CSEC Add Maths

Paper 2

June 2022

Solutions

SECTION I

ALGEBRA, SEQUENCES AND SERIES

ALL working must be clearly shown.

1. (a) Consider the quadratic equation $qx^2 - (4p)x + pq^2 = 0$, where p and q are both **positive** integers.

(i) Express the sum AND product of the roots of the equation in terms of p and q. [3]

$qx^2 - (4p)x + pq^2 = 0$

which is in the form $ax^2 + bx + c = 0$,

where $a = q, b = -4p$ and $c = pq^2$.

$\alpha + \beta = -\frac{b}{a}$

$= \frac{-(-4p)}{q}$

$= \frac{4p}{q}$

$\alpha\beta = \frac{c}{a}$

$= \frac{pq^2}{q}$

$= pq$

$\therefore$ The sum of the roots is $\frac{4p}{q}$ and the product of the roots is pq.

(ii) Determine the value for q such that the sum of the roots is equal to the product of the roots. [2]

Sum of the roots = Product of the roots

$$\alpha + \beta = \alpha\beta$$

$$\frac{4p}{q} = pq$$

$$\frac{4p}{q} = \frac{pq}{1}$$

$$pq^2 = 4p$$

$$q^2 = 4$$

$$q^2 - 4 = 0$$

$$(q + 2)(q - 2) = 0$$

Either $q + 2 = 0$ or $q - 2 = 0$

$q = -2$ $q = 2$

Since p and q are both positive integers, then $q = 2$.

$\therefore$ The value for q such that the sum of the roots is equal to the product of the roots is $q = 2$.

(iii) If the sum of the roots of the equation is 20, use your answer from (a)(ii) to determine a value for p. [1]

$\alpha + \beta = 20$

$\frac{4p}{q} = 20$

Substituting $q = 2$ gives,

$\frac{4p}{2} = 20$

$4p = 2 \times 20$

$4p = 40$

$p = \frac{40}{4}$

$p = 10$

$\therefore p = 10$

(iv) Hence, express the given quadratic equation in terms of its numerical coefficients. [1]

$qx^2 - (4p)x + pq^2 = 0$

Since $p = 10$ and $q = 2$, then

$(2)x^2 - 4(10)x + (10)(2)^2 = 0$

$2x^2 - 40x + (10)(4) = 0$

$2x^2 - 40x + 40 = 0$

(b) A series is given by

$$25 - 5 + 1 - \frac{1}{5} + \frac{1}{25} \dots$$

(i) Show that the series is geometric. [2]

From the series given,

$T_1 = 25 \quad \rightarrow \quad a$

$T_2 = -5 \quad \rightarrow \quad ar$

$T_3 = 1 \quad \rightarrow \quad ar^2$

$T_4 = -\frac{1}{5} \quad \rightarrow \quad ar^3$

$T_5 = \frac{1}{25} \quad \rightarrow \quad ar^4$

Now,

$$r = \frac{T_2}{T_1} = \frac{-5}{25} = -\frac{1}{5}$$

$$r = \frac{T_3}{T_2} = \frac{1}{-5} = -\frac{1}{5}$$

$$r = \frac{T_4}{T_3} = \frac{-\frac{1}{5}}{1} = -\frac{1}{5}$$

$$r = \frac{T_5}{T_4} = \frac{\frac{1}{25}}{-\frac{1}{5}} = -\frac{1}{5}$$

So, we have that first term, $a = 25$ and the common ratio, $r = -\frac{1}{5}$.

$\therefore$ The series is geometric.

Q.E.D.

(ii) Calculate the sum to infinity of the series, giving the answer to **2 decimal places.** [2]

We know that $a = 25$ and $r = -\frac{1}{5}$.

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{25}{1-\left(-\frac{1}{5}\right)}$$

$$= \frac{25}{1+\frac{1}{5}}$$

$$= \frac{25}{\frac{6}{5}}$$

$$= 20\frac{5}{6}$$

$$= 20.83 \quad \text{(to 2 decimal places)}$$

$\therefore$ The sum to infinity of the series is $S_\infty = 20\frac{5}{6}$.

(c) A recent university graduate was offered a starting salary of \$720 000 for the first year, with increases of \$5000 at the start of every year thereafter. Determine the number of years (to the nearest whole number) that it would take for her annual salary to be 20% greater than her salary in the first year. [4]

The starting salary is \$720 000.

