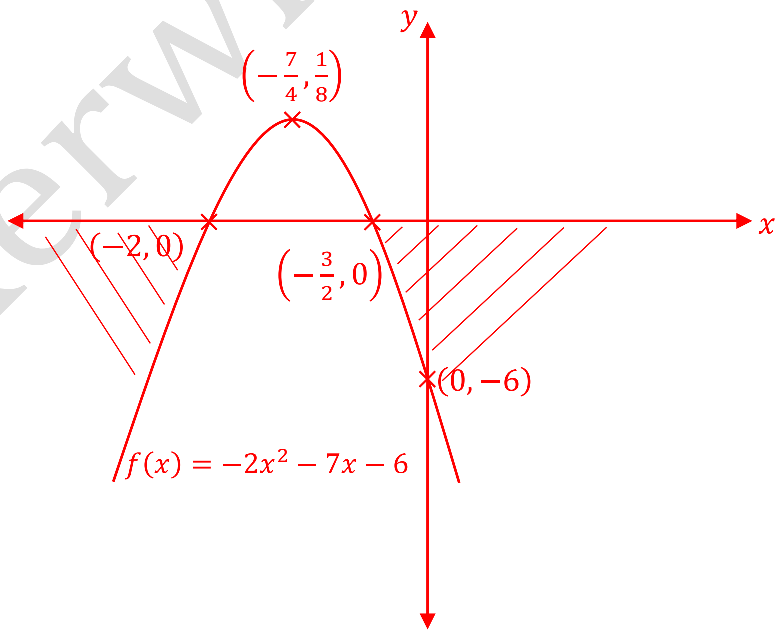
$f(x) = -2\left(x + \frac{7}{4}\right)^2 + \frac{1}{8}$

The maximum point is $\left(-\frac{7}{4}, \frac{1}{8}\right)$.

$f(x) = -2x^2 - 7x - 6$

The y-intercept is $c = -6$. The point is $(0, -6)$.

The sketch of $f(x)$ is shown below:



Consider when $f(x) < 0$.

We look where the graph is negative, that is, below the x-axis.

$\therefore$ The solution set is $\left\{x: x < -2 \cup x > -\frac{3}{2}\right\}$.

(b) A geometric series can be represented by $\frac{y}{x} + \frac{y^2}{x^3} + \frac{y^3}{x^5} + \cdots$

Prove that $S_\infty = xy(x^2 - y)^{-1}$. [4]

$\frac{y}{x} + \frac{y^2}{x^3} + \frac{y^3}{x^5} + \cdots$

$a = \frac{y}{x}$

$ar = \frac{y^2}{x^3}$

So, we have,

$r = \frac{ar}{a}$

$= \frac{y^2}{x^3} \div \frac{y}{x}$

$= \frac{y^2}{x^3} \times \frac{x}{y}$

$= \frac{y}{x^2}$

Now,

$S_\infty = \frac{a}{1-r}$

Substituting $a = \frac{y}{x}$ and $r = \frac{y}{x^2}$ gives:

$$S_\infty = \frac{\left(\frac{y}{x}\right)}{1-\left(\frac{y}{x^2}\right)}$$

$$= \frac{y}{x} \div \left(1 - \frac{y}{x^2}\right)$$

$$= \frac{y}{x} \div \left(\frac{x^2-y}{x^2}\right)$$

$$= \frac{y}{x} \times \frac{x^2}{x^2-y}$$

$$= \frac{xy}{x^2-y}$$

$$= xy(x^2 - y)^{-1}$$

$$\therefore S_\infty = xy(x^2 - y)^{-1}$$

Q.E.D.

Total: 14 marks

SECTION II

Answer BOTH questions.

ALL working must be clearly shown.

3. (a) A circle with centre $(1, -1)$ passes through the point $(4, 3)$.

(i) Calculate the radius of the circle. [2]

The points are $(1, -1)$ and $(4, 3)$.

$$\text{Radius of the circle} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-1)^2 + \big(3-(-1)\big)^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

(ii) Write the equation of the circle in the form

$x^2 + y^2 + 2fx + 2gy + c = 0.$ [2]

The centre is $C(1, -1)$ and $r = 5$.

