CSEC Add Maths

Paper 2

June 2024

Solutions

SECTION I

ALGEBRA, SEQUENCES AND SERIES

ALL working must be clearly shown.

1. (a) (i) Determine the other linear factors of the polynomial $3x^3 + 8x^2 - 20x - 16$, given that $x - 2$ is a factor. [3]

Let $f(x) = 3x^3 + 8x^2 - 20x - 16$.

We are given that $(x - 2)$ is a factor of $f(x)$.

By long division,

$$
\begin{array}{r|l}
 & \phantom{3x^3 + {}}3x^2 + 14x + 8 \\
\cline{2-2}
x - 2 & 3x^3 + 8x^2 - 20x - 16 \\
 & 3x^3 - 6x^2 \\
\cline{2-2}
 & \phantom{3x^3 + {}}14x^2 - 20x \\
 & \phantom{3x^3 + {}}14x^2 - 28x \\
\cline{2-2}
 & \phantom{3x^3 + 8x^2 - {}}8x - 16 \\
 & \phantom{3x^3 + 8x^2 - {}}8x - 16 \\
\cline{2-2}
 & \phantom{3x^3 + 8x^2 - 20x - {}}0
\end{array}
$$

So, we have,

$f(x) = (x - 2)(3x^2 + 14x + 8)$

$\quad\;\; = (x - 2)(3x + 2)(x + 4)$

$\therefore$ The other linear factors besides $(x - 2)$ are $(3x + 2)$ and $(x + 4)$.

(ii) Hence, simplify the multiplication

$$\frac{3x^3+8x^2-20x-16}{x^2-4} \times \frac{x+2}{x+4}$$ [2]

From above, $3x^3 + 8x^2 - 20x - 16 = (x-2)(3x+2)(x+4)$.

$$= \frac{3x^3+8x^2-20x-16}{x^2-4} \times \frac{x+2}{x+4}$$

$$= \frac{\cancel{(x-2)}(3x+2)\cancel{(x+4)}}{\cancel{(x+2)}\cancel{(x-2)}} \times \frac{\cancel{x+2}}{\cancel{x+4}}$$

$$= 3x + 2$$

(b) The equation $kx^2 + x - 15 = 10$ has roots α and β, where $k \in \mathbb{W}$.

(i) Determine expressions for

- $\alpha + \beta$
- $\alpha\beta$ [2]

$$kx^2 + x - 15 = 10$$

$$kx^2 + x - 15 - 10 = 0$$

$$kx^2 + x - 25 = 0$$

which is in the form $ax^2 + bx + c = 0$,

where $a = k, b = 1$ and $c = -25$.

$$\alpha + \beta = -\frac{b}{a}$$

$$= -\frac{1}{k}$$

$$\alpha\beta = \frac{c}{a}$$

$$= -\frac{25}{k}$$

(ii) Given that $\alpha^2 + \beta^2 = \frac{61}{4}$, use the expressions in (b)(i) to determine the value of k. [4]

$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\frac{61}{4} = \left(-\frac{1}{k}\right)^2 - 2\left(-\frac{25}{k}\right)$$

$$\frac{61}{4} = \frac{1}{k^2} + \frac{50}{k}$$

$(\times\ 4k^2)$

$$61k^2 = 4 + 200k$$

$61k^2 - 200k - 4 = 0$ which is in the form $ak^2 + bk + c$,

where $a = 61, b = -200$ and $c = -4$.

Using the quadratic equation,

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-200) \pm \sqrt{(-200)^2 - 4(61)(-4)}}{2(61)}$$

$$= \frac{200 \pm \sqrt{40976}}{122}$$

Either $k = \frac{200 - \sqrt{40976}}{122}$ or $k = \frac{200 + \sqrt{40976}}{122}$

$= -0.02$ (to 3 s.f.) $= 3.3$ (to 3 s.f.)

(Since $k \in \mathbb{W}$, an error lies in the question and not in the solution)

Since $k \in \mathbb{W}$, the only possible whole number close to 3.3 is $k = 3$.

$\therefore$ The value of k is $k = 3$.