Let $a = \$720\ 000$.

Now, there are increases of \$5000 at the start of every year thereafter.

Notice that this is an arithmetic progression where $d = \$5000$.

Now, her expected annual salary is 20% greater than her salary in the first year.

So, we have,

Expected annual salary $= 120\%$ of \$720 000

$$= \frac{120}{100} \times \$720\,000$$

$$= \$864\,000$$

We need to determine how many years it would take for her annual salary to reach \$864 000.

Consider,

$$T_n = a + (n-1)d$$

$$864\,000 = 720\,000 + (n-1)(5000)$$

$$864\,000 - 720\,000 = (n-1)(5000)$$

$$144\,000 = (n-1)(5000)$$

$$\frac{144\,000}{5000} = n - 1$$

$$28.8 = n - 1$$

$$n = 28.8 + 1$$

$$n = 29.8$$

$$n = 30 \text{ years} \quad \text{(to the nearest whole number)}$$

$\therefore$ The number of required years is 30 years.

Total: 15 marks

2. (a) When the polynomial expression $2x^3 - 3x^2 - cx + d$ is divided by $(x + 1)$ and $(x - 2)$, the same remainder of 64 is obtained.

Determine the value of c and d. [4]

Let $f(x) = 2x^3 - 3x^2 - cx + d$.

When $f(x)$ is divided by $(x + 1)$, it leaves a remainder of 64. Therefore, by the Remainder Theorem, $f(-1) = 64$.

$$f(-1) = 2(-1)^3 - 3(-1)^2 - c(-1) + d$$

$$64 = 2(-1) - 3(1) - c(-1) + d$$

$$64 = -2 - 3 + c + d$$

$$64 + 2 + 3 = c + d$$

$$69 = c + d$$

$$d = 69 - c \qquad \rightarrow \text{Equation 1}$$

When $f(x)$ is divided by $(x - 2)$, it leaves a remainder of 64. Therefore, by the Remainder Theorem, $f(2) = 64$.

$$f(2) = 2(2)^3 - 3(2)^2 - c(2) + d$$

$$64 = 2(8) - 3(4) - 2c + d$$

$$64 = 16 - 12 - 2c + d$$

$$64 = 4 - 2c + d$$

$$64 - 4 = -2c + d$$

$$60 = -2c + d \qquad \rightarrow \text{Equation 2}$$

Substituting Equation 1 into Equation 2 gives:

$$60 = -2c + (69 - c)$$

$$60 - 69 = -2c - c$$

$$-9 = -3c$$

$$c = \frac{-9}{-3}$$

$$c = 3$$

Substituting $c = 3$ into Equation 1 gives:

$$d = 69 - 3$$

$$= 66$$

$\therefore$ The value of $c = 3$ and $d = 66$.

(b) Show that the expression $\frac{\sqrt{25}}{\sqrt{45}}$ is the same as $\frac{\sqrt{5}}{3}$. [2]

$$\frac{\sqrt{25}}{\sqrt{45}} = \frac{\sqrt{5\times5}}{\sqrt{9\times5}}$$

$$= \frac{\sqrt{5}\sqrt{5}}{\sqrt{9}\sqrt{5}}$$

$$= \frac{\sqrt{5}}{\sqrt{9}}$$

$$= \frac{\sqrt{5}}{3}$$

$\therefore \frac{\sqrt{25}}{\sqrt{45}}$ is the same as $\frac{\sqrt{5}}{3}$.

Q.E.D.

(c) (i) Given $g(x) = 6x^2 + 12x - 18$, express $g(x)$ in the form

$a(x+h)^2 + k$. [3]

$$6x^2 + 12x - 18 = 6(x^2 + 2x) - 18$$

$$= 6(x^2 + 2x + 1) - 18 - 6(1)$$

$$= 6(x+1)^2 - 18 - 6$$

$= 6(x+1)^2 - 24$ which is in the form $a(x+h)^2 + k$,

where $a = 6, h = 1$ and $k = -24$.