The equation of the circle is,

$$(x-1)^2 + (y+1)^2 = (5)^2$$

$$x^2 - 2x + 1 + y^2 + 2y + 1 = 25$$

$$x^2 + y^2 - 2x + 2y + 1 + 1 - 25 = 0$$

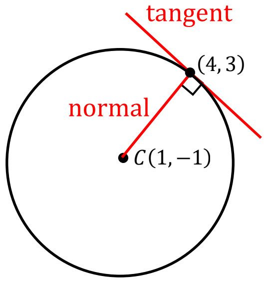
$$x^2 + y^2 - 2x + 2y - 23 = 0$$

which is in the form $x^2 + y^2 + 2fx + 2gy + c = 0$,

where $f = -1$, $g = 1$ and $c = -23$.

(iii) Determine the equation of the tangent to the circle at the point $(4, 3)$. [3]

Consider the sketch below.



Points are $(1, -1)$ and $(4, 3)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-(-1)}{4-1}$

$= \frac{4}{3}$

Gradient of normal $= \frac{4}{3}$

Gradient of tangent $= -\frac{3}{4}$

Substituting $m = -\frac{3}{4}$ and point $(4, 3)$ into $y - y_1 = m(x - x_1)$ gives:

$y - 3 = -\frac{3}{4}(x - 4)$

$y - 3 = -\frac{3}{4}x + 3$

$y = -\frac{3}{4}x + 3 + 3$

$y = -\frac{3}{4}x + 6$ $\quad \rightarrow$ Equation of tangent

(b) Two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $\boldsymbol{p} = 8\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$ and $\boldsymbol{q} = \hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}}$.

(i) Calculate $\boldsymbol{p} \cdot \boldsymbol{q}$. [2]

$\boldsymbol{p} = 8\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$ and $\boldsymbol{q} = \hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}}$

$= \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

The dot product is,

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$= (8)(1) + (2)(-4)$$

$$= 8 - 8$$

$$= 0$$

(ii) State the angle between the two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$. [1]

Since $\boldsymbol{p} \cdot \boldsymbol{q} = 0$, then $\boldsymbol{p}$ is perpendicular to $\boldsymbol{q}$.

$\therefore$ The angle between the two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ is $90°$.

(c) The position vector $\boldsymbol{a} = 4\hat{\boldsymbol{\imath}} - 7\hat{\boldsymbol{\jmath}}$. Find the unit vector in the direction of $\boldsymbol{a}$. [2]

$\boldsymbol{a} = 4\hat{\boldsymbol{\imath}} - 7\hat{\boldsymbol{\jmath}}$

$$|\boldsymbol{a}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(4)^2 + (-7)^2}$$

$$= \sqrt{16 + 49}$$

$$= \sqrt{65}$$

The unit vector in the direction of $\boldsymbol{a} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$

$$= \frac{4\hat{\boldsymbol{\imath}} - 7\hat{\boldsymbol{\jmath}}}{\sqrt{65}}$$

$$= \frac{4}{\sqrt{65}}\hat{\boldsymbol{\imath}} - \frac{7}{\sqrt{65}}\hat{\boldsymbol{\jmath}}$$

Total: 12 marks

4. (a) A compass is used to draw a sector of radius 6 cm and area 11.32 cm^2.

(i) Determine the angle of the sector in radians. [3]

$$A = \frac{1}{2}r^2\theta$$

$$11.32 = \frac{1}{2}(6)^2\theta$$

$$11.32 = \frac{1}{2}(36)\theta$$

$$11.32 = 18\theta$$

$$\theta = \frac{11.32}{18}$$

$\theta = 0.629^c$ (to 3 significant figures)

(ii) Calculate the perimeter of the sector. [2]

Length of arc $= r\theta$

Length of arc $= 6 \times \frac{11.32}{18}$

Length of arc $= 3.773\ cm$ (to 3 decimal places)

Perimeter of the sector $= (2 \times \text{radius}) + \text{Length of arc}$

$= (2 \times 6) + 3.773$

$= 12 + 3.773$

$= 15.773\ cm$

$\therefore$ The perimeter of the sector is 15.773 cm.

(b) A right-angled triangle XYZ has an angle, θ, where $\sin\theta = \frac{\sqrt{5}}{5}$.