(c) For the quadratic function $g(x) = -5x^2 - 4x + 2$, determine the exact value of the maximum point **and** the range using the method of completing the square, or otherwise. [4]

$$-5x^2 - 4x + 2$$

$$= -5\left(x^2 + \frac{4}{5}x\right) + 2$$

$$= -5\left(x^2 + \frac{4}{5}x + \frac{4}{25}\right) + 2 + 5\left(\frac{4}{25}\right)$$

$$= -5\left(x + \frac{2}{5}\right)^2 + 2 + \frac{4}{5}$$

$$= -5\left(x + \frac{2}{5}\right)^2 + \frac{14}{5}$$ which is in the form $a(x+h)^2 + k$,

where $a = -5, h = \frac{2}{5}$ and $k = \frac{14}{5}$.

The maximum point is of the form $(-h, k)$.

So, the exact value of the maximum point is $\left(-\frac{2}{5}, \frac{14}{5}\right)$.

The range is $g(x) \leq \frac{14}{5}$.

In other words, the range is $\left(-\infty, \frac{14}{5}\right)$.

Total: 15 marks

2. (a) Write the expression $2\log_3 x + 2 - \log_3 y$ as a single term. [3]

$2\log_3 x + 2 - \log_3 y$

$= \log_3 x^2 + 2\log_3 3 - \log_3 y$ $\quad [\because \log_3 3 = 1]$

$= \log_3 x^2 + \log_3 3^2 - \log_3 y$ $\quad [\because n\log_a b = \log_a b^n]$

$= \log_3 x^2 + \log_3 9 - \log_3 y$

$= \log_3 9x^2 - \log_3 y$ $\quad [\because \log_x a + \log_x b = \log_x ab]$

$= \log_3 \frac{9x^2}{y}$ $\quad \left[\because \log_x a - \log_x b = \log_x\left(\frac{a}{b}\right)\right]$

(b) (i) By using logarithms, express the relationship $V = 7 \times 5^t$ in linear form. [2]

$V = 7 \times 5^t$

Taking logs on both sides:

$\log V = \log(7 \times 5^t)$

$\log V = \log 7 + \log 5^t$ $\quad [\because \log_x ab = \log_x a + \log_x b]$

$\log V = \log 7 + t\log 5$ $\quad [\because \log_x a^n = n\log_x a]$

$\log V = t\log 5 + \log 7$

which is in the form $Y = mX + c$,

where $Y = \log V$, $m = \log 5$, $X = t$ and $c = \log 7$.

(ii) Hence, state the value of the gradient of the line which represents the relationship in (b)(i). [1]

The gradient of the line is $m = \log 5$.

(c) Rationalize the denominator of the expression $\frac{1+\sqrt{2}}{3-\sqrt{2}}$. [4]

$$\frac{1+\sqrt{2}}{3-\sqrt{2}} = \frac{1+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{(1+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

$$= \frac{3+\sqrt{2}+3\sqrt{2}+2}{9-2}$$

$$= \frac{5+4\sqrt{2}}{7}$$

(d) Evaluate $\sum_{i=0}^{4} 5^{i-2}$. [3]

$$\sum_{i=0}^{4} 5^{i-2} = 5^{0-2} + 5^{1-2} + 5^{2-2} + 5^{3-2} + 5^{4-2}$$

$$= 5^{-2} + 5^{-1} + 5^{0} + 5^{1} + 5^{2}$$

$$= \frac{1}{25} + \frac{1}{5} + 1 + 5 + 25$$

$$= \frac{781}{25}$$

$$= 31.24$$

(e) Determine whether the following sequence is divergent or convergent.

Justify your response. [2]

$$1, \quad -\frac{2}{3}, \quad \frac{4}{9}, \quad -\frac{8}{27}, \ldots$$

Let $a = 1$ and $r = -\frac{2}{3}$.

So, we have,

$T_1 = 1$

$T_2 = 1\left(-\frac{2}{3}\right)^1 = -\frac{2}{3}$

$T_3 = 1\left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

$T_4 = 1\left(-\frac{2}{3}\right)^3 = -\frac{8}{27}$

The nth term of the series can be expressed as $T_n = ar^{n-1}$.

The series is a geometric progression where $a = 1$ and the common ratio is $r = -\frac{2}{3}$.