(ii) Using the expression derived in (c)(i), determine the roots of $g(x)$. [3]

To find the roots, let $g(x) = 0$.

$$6(x+1)^2 - 24 = 0$$

$$6(x+1)^2 = 24$$

$$(x+1)^2 = \frac{24}{6}$$

$$(x+1)^2 = 4$$

$$x + 1 = \pm 2$$

$$x = -1 \pm 2$$

Either $x = -1 - 2$ or $x = -1 + 2$

$x = -3$ $x = 1$

$\therefore$ The roots of $g(x)$ are $x = -3$ and $x = 1$.

(iii) Hence, sketch the graph of $g(x)$ on the following grid. [3]

$g(x) = 6x^2 + 12x - 18$

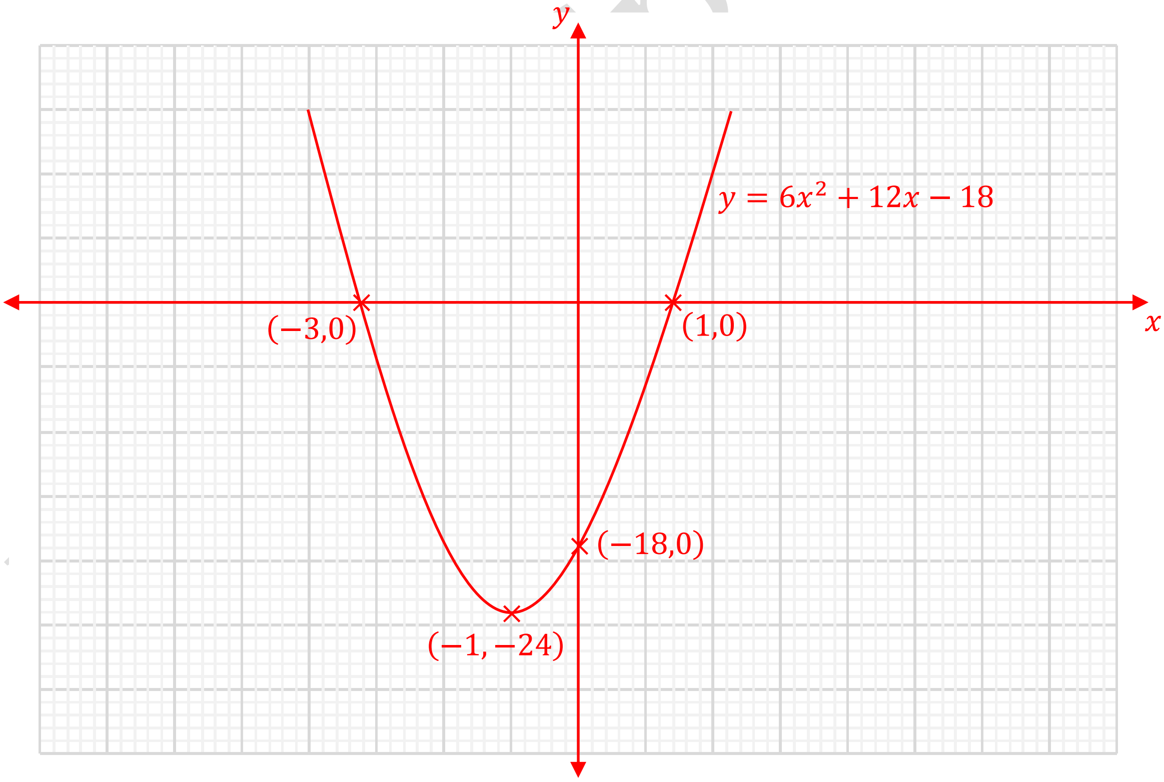
The y-intercept is $c = -18$.

The roots are $x = -3$ and $x = 1$.

$g(x) = 6(x+1)^2 - 24$

The minimum point is $(-1, -24)$.

The sketch of the graph of $g(x)$ is as follows:



Total: 15 marks

SECTION II

COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

ALL working must be clearly shown.

3. (a) The equation of a circle is $x^2 + y^2 + 4x - 8y + 10 = 0$.