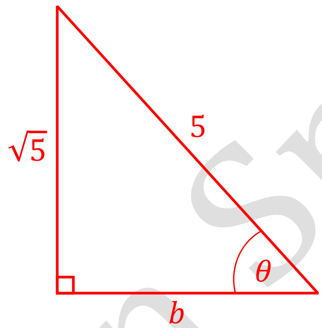
Without evaluating θ, calculate the exact value (in surd form if applicable) of

(i) $\cos\theta$ [2]

$\sin\theta = \frac{\sqrt{5}}{5}$

$= \frac{opp}{hyp}$

Consider the sketch below:



Using Pythagoras' Theorem,

$a^2 + b^2 = c^2$

$\left(\sqrt{5}\right)^2 + b^2 = (5)^2$

$5 + b^2 = 25$

$b^2 = 25 - 5$

$b^2 = 20$

$b = \sqrt{20}$

$b = \sqrt{4 \times 5}$

$b = \sqrt{4}\sqrt{5}$

$b = 2\sqrt{5}$

Now,

$\cos\theta = \frac{adj}{hyp}$

$= \frac{2\sqrt{5}}{5}$

(ii) $\sin 2\theta$ [2]

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{5}}{5}\right)\left(\frac{2\sqrt{5}}{5}\right)$

$= 4 \times \frac{5}{25}$

$= \frac{4}{5}$

(c) Show that $\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$. [3]

Taking L.H.S:

$\tan^2\theta + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

$=$ R.H.S.

$\therefore \tan^2\theta + 1 = \frac{1}{\cos^2\theta}$

Q.E.D.

Total: 12 marks

SECTION III

Answer BOTH questions.

ALL working must be clearly shown.

5. (a) The stationary points of a curve are given by $\left(5, 11\frac{2}{3}\right)$ and $(3, 15)$.

(i) Derive an expression for $\frac{dy}{dx}$. [2]

The stationary points are $\left(5, 11\frac{2}{3}\right)$ and $(3, 15)$.

$\frac{dy}{dx} = (x-5)(x-3)$

$= x^2 - 5x - 3x + 15$

$= x^2 - 8x + 15$

$\therefore$ An expression for $\frac{dy}{dx} = x^2 - 8x + 15$.

(ii) Determine the nature of the stationary points. [5]

To determine the nature of the stationary points, we will use the second derivative test.

$\frac{dy}{dx} = x^2 - 8x + 15$

$\frac{d^2y}{dx^2} = 2x - 8$

When $x = 5$,

$\frac{d^2y}{dx^2} = 2(5) - 8$

$= 10 - 8$

$= 2 \quad (> 0)$

Since $\frac{d^2y}{dx^2} > 0$, then the point $\left(5, 11\frac{2}{3}\right)$ is a minimum point.

When $x = 3$,

$\frac{d^2y}{dx^2} = 2(3) - 8$

$= 6 - 8$

$= -2 \quad (< 0)$

Since $\frac{d^2y}{dx^2} < 0$, then the point $(3, 15)$ is a maximum point.

(iii) Determine the equation of the curve. [4]

$\frac{dy}{dx} = x^2 - 8x + 15$

$y = \int \frac{dy}{dx}\, dx$

$y = \int (x^2 - 8x + 15)\, dx$

$y = \frac{x^3}{3} - \frac{8x^2}{2} + 15x + c$

$y = \frac{x^3}{3} - 4x^2 + 15x + c$

Substituting $(3, 15)$ into the equation gives:

$$15 = \frac{(3)^3}{3} - 4(3)^2 + 15(3) + c$$

$$15 = 9 - 36 + 45 + c$$

$$15 = 18 + c$$

$$c = 15 - 18$$

$$c = -3$$

$\therefore$ The equation of the curve is: $y = \frac{x^3}{3} - 4x^2 + 15x - 3$

(b) Differentiate $\sqrt[3]{(2x+3)^2}$ with respect to x, giving your answer in its simplest form. [3]

Let $y = \sqrt[3]{(2x+3)^2}$

$$y = (2x+3)^{\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{2}{3}(2)(2x+3)^{\frac{2}{3}-1}$$

$$= \frac{4}{3}(2x+3)^{-\frac{1}{3}}$$

$$= \frac{4}{3\left(\sqrt[3]{2x+3}\right)}$$

Total: 14 marks

6. (a) Integrate $3\cos x + 2\sin x$. [2]

$$\int (3\cos x + 2\sin x)\,dx = 3\sin x + 2(-\cos x) + c$$

$$= 3\sin x - 2\cos x + c$$

(b) Evaluate $\int_1^4 \frac{2\sqrt{x}}{x}\,dx$. [4]

$$\int_1^4 \frac{2\sqrt{x}}{x}\,dx = \int_1^4 \frac{2\sqrt{x}}{\sqrt{x}\sqrt{x}}\,dx$$