We can deduced that $|r| < 1$.

The sum to infinity is:

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1}{1-\left(-\frac{2}{3}\right)}$$

$$= \frac{1}{1+\frac{2}{3}}$$

$$= \frac{1}{\frac{5}{3}}$$

$$= \frac{3}{5}$$

So, the sequence converges to a limit of $\frac{3}{5}$.

$\therefore$ The sequence is convergent.

Total: 15 marks

SECTION II

COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

ALL working must be clearly shown.

3. (a) Determine the points of intersection of the circle $x^2 + y^2 - 4x + 6y + 8 = 0$ and the line $y = x - 6$. [5]

$x^2 + y^2 - 4x + 6y + 8 = 0 \quad \rightarrow$ Equation 1

$y = x - 6 \quad \rightarrow$ Equation 2

Substituting Equation 2 into Equation 1 gives:

$$x^2 + (x-6)^2 - 4x + 6(x-6) + 8 = 0$$

$$x^2 + x^2 - 12x + 36 - 4x + 6x - 36 + 8 = 0$$

$$2x^2 - 10x + 8 = 0$$

$$(\div 2)$$

$$x^2 - 5x + 4 = 0$$

$$(x-1)(x-4) = 0$$

Either $x - 1 = 0$ or $x - 4 = 0$

$x = 1 \qquad x = 4$

When $x = 1$,

$y = 1 - 6$

$= -5$

When $x = 4$,

$y = 4 - 6$

$= -2$

$\therefore$ The points of intersection are $(1, -5)$ and $(4, -2)$.

(b) (i) Given that the coordinates of the points A and B are $(7, -3)$ and $(2, 1)$ respectively, state the position vectors corresponding to the points A and B in the form $x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$. [1]

The point A is $(7, -3)$.

So, the position vector is

$\overrightarrow{OA} = 7\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}}$ which is in the form $x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$,

where $x = 7$ and $y = -3$.

The point B is $(2, 1)$.

So, the position vector is

$\overrightarrow{OB} = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$ which is in the form $x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$,

where $x = 2$ and $y = 1$.

$\therefore \overrightarrow{OA} = 7\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}}$

$\overrightarrow{OB} = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$

(ii) Determine $\overrightarrow{AB}$. [2]

Using the triangle law,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} - (7\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}})$$

$$= 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} - 7\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}}$$

$= -5\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}}$ which is in the form $x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$,

where $x = -5$ and $y = 4$.

(iii) Calculate the value of the scalar product $\boldsymbol{OA} \cdot \boldsymbol{OB}$. [2]

$\overrightarrow{OA} = 7\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}}$ and $\overrightarrow{OB} = 2\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$

$= \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

The scalar product is,

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

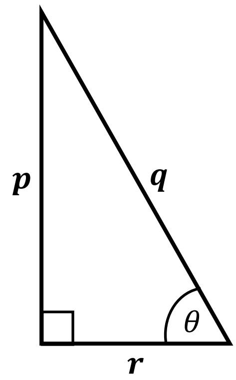
$$= (7)(2) + (-3)(1)$$

$$= 14 - 3$$

$$= 11$$

$\therefore$ The value of the scalar product $\overrightarrow{OA} \cdot \overrightarrow{OB} = 11$.

(c) (i) Using the letters p, q or r, write an expression for EACH of the following trigonometric ratios. [2]



$\sin\theta =$

$\sin\theta = \frac{opp}{hyp}$

$= \frac{p}{q}$

$\cos\theta =$

$\cos\theta = \frac{adj}{hyp}$

$= \frac{r}{q}$

(ii) Using your answers in (c)(i), determine the value of $\sin^2\theta + \cos^2\theta$.

Recall that $\sin^2\theta = (\sin\theta)^2$. [4]

We have: $\sin\theta = \frac{p}{q}$ and $\cos\theta = \frac{r}{q}$

Using Pythagoras' Theorem,

$p^2 + r^2 = q^2$ $\rightarrow$ Equation 1

Now,

$$\sin^2\theta + \cos^2\theta = (\sin\theta)^2 + (\cos\theta)^2$$

$$= \left(\frac{p}{q}\right)^2 + \left(\frac{r}{q}\right)^2$$

$$= \frac{p^2}{q^2} + \frac{r^2}{q^2}$$

$$= \frac{p^2+r^2}{q^2}$$

$$= \frac{q^2}{q^2} \qquad \text{[from Equation 1]}$$

$$= 1$$

$\therefore$ The value of $\sin^2\theta + \cos^2\theta = 1$.