(i) Determine the coordinates of its centre AND the length of the radius of the circle. [3]

$$x^2 + y^2 + 4x - 8y + 10 = 0$$

$$x^2 + 4x + y^2 - 8y = -10$$

$$x^2 + 4x + 4 + y^2 - 8y + 16 = -10 + 4 + 16$$

$$(x + 2)^2 + (y - 4)^2 = 10$$

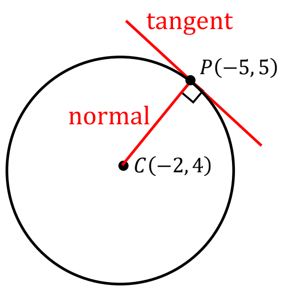
which is in the form $(x - a)^2 + (y - b)^2 = r^2$,

where $C(-2, 4)$ and radius, $r = \sqrt{10}$ units.

$\therefore$ The coordinates of the centre is $C(-2, 4)$ and the length of the radius, $r = \sqrt{10}$ units.

(ii) Determine the equation of the tangent to the circle at the point $P(-5, 5)$. [3]

Consider the sketch below.



Points are $C(-2, 4)$ and $P(-5, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-4}{-5-(-2)}$$

$$= \frac{1}{-3}$$

Gradient of the normal $= -\frac{1}{3}$

Gradient of the tangent $= 3$

Substituting $m = 3$ and point $(-5, 5)$ into $y - y_1 = m(x - x_1)$ gives:

$y - 5 = 3(x - (-5))$

$y - 5 = 3(x + 5)$

$y - 5 = 3x + 15$

$y = 3x + 15 + 5$

$y = 3x + 20$

$\therefore$ The equation of the tangent is: $y = 3x + 20$

(b) The vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are such that $\overrightarrow{OX} = 4\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ and $\overrightarrow{OY} = \hat{\boldsymbol{\imath}} - 4\hat{\boldsymbol{\jmath}}$. Show that the vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are perpendicular. [5]

To show that the vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are perpendicular, we need to show that the angle between them is equal to 90°.

$\overrightarrow{OX} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\overrightarrow{OY} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$$\overrightarrow{OX} \cdot \overrightarrow{OY} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$= (4)(1) + (1)(-4)$$

$$= 4 - 4$$

$$= 0$$

$$|\overrightarrow{OX}| = \sqrt{(4)^2 + (1)^2}$$

$$= \sqrt{16 + 1}$$

$$= \sqrt{17}$$

$$|\overrightarrow{OY}| = \sqrt{(1)^2 + (-4)^2}$$

$$= \sqrt{1 + 16}$$

$$= \sqrt{17}$$

$$\cos\theta = \frac{\overrightarrow{OX} \cdot \overrightarrow{OY}}{|\overrightarrow{OX}||\overrightarrow{OY}|}$$

$$\cos\theta = \frac{0}{\sqrt{17}\sqrt{17}}$$

$$\cos\theta = 0$$

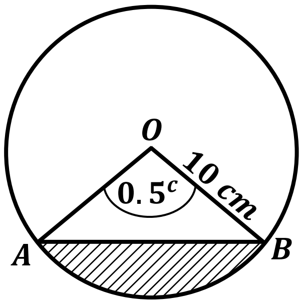
$$\theta = \cos^{-1}(0)$$

$$\theta = 90°$$

$\therefore$ The vectors $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are perpendicular.

Q.E.D.

(c) The diagram below, **not drawn to scale**, shows a chord AB which subtends an angle of 0.5^c (0.5 radians) at the centre, O, of a circle of radius 10 cm. Given that the area of triangle $AOB = \frac{1}{2}r^2 \sin\theta$, calculate the area of the shaded region. [3]



Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(10)^2(0.5)$

$= 25\ cm^2$

Area of triangle $= \frac{1}{2}r^2 \sin\theta$

$= \frac{1}{2}(10)^2 \sin(0.5^c)$

$= 23.97\ cm^2$ (to 2 decimal places)

Now,

Area of shaded region = Area of sector – Area of triangle

$= 25 - 23.97$

$= 1.03\ cm^2$

$\therefore$ The area of shaded region is $1.03\ cm^2$.

THE STUDENT HUB

(d) (i) Show that $\cos 2\theta = 2\cos^2\theta - 1$. [3]

Taking L.H.S:

$$\cos 2\theta = \cos(\theta + \theta)$$

$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$= \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - (1 - \cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1$$

$=$ R.H.S.

$\therefore \cos 2\theta = 2\cos^2\theta - 1$

Q.E.D.