$$= \int_1^4 \frac{2}{\sqrt{x}}\,dx$$

$$= 2\int_1^4 x^{-\frac{1}{2}}\,dx$$

$$= 2\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^4$$

$$= 2\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4$$

$$= 2(2)\left[x^{\frac{1}{2}}\right]_1^4$$

$$= 4\left[x^{\frac{1}{2}}\right]_1^4$$

$$= 4\left[(4)^{\frac{1}{2}} - (1)^{\frac{1}{2}}\right]$$

$$= 4(2-1)$$

$$= 4(1)$$

$$= 4$$

(c) The point $(2, 4)$ lies on the curve whose gradient is given by $\frac{dy}{dx} = -2x + 1$.

Determine

(i) the equation of the curve [4]

$$\frac{dy}{dx} = -2x + 1$$

$$y = \int \frac{dy}{dx}\, dx$$

$$y = \int (-2x + 1)\, dx$$

$$y = \frac{-2x^2}{2} + x + c$$

$$y = -x^2 + x + c$$

Substituting the point $(2, 4)$ into the equation gives:

$$4 = -(2)^2 + (2) + c$$

$$4 = -4 + 2 + c$$

$$4 = -2 + c$$

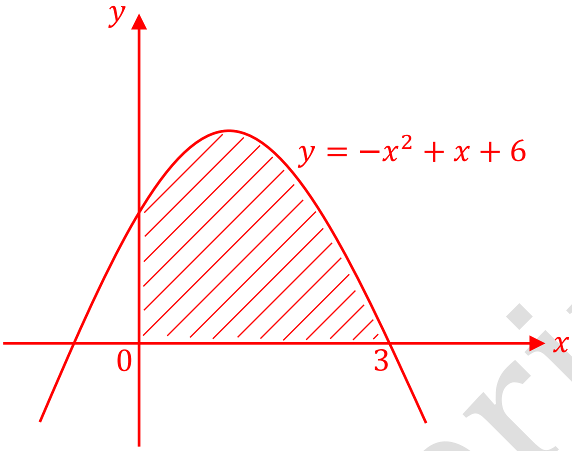
$$c = 4 + 2$$

$$c = 6$$

∴ The equation of the curve is $y = -x^2 + x + 6$.

(ii) the area under the curve in the finite region in the first quadrant between 0 and 3 on the x-axis. [4]

Consider the sketch below:



Area under the curve $= \int_a^b y \; dx$

$$= \int_0^3 (-x^2 + x + 6)\, dx$$

$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_0^3$$

$$= \left[-\frac{(3)^3}{3} + \frac{(3)^2}{2} + 6(3)\right] - \left[-\frac{(0)^3}{3} + \frac{(0)^2}{2} + 6(0)\right]$$

$$= \left(-9 + \frac{9}{2} + 18\right) - 0$$

$$= \frac{27}{2} \text{ units}^2$$

$\therefore$ The area under the curve is $\frac{27}{2}$ units^2.

Total: 14 marks

SECTION IV

Answer only ONE question.

ALL working must be clearly shown.

7. (a) The weights, in kg, of students in a Grade 5 class are displayed in the following stem and leaf diagram.

Boys		Girls
9 8	2	8 8 9
9 9 9 7 3	3	2 2 3 5 8 8 8
5 1 1 1 1 0 0	4	0 1 1 2
1	5	

Key:

Boys 8|2 means 28 kg

Girls 2|8 means 28 kg

(i) State the number of students in the class. [1]

Number of boys $= 15$

Number of girls $= 14$

Number of students $= 15 + 14$

$= 29$

$\therefore$ The number of student is 29 students.

(ii) Construct ONE box-and-whisker plot for the entire Grade 5 class (boys and girls combined). [4]

Lowest score = 28

Highest score = 51

The raw data is:

~~28~~	~~28~~	~~28~~	~~29~~	~~29~~	~~32~~	~~32~~	~~33~~	~~33~~	~~35~~
~~37~~	~~38~~	~~38~~	~~38~~	(39)	~~39~~	~~39~~	~~40~~	~~40~~	~~40~~
~~41~~	~~41~~	~~41~~	~~41~~	~~41~~	~~41~~	~~42~~	~~45~~	~~51~~	

The median is the 15th value.

$Q_2 = 39$

The lower quartile is the mean of the 7th and 8th values.