(d) Prove the identity $\tan A + \frac{1}{\tan A} = \frac{1}{\sin A \cos A}$. [4]

Taking L.H.S:

$$\tan A + \frac{1}{\tan A} = \frac{\sin A}{\cos A} + \frac{1}{\left(\frac{\sin A}{\cos A}\right)}$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

$$= \text{R.H.S}$$

$$\therefore \tan A + \frac{1}{\tan A} = \frac{1}{\sin A \cos A}$$

Q.E.D.

Total: 20 marks

SECTION III

INTRODUCTORY CALCULUS

ALL working must be clearly shown.

4. (a) (i) Use the definition of the derivative as a limit to find $f'(x)$ for the function $f(x) = x^2 - 3$. [4]

$f(x) = x^2 - 3$

By the definition of the derivative as a limit,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3 - (x^2 - 3)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3 - x^2 + 3}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} (2x + h)$$

$$= 2x + 0$$

$$= 2x$$

$\therefore f'(x) = 2x$

(ii) Hence, determine the value of $f''(5)$. [2]

$f'(x) = 2x$

$f''(x) = 2$

So, for any value of x, $f''(x) = 2$.

$\therefore\ f''(5) = 2$

(b) Given that $y = \frac{\sin x}{\cos x}$, show that $\frac{dy}{dx} = \frac{1}{\cos^2 x}$. [4]

$y = \frac{\sin x}{\cos x}$ which is in the form $y = \frac{u}{v}$.

Let $u = \sin x$ and $v = \cos x$

$\frac{du}{dx} = \cos x$ $\frac{dv}{dx} = -\sin x$

Using the quotient rule,

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos^2 x}$$

Q.E.D.

(c) The equation of a curve is given by $y = (5x^2 - 7)^4$. Determine the equation of the gradient of the curve. [5]

$y = (5x^2 - 7)^4$

Let $t = 5x^2 - 7$, $y = t^4$

$\frac{dt}{dx} = 10x$ $\frac{dy}{dt} = 4t^3$

Using chain rule,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= 4t^3 \times 10x$

$= 4(5x^2 - 7)^3 \times 10x$

$= 40x(5x^2 - 7)^3$

Alternatively,

$y = (5x^2 - 7)^4$

$\frac{dy}{dx} = 4(10x)(5x^2 - 7)^{4-1}$

$= 40x(5x^2 - 7)^3$

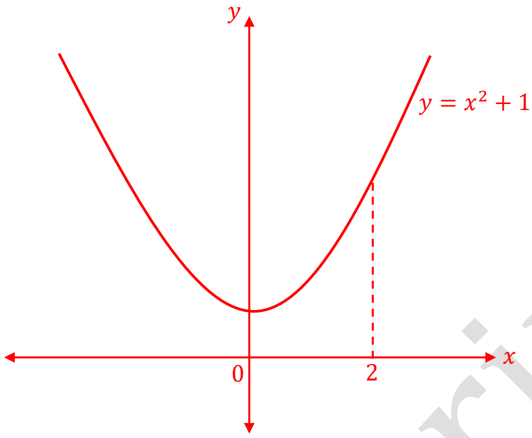
$\therefore$ The equation of the gradient of the curve is $\frac{dy}{dx} = 40x(5x^2 - 7)^3$.

Total: 15 marks

5. (a) The region enclosed between the curve $y = x^2 + 1$, the x-axis, the y-axis and the line $x = 2$ is rotated about the x-axis through an angle of 360°.