(ii) Hence, solve the equation $\cos 2\theta + \cos\theta + 1 = 0$ for $0 < \theta < 2\pi$. [3]

$$\cos 2\theta + \cos\theta + 1 = 0$$

$$2\cos^2\theta - 1 + \cos\theta + 1 = 0$$

$$2\cos^2\theta + \cos\theta = 0$$

$$\cos\theta\,(2\cos\theta + 1) = 0$$

Either $\cos\theta = 0$ or $2\cos\theta + 1 = 0$

$$2\cos\theta = -1$$

$$\cos\theta = -\tfrac{1}{2}$$

Consider,

$\cos\theta = 0$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Consider,

$\cos\theta = -\frac{1}{2}$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$\therefore \theta = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}$ for $0 < \theta < 2\pi$.

Total: 20 marks

SECTION III

INTRODUCTORY CALCULUS

ALL working must be clearly shown.

4. (a) Given that $f(x) = x(5-x)^2$, determine $f''(x)$. [3]

$$f(x) = x(5-x)^2$$

$$= x(25 - 10x + x^2)$$

$$= 25x - 10x^2 + x^3$$

Now,

$$f'(x) = 25 - 20x + 3x^2$$

$$f''(x) = -20 + 6x$$

$$\therefore f''(x) = -20 + 6x$$

(b) Differentiate EACH of the following expressions with respect to x, simplifying your answer where possible.

(i) $2 \sin 3x + \cos x$ [3]

$$\frac{d}{dx}(2\sin 3x + \cos x)\, dx = 2(3\cos 3x) + (-\sin x)$$

$$= 6\cos 3x - \sin x$$

(ii) $(1+2x)^3(x+2)$ [3]

Let $y = (1+2x)^3(x+2)$.

Let $u = (1+2x)^3$, $v = x+2$

$\frac{du}{dx} = 3(2)(1+2x)^{3-1}$ $\frac{dv}{dx} = 1$

$= 6(1+2x)^2$

Using the product rule,

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$= (x+2)(6)(1+2x)^2 + (1+2x)^3(1)$

$= 6(x+2)(1+2x)^2 + (1+2x)^3$

$\therefore \frac{d}{dx}[(1+2x)^3(x+2)] = 6(x+2)(1+2x)^2 + (1+2x)^3$

(c) (i) Determine the stationary points on the curve $y = x^3 - 4x^2 + 4x$. [4]

$y = x^3 - 4x^2 + 4x$

$\frac{dy}{dx} = 3x^2 - 8x + 4$

At stationary points, $\frac{dy}{dx} = 0$.

When $\frac{dy}{dx} = 0$,

$3x^2 - 8x + 4 = 0$

$(3x-2)(x-2) = 0$

Either $3x - 2 = 0$ or $x - 2 = 0$

$3x = 2$ $x = 2$

$x = \frac{2}{3}$

When $x = \frac{2}{3}$,

$y = \left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$

$= \frac{8}{27} - \frac{16}{9} + \frac{8}{3}$

$= \frac{32}{27}$

When $x = 2$,

$y = (2)^3 - 4(2)^2 + 4(2)$

$= 8 - 16 + 8$

$= 0$

$\therefore$ The stationary points are $\left(\frac{2}{3}, \frac{32}{27}\right)$ and $(2, 0)$.

(ii) Providing details, determine the nature of EACH stationary point in (c)(i). [2]

To determine the nature of each stationary point, we will use the second derivative test.

$\frac{dy}{dx} = 3x^2 - 8x + 4$

$\frac{d^2y}{dx^2} = 6x - 8$

When $x = \frac{2}{3}$,

$$\frac{d^2y}{dx^2} = 6\left(\frac{2}{3}\right) - 8$$

$$= 4 - 8$$

$$= -4 \qquad (< 0)$$

Since $\frac{d^2y}{dx^2} < 0$, then the point $\left(\frac{2}{3}, \frac{32}{27}\right)$ is a maximum point.

When $x = 2$,

$$\frac{d^2y}{dx^2} = 6(2) - 8$$

$$= 12 - 8$$

$$= 4 \qquad (> 0)$$

Since $\frac{d^2y}{dx^2} > 0$, then the point $(2, 0)$ is a minimum point.