$Q_1 = \frac{32+33}{2}$

$= \frac{65}{2}$

$= 32.5$

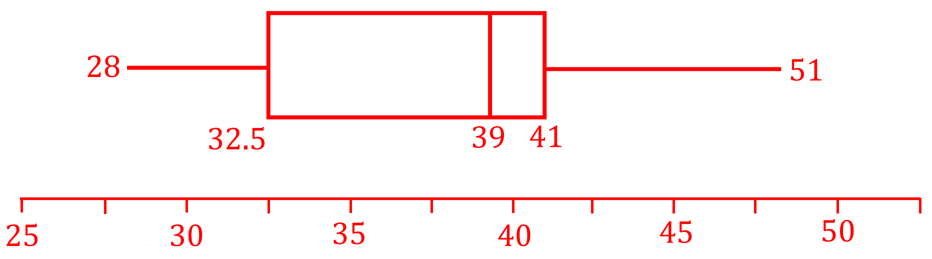
The upper quartile is the mean of the 22nd and 23rd value.

$Q_3 = \frac{41+41}{2}$

$= \frac{82}{2}$

$= 41$

The box-and-whisker plot is shown below:



(iii) The standard deviation of the weights of the boys is 5.53 kg.

Determine the standard deviation of the weights of the girls. Provide an interpretation of your answer for the girls compared to that given for the boys. [5]

Mean, $\overline{x} = \frac{\sum x}{n}$

$= \frac{28+28+29+32+32+33+35+38+38+38+40+41+41+42}{14}$

$= \frac{495}{14}$

$= 35.36$ (to 2 decimal places)

Now,

x	$x - \overline{x}$	$(x - \overline{x})^2$
28	-7.36	54.17
28	-7.36	54.17
29	-6.36	40.45
32	-3.36	11.29
32	-3.36	11.29
33	-2.36	5.57
35	-0.36	0.13
38	2.64	6.97
38	2.64	6.97
38	2.64	6.97
40	4.64	21.53
41	5.64	31.81
41	5.64	31.81
42	6.64	44.09
		$\sum = 327.22$

The standard deviation, $S = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{327.22}{14}}$

$= 4.83\ kg$ (to 2 decimal places)

The standard deviation of the weights of the girls ($4.83\ kg$) is less than that of the boys ($5.53\ kg$).

This means that the data showing the weights of the girls has a less spread or is of a lesser variability than that for the boys. In the case of the girls, their weights are more clustered around the mean.

(iv) Determine the number of students above the 20th percentile for this class. [2]

There is a total of 29 students in the class.

The 20th percentile occurs at $\frac{20}{100} \times 29 = 5.8^{th}$ value.

We take the nearest whole number which is the 6th value.

The 6th value is 32.

Number of students who scored 32 and less $= 7$ students.

Number of students who scored more than 32 $= 22$ students.

$\therefore$ The number of students above the 20th percentile is 22 students.

(b) A vendor has 15 apples on a tray: 5 red, 6 green and 4 yellow. A customer requests 3 apples but does NOT specify a colour.

Determine the probability that the apples chosen

(i) contain one of EACH colour [4]

There are 6 possible ways that this can happen.

The customer can get:

- RGY
- GRY
- YRG
- RYG
- GYR
- YGR

$$P(RGY) = P(R \text{ and } G \text{ and } Y) = \frac{5}{15} \times \frac{6}{14} \times \frac{4}{13} = \frac{4}{91}$$

$$P(RYG) = P(R \text{ and } Y \text{ and } G) = \frac{5}{15} \times \frac{4}{14} \times \frac{6}{13} = \frac{4}{91}$$

$$P(GRY) = P(G \text{ and } R \text{ and } Y) = \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} = \frac{4}{91}$$

$$P(GYR) = P(G \text{ and } Y \text{ and } R) = \frac{6}{15} \times \frac{4}{14} \times \frac{5}{13} = \frac{4}{91}$$

$$P(YRG) = P(Y \text{ and } R \text{ and } G) = \frac{4}{15} \times \frac{5}{14} \times \frac{6}{13} = \frac{4}{91}$$

$$P(YGR) = P(Y \text{ and } G \text{ and } R) = \frac{4}{15} \times \frac{6}{14} \times \frac{5}{13} = \frac{4}{91}$$

$$\text{Probability} = 6 \times \frac{4}{91}$$

$$= \frac{24}{91}$$

$\therefore$ The probability that the apples chosen contain one of each colour is $\frac{24}{91}$.