Calculate the volume of the solid of revolution that is formed. [5]

Consider the diagram below:



$y = x^2 + 1$

$y^2 = (x^2 + 1)^2$

$y^2 = x^4 + 2x^2 + 1$

Volume about x-axis $= \pi \int_a^b y^2 \, dx$

$$= \pi \int_0^2 (x^4 + 2x^2 + 1)^2 \, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_0^2$$

$$= \pi \left[\left(\frac{(2)^5}{5} + \frac{2(2)^3}{3} + (2)\right) - \left(\frac{(0)^5}{5} + \frac{2(0)^3}{3} + (0)\right)\right]$$

$$= \pi \left[\left(\frac{32}{5} + \frac{16}{3} + 2\right) - 0\right]$$

$$= \frac{206\pi}{15} \text{ units}^3$$

$\therefore$ The volume of the solid of revolution that is formed is $\frac{206\pi}{15}$ units^3.

(b) As a particle moves along a straight line, its displacement is measured from a fixed point, O, on the line. At time t seconds, the acceleration, a, is given by $a = 24t - 14$.

(i) Find the expression for the velocity, v, of the particle, given that when $t = 1, v = 3\ ms^{-1}$. [3]

$a = 24t - 14$

$v = \int a\, dt$

$= \int (24t - 14)\, dt$

$= \frac{24t^2}{2} - 14t + c$

$= 12t^2 - 14t + c$

When $t = 1$ and $v = 3\ ms^{-1}$,

$3 = 12(1)^2 - 14(1) + c$

$3 = 12 - 14 + c$

$3 = -2 + c$

$3 + 2 = c$

$5 = c$

$\therefore$ The expression for the velocity, $v = 12t^2 - 14t + 5$.

(ii) Using your answer in (b)(i), find an expression for the displacement, s, of the particle, given that when $t = 1, s = 10\ m$. [3]

$v = 12t^2 - 14t + 5$

$s = \int v\, dt$

$= \int (12t^2 - 14t + 5)\, dt$

$= \frac{12t^3}{3} - \frac{14t^2}{2} + 5t + c$

$= 4t^3 - 7t^2 + 5t + c$

When $t = 1$ and $s = 10\ m$,

$10 = 4(1)^3 - 7(1)^2 + 5(1) + c$

$10 = 4 - 7 + 5 + c$

$10 = 2 + c$

$10 - 2 = c$

$8 = c$

$\therefore$ The expression for the displacement, $s = 4t^3 - 7t^2 + 5t + 8$.

(c) The gradient function of the curve C is given by $7 - 2x$. If the curve passes through the point $(3, 8)$, find the equation of the curve. [4]

$\frac{dy}{dx} = 7 - 2x$

$y = \int \frac{dy}{dx} dx$

$= \int (7 - 2x)\, dx$

$= 7x - \frac{2x^2}{2} + c$

$= 7x - x^2 + c$

Substituting point $(3, 8)$ into $y = 7x - x^2 + c$ gives:

$(8) = 7(3) - (3)^2 + c$

$8 = 21 - 9 + c$

$8 = 12 + c$

$8 - 12 = c$

$-4 = c$

$\therefore$ The equation of the curve is: $y = 7x - x^2 - 4$

Total: 15 marks

SECTION IV

PROBABILITY AND STATISTICS

ALL working must be clearly shown.

6. (a) The heights of 35 children at a nursery were recorded to the nearest centimetre. The data is shown below.

61	118	79	90	83	70	80
95	75	76	92	62	115	79
71	103	109	84	65	86	92
111	78	94	74	99	81	108
108	67	109	92	79	116	62

(i) Display the data shown above on a stem-and-leaf diagram. [3]

The stem-and-leaf diagram is shown below:

6	1	2	2	5	7				
7	0	1	4	5	6	8	9	9	9
8	0	1	3	4	6				
9	0	2	2	2	4	5	9		
10	3	8	8	9	9				
11	1	5	6	8					

Key: 6|1 means 61

(ii) From the stem-and-leaf diagram in (a)(i), determine the value of the

- median

The median occurs at $= \frac{n+1}{2}$

$= \frac{35+1}{2}$

$= \frac{36}{2}$

$= 18^{\text{th}}$ value

From the stem-and-leaf diagram, the 18^{th} value is 84.