Total: 15 marks

5. (a) Determine $\int (4\cos\theta - 6\sin\theta)\ d\theta$. [2]

$$\int (4\cos\theta - 6\sin\theta)\ d\theta = 4\sin\theta - (-6\cos\theta) + c$$

$$= 4\sin\theta + 6\cos\theta + c$$

(b) Evaluate $\int_1^2 (3-x)^2\ dx$. [3]

$$\int_1^2 (3-x)^2\ dx = \int_1^2 (9 - 6x + x^2)\ dx$$

$$= \left[9x - \frac{6x^2}{2} + \frac{x^3}{3}\right]_1^2$$

$$= \left[9x - 3x^2 + \frac{x^3}{3}\right]_1^2$$

$$= \left[9(2) - 3(2)^2 + \frac{(2)^3}{3}\right] - \left[9(1) - 3(1)^2 + \frac{(1)^3}{3}\right]$$

$$= \left(18 - 12 + \frac{8}{3}\right) - \left(9 - 3 + \frac{1}{3}\right)$$

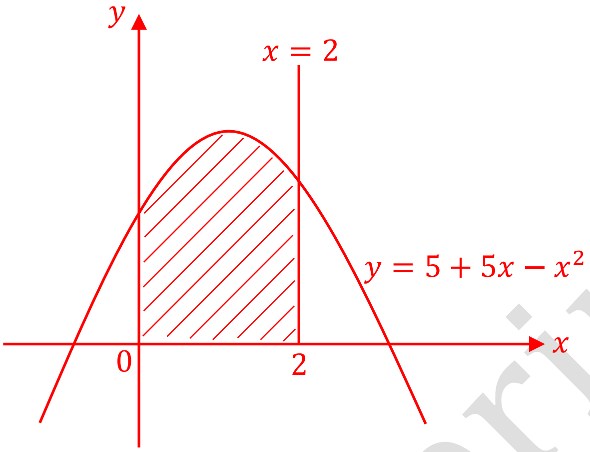
$$= \frac{26}{3} - \frac{19}{3}$$

$$= \frac{7}{3}$$

$$\therefore \int_1^2 (3-x)^2\ dx = \frac{7}{3}$$

(c) Determine the area of the region bounded by the curve $y = 5 + 5x - x^2$, the x-axis, the y-axis and the line $x = 2$. [4]

Consider the sketch below:



$$\text{Area of shaded region} = \int_a^b y\,dx$$

$$= \int_0^2 (5 + 5x - x^2)\,dx$$

$$= \left[5x + \frac{5x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$= \left[5(2) + \frac{5(2)^2}{2} - \frac{(2)^3}{3}\right] - \left[5(0) + \frac{5(0)^2}{2} - \frac{(0)^3}{3}\right]$$

$$= \left(10 + 10 - \frac{8}{3}\right) - 0$$

$$= \frac{52}{3}\ \text{units}^2$$

$\therefore$ The area of the region is $\frac{52}{3}$ units^2.

(d) A particle moves in a straight line so that t seconds after passing through a fixed point, O, its acceleration, a, is given by $a = (3t - 1)\ ms^{-2}$. When $t = 2$, the particle has a velocity, v, of $4\ ms^{-1}$, and a displacement of $6\ m$ from O.

Determine the velocity when $t = 4$. [6]

$a = 3t - 1$

$v = \int a\ dt$

$v = \int (3t - 1)\, dt$

$v = \frac{3t^2}{2} - t + c$

When $t = 2, v = 4$. So, we have,

$4 = \frac{3(2)^2}{2} - (2) + c$

$4 = 6 - 2 + c$

$4 = 4 + c$

$0 = c$

Hence, $v = \frac{3t^2}{2} - t$.

When $t = 4$,

$v = \frac{3(4)^2}{2} - (4)$

$= 24 - 4$

$= 20\ ms^{-1}$

$\therefore$ The velocity when $t = 4$ is $v = 20\ ms^{-1}$.