(ii) are ALL of the same colour [4]

$P(3\ apples\ drawn\ are\ the\ same\ colour)$

$= P(RRR) + P(GGG) + P(YYY)$

$= \left(\frac{5}{15} \times \frac{4}{14} \times \frac{3}{13}\right) + \left(\frac{6}{15} \times \frac{5}{14} \times \frac{4}{13}\right) + \left(\frac{4}{15} \times \frac{3}{14} \times \frac{2}{13}\right)$

$= \frac{60}{2730} + \frac{120}{2730} + \frac{24}{2730}$

$= \frac{60+120+24}{2730}$

$= \frac{204}{2730}$

$= \frac{35}{455}$

∴ The probability that the apples chosen are all of the same colour is $\frac{35}{455}$.

Total: 20 marks

8. (a) A car has stopped at a traffic light. When the light turns green, it accelerates uniformly to a speed of 28 ms^{-1} in 15 seconds. The car continues to travel at this speed for another 35 seconds, before it has to stop 10 seconds later at another traffic light.

(i) On the grid provided **on page 25**, draw a speed-time graph showing the information above. [3]

When the light turns green, it accelerates uniformly to a speed of 28 ms^{-1} in 15 seconds.

Point is $(15, 28)$.

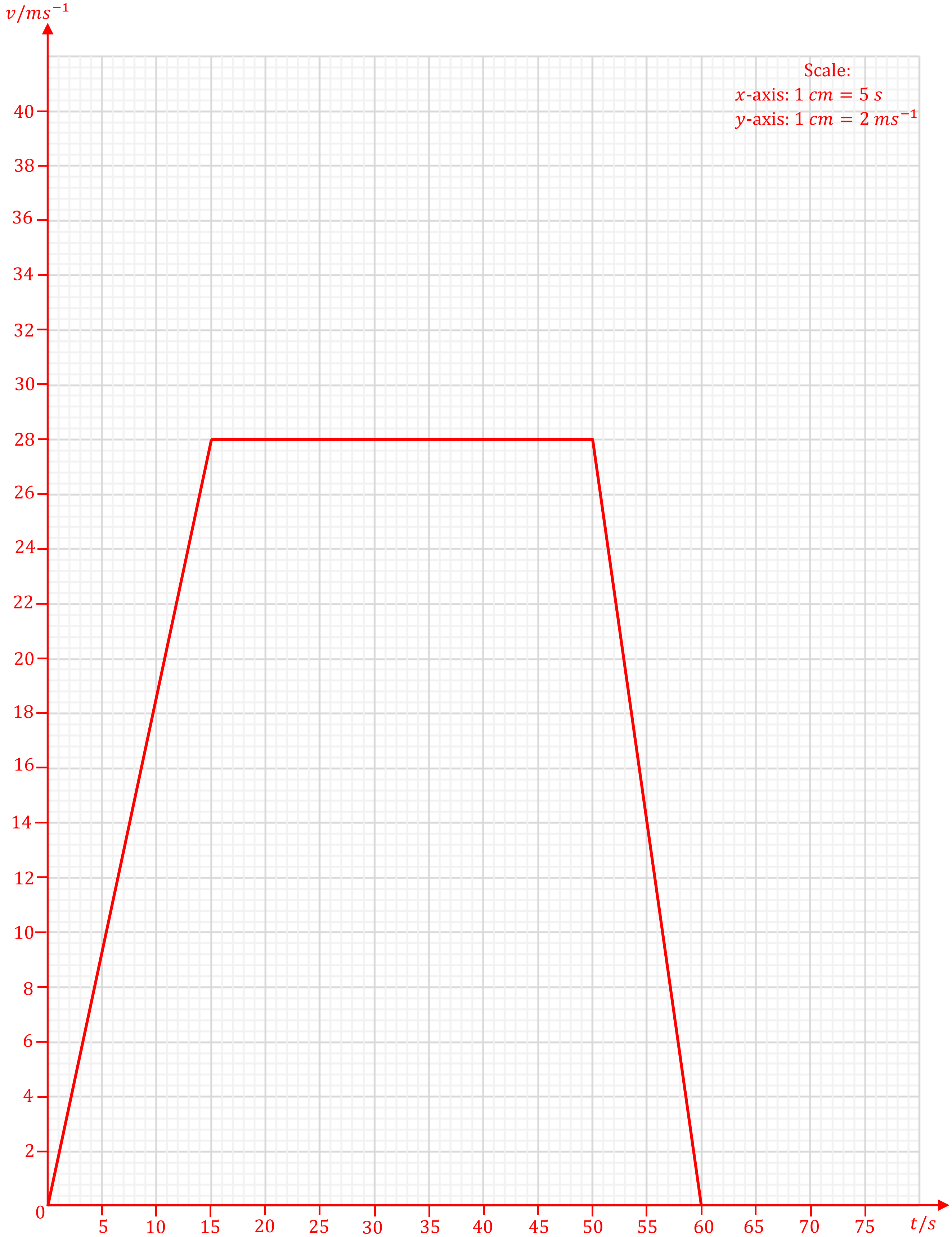
The car continues to travel at this speed for another 35 seconds.