$\therefore$ Median $= 84$

- lower quartile

The value of Q_1 occurs at $= \frac{n+1}{4}$

$= \frac{35+1}{4}$

$= \frac{36}{4}$

$= 9^{\text{th}}$ value

From the stem-and-leaf diagram, the 9^{th} value is 75.

$\therefore$ The value of the lower quartile is $Q_1 = 75$.

- upper quartile [3]

The value of Q_3 occurs at $= \frac{3(n+1)}{4}$

$= \frac{3(35+1)}{4}$

$= \frac{3(36)}{4}$

$= \frac{108}{4}$

$= 27^{\text{th}}$ value

From the stem-and-leaf diagram, the 27^{th} value is 103.

$\therefore$ The value of the upper quartile is $Q_3 = 103$.

(iii) At another nearby nursery, the heights of children of the same age were recorded. The median height was 79 cm and the interquartile range was 24 cm.

Compare the characteristics of the two groups of children at the two nurseries and describe ONE distinct observation about the distributions. [2]

For the first nursery,

$IQR = Q_3 - Q_1$

$= 103 - 75$

$= 28\ cm$

The first nursery has a larger IQR than the second nursery which indicates that the data in the first nursery has a greater spread than the data in the second nursery.

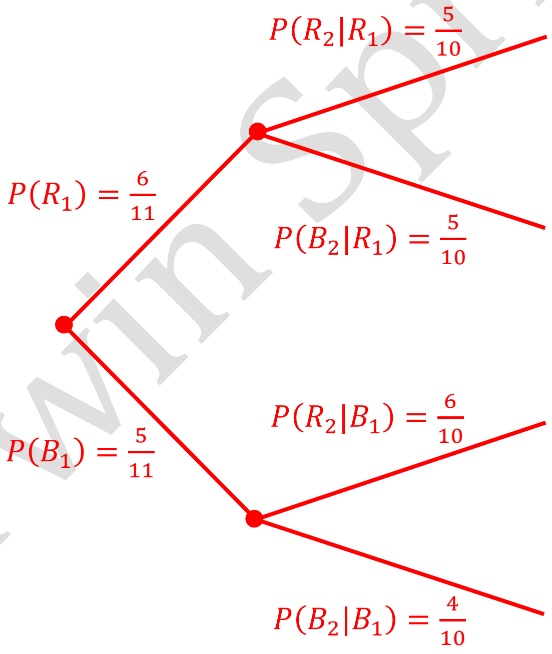
Comparing the medians, the median of the data for the first nursery is greater than the median for the second nursery. It can be deduced that the children in the first nursery are taller when compared to the children in the second nursery.

(b) (i) A bag contains 6 red marbles and 5 black marbles. During 2 rounds of a marble game, a student is required to randomly draw 1 marble without replacement for each round. Construct a probability tree diagram to represent this information. [4]

Let R represent the event of choosing a red marble.

Let B represent the event of choosing a black marble.

The probability tree diagram is shown below:



(ii) Using your answer in (b)(i), find the probability that the marbles drawn are ALL the SAME colour. [4]

$$P(same\ colour) = P(R_1R_2) + P(B_1B_2)$$

$$= P(R_1)P(R_2|R_1) + P(B_1)P(B_2|B_1)$$

$$= \left(\frac{6}{11}\right)\left(\frac{5}{10}\right) + \left(\frac{5}{11}\right)\left(\frac{4}{10}\right)$$

$$= \frac{30}{110} + \frac{20}{110}$$

$$= \frac{50}{110}$$

$$= \frac{5}{11}$$

$\therefore$ The probability that the marbles drawn are all the same colour is $\frac{5}{11}$.

(c) Given two events, A and B, $P(A \cup B) = 0.7, P(A) = 0.3$ and $P(B') = 0.6$.

(i) Calculate $P(A \cap B)$. [3]

$$P(B) = 1 - P(B')$$

$$= 1 - 0.6$$

$$= 0.4$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.3 + 0.4 - P(A \cap B)$$

$$0.7 = 0.7 - P(A \cap B)$$

$$P(A \cap B) = 0.7 - 0.7$$

$$P(A \cap B) = 0$$

(ii) What is the relationship between Events A and B? [1]

Since $P(A \cap B) = 0$, then events A and B are mutually exclusive events.

Total: 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.