Total: 15 marks

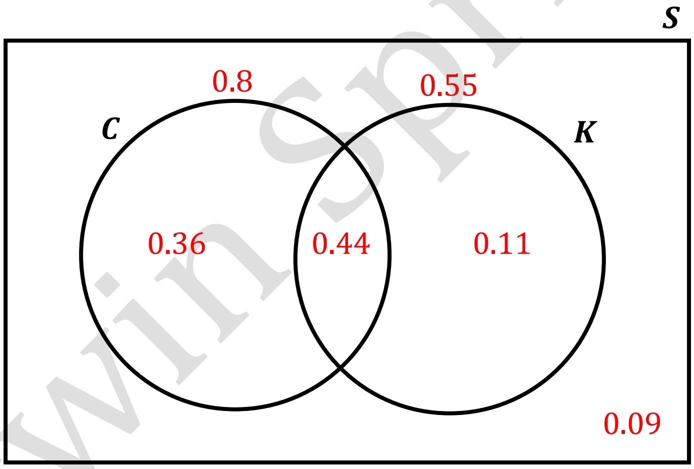
SECTION IV

PROBABILITY AND STATISTICS

ALL working must be clearly shown.

6. (a) At a school canteen, 80% of the students (S) purchase chips (C) and 55% purchase chicken (K). Of the students who purchase chicken, 11% do not purchase chips.

(i) Complete the following Venn diagram to illustrate this information. [4]



Purchased chips $= 0.8$

Purchased chicken $= 0.55$

Purchased only chicken $= 0.11$

Purchased chicken and chips $= 0.55 - 0.11$

$= 0.44$

Purchased only chips $= 0.8 - 0.44$

$= 0.36$

Purchased neither chips not chicken $= 1 - (0.36 + 0.44 + 0.11)$

$= 1 - 0.91$

$= 0.09$

(ii) Determine the probability that a student chosen at random purchases ONLY chicken or ONLY chips. [1]

$P(K \text{ only or } C \text{ only}) = 0.36 + 0.11$

$= 0.47$

$\therefore$ The probability that a student chosen at random purchases ONLY chicken or ONLY chips is 0.47.

(b) The probabilities of the occurrence of two events, A and B, are given by $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{7}{10}$. Determine

(i) $P(A \cap B)$ [3]

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{7}{10} = \frac{1}{4} + \frac{3}{5} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{4} + \frac{3}{5} - \frac{7}{10}$$

$$P(A \cap B) = \frac{3}{20}$$

(ii) $P(A|B)$ [3]

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\left(\frac{3}{20}\right)}{\left(\frac{3}{5}\right)}$$

$$= \frac{3}{20} \div \frac{3}{5}$$

$$= \frac{3}{20} \times \frac{5}{3}$$

$$= \frac{1}{4}$$

$$\therefore\ P(A|B) = \frac{1}{4}$$

(c) State, with a reason, whether Events A and B are independent. [1]

$$P(A \cap B) = \frac{3}{20}$$

$$P(A) \times P(B) = \frac{1}{4} \times \frac{3}{5}$$

$$= \frac{3}{20}$$

Since $P(A \cap B) = P(A) \times P(B)$, then the events A and B are independent.

(d) The following table shows the marks obtained by 27 students in a Mathematics test.

30	35	39	42	45	45	52	59	61
61	65	69	70	71	75	77	79	81
83	83	85	87	89	90	90	95	98

(i) Construct a stem-and-leaf diagram to display this data. [4]

The stem-and-leaf diagram is shown below:

3	0	5	9			
4	2	5	5			
5	2	9				
6	1	1	5	9		
7	0	1	5	7	9	
8	1	3	3	5	7	9
9	0	0	5	8		

Key: 3|0 means 30

(ii) State ONE advantage of using a stem-and-leaf diagram to display the data. [1]

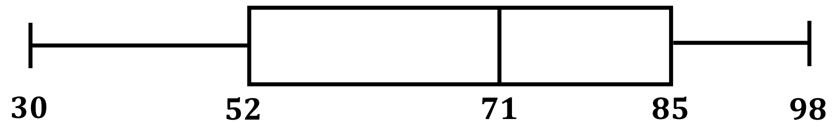
One advantage of using a stem-and-leaf diagram to display the data is that it maintains the original data.

(iii) State the range of values of the marks obtained by the students. [1]

$$\begin{aligned}\text{Range} &= \text{Highest Value} - \text{Lowest Value}\\ &= 98 - 30\\ &= 68\end{aligned}$$

$\therefore$ The range of values of the marks obtained by the students is 68 marks.

(iv) The data are displayed in the following box-and-whisker plot.



State TWO distinct observations about the data as seen in the box-and-whisker plot. [2]

Median = 71

Lower quartile = 52

Upper quartile = 85

Two distinct observations are:

1. Data is asymmetric.
2. Data is negatively skewed.

Total: 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.