Point is $(50, 28)$.

It has to stop 10 seconds later at another traffic light.

Point is $(60, 0)$.

Title: Graph showing v/ms^{-1} vs. t/s



(ii) Calculate the distance the car travelled between the two traffic lights. [3]

Distance travelled = Area under the curve

$$= \frac{1}{2}(a + b)h$$

$$= \frac{1}{2}(35 + 60)(28)$$

$$= \frac{1}{2}(95)(28)$$

$$= 1330\ m$$

$\therefore$ The distance the car travelled between the two traffic lights is $1330\ m$.

(iii) Calculate the average speed of the car over this journey, giving your answer in $\boldsymbol{kmh^{-1}}$. [3]

Total distance covered $= 1330\ m$

$$= \frac{1330}{1000}\ km$$

$$= 1.33\ km$$

Total time taken $= 60\ s$

$$= \frac{60}{3600}\ h$$

$$= \frac{1}{60}\ h$$

Now,

$$\text{Average speed} = \frac{Total\ distance\ covered}{Total\ time\ taken}$$

$$= \frac{1.33}{\left(\frac{1}{60}\right)}$$

$$= 79.8\ kmh^{-1}$$

∴ The average speed of the car over this journey is 79.8 kmh^{-1}.

(b) A particle moves in a straight line such that t seconds after passing a fixed point, O, its acceleration, a, in ms^{-2}, is given by $a = 12t - 17$. Given that its speed at O is 10 ms^{-1}, determine

(i) the values of t for which the particle is stationary [5]

$a = 12t - 17$

$v = \int a\ dt$

$= \int (12t - 17)\ dt$

$= \frac{12t^2}{2} - 17t + c$

$= 6t^2 - 17t + c$

When $v = 10, t = 0$,

$(10) = 6(0)^2 - 17(0) + c$

$10 = 0 - 0 + c$

$10 = c$

$c = 10$

So, we have $v = 6t^2 - 17t + 10$.

At a stationary point, $v = 0$.

$6t^2 - 17t + 10 = 0$

$(6t - 5)(t - 2) = 0$

Either $\quad 6t - 5 = 0 \quad$ or $\quad t - 2 = 0$

$6t = 5 \qquad t = 2$

$t = \frac{5}{6}$

$\therefore$ The particle is stationary when $t = \frac{5}{6}$ seconds or $t = 2$ seconds.

(ii) the distance the particle travels in the fourth second [6]

$s = \int v \, dt$

$= \int (6t^2 - 17t + 10) \, dt$

$= \frac{6t^3}{3} - \frac{17t^2}{2} + 10t + k$

$= 2t^3 - \frac{17t^2}{2} + 10t + k$

When $s = 0, t = 0$,

$0 = 2(0)^3 - \frac{17(0)^2}{2} + 10(0) + k$

$0 = 0 - 0 + k$

$k = 0$

So, we have, $s = 2t^3 - \frac{17t^2}{2} + 10t$.

When $t = 3$,

$$s = 2(3)^3 - \frac{17(3)^2}{2} + 10(3)$$

$$= 54 - \frac{17(9)}{2} + 30$$

$$= 7.5\ m$$

When $t = 4$,

$$s = 2(4)^3 - \frac{17(4)^2}{2} + 10(4)$$

$$= 128 - 136 + 40$$

$$= 32\ m$$

Distance travelled in the 4^{th} second $= 32 - 7.5$

$$= 24.5\ m$$

$\therefore$ The distance the particle travels in the fourth second is $24.5\ m$.

Total: 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.