CSEC Mathematics

January 2010 – Paper 2

Solutions

Kerwin Springer

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the exact value of

$$\frac{2.76}{0.8} + 8.7^2$$ [3]

Using a calculator,

$\frac{2.76}{0.8} + 8.7^2 = 3.45 + 75.69$

$= 79.14$

(b) In a certain company, a salesman is paid a fixed salary of $3 140 per month plus an annual commission of 2% on the TOTAL value of cars sold for the year. If the salesman sold cars valued at $720 000 in 2009, calculate

(i) his fixed salary for the year [1]

Fixed salary for the year = Monthly salary × 12

= \$3140 × 12

= \$37 680

(ii) the amount he received in commission for the year [1]

Amount in commission = 2% of \$720 000

$= \frac{2}{100} \times \$720\,000$

= \$14 400

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(iii) his TOTAL income for the year [1]

Total income = Fixed salary + Amount earned in commission

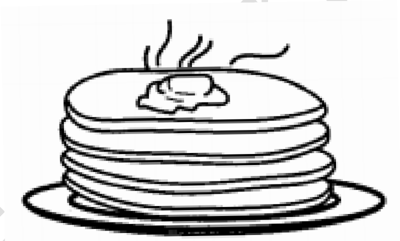
= \$37 680 + \$14 400

= \$52 080

∴ His total income for the year is \$52 080.

(c) The ingredients for making pancakes are shown in the diagram below.

Ingredients for making 8 pancakes



2 cups pancake mix

$1\frac{1}{3}$ cups milk

(i) Ryan wishes to make 12 pancakes using the instructions given above.

Calculate the number of cups of pancake mix he must use. [2]

We are given that 8 pancakes are produced from using 2 cups of mix.

So, we have,

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$8 \text{ pancakes} = 2 \text{ cups of mix}$

$1 \text{ pancake} = \frac{2}{8} \text{ cups of mix}$

$12 \text{ pancakes} = 12 \times \frac{2}{8}$

$= 3 \text{ cups of mix}$

$\therefore$ He must use 3 cups of pancake mix.

(ii) Neisha used 5 cups of milk to make pancakes using the same instructions. How many pancakes did she make? [3]

We are given that $1\frac{1}{3}$ cups of milk produce 8 pancakes.

So, we have,

$1\frac{1}{3} \text{ cups of milk} = 8 \text{ pancakes}$

$1 \text{ cup of milk} = \frac{8}{1\frac{1}{3}}$

$= 6 \text{ pancakes}$

$5 \text{ cups of milk} = 5 \times 6$

$= 30 \text{ pancakes}$

$\therefore$ Neisha made 30 pancakes.

Total: 11 marks

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2. (a) Given that $a = 6, b = -4$ and $c = 8$, calculate the value of [3]

$$\frac{a^2+b}{c-b}$$

We are given that $a = 6, b = -4$ and $c = 8$.

$$\frac{a^2+b}{c-b} = \frac{(6)^2+(-4)}{(8)-(-4)}$$

$$= \frac{36-4}{8+4}$$

$$= \frac{32}{12}$$

$$= \frac{8}{3}$$

(b) Simplify the expression:

(i) $3(x-y) + 4(x+2y)$ [2]

$$3(x-y) + 4(x+2y)$$

$$= 3x - 3y + 4x + 8y$$

$$= 7x + 5y$$

(ii) $\frac{4x^2 \times 3x^4}{6x^3}$ [3]

$$\frac{4x^2 \times 3x^4}{6x^3}$$

$$= \frac{12x^6}{6x^3}$$

$$= 2x^3$$

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(c) (i) Solve the inequality

$$x - 3 < 3x - 7$$ [3]

$x - 3 < 3x - 7$

$7 - 3 < 3x - x$

$4 < 2x$

$2x > 4$

$x > \frac{4}{2}$

$x > 2$

(ii) If x is an integer, determine the SMALLEST value of x that satisfies the inequality in (c)(i) above. [1]

$x > 2$ and $x \in \mathbb{Z}$.

$\mathbb{Z} = \{\ldots - 3, -2, -2, 0, 1, 2, 3, \ldots\}$

$\therefore$ The smallest integer that satisfies the inequality is $x = 3$.

Total: 12 marks

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3. (a) T and E are subsets of a universal set, U, such that:

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$T = \{\text{multiples of } 3\}$

$E = \{\text{even numbers}\}$

(i) Draw a Venn diagram to represent this information. [4]

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

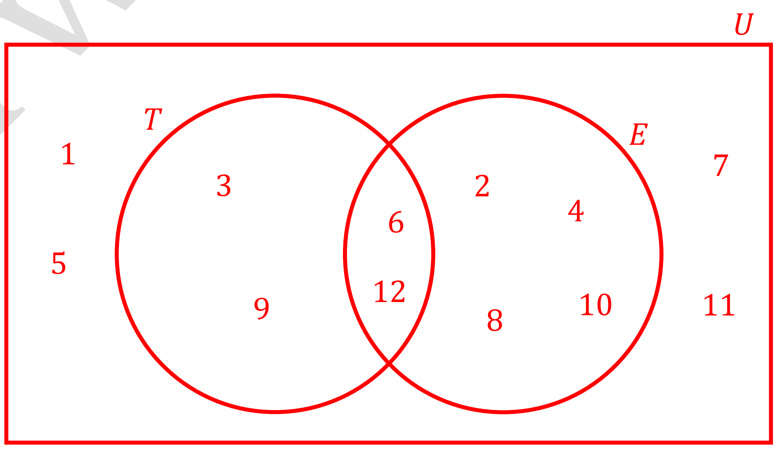
$T = \{multiples\ of\ 3\}$

$= \{3, 6, 9, 12\}$

$E = \{even\ numbers\}$

$= \{2, 4, 6, 8, 10, 12\}$

The Venn diagram is as follows:



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(ii) List the members of the set

(a) $T \cap E$ [1]

$T \cap E = \{6, 12\}$

(b) $(T \cup E)'$ [1]

$(T \cup E)' = \{1, 5, 7, 11\}$

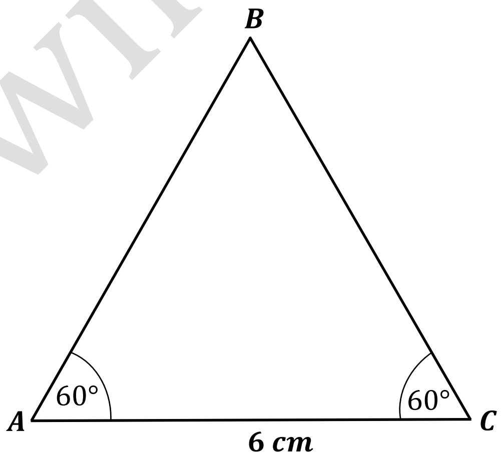
(b) **Using a pencil, a ruler, and a pair of compasses only:**

(i) Construct, accurately, the triangle ABC shown below, where,

$AC = 6\ cm$

$\angle ACB = 60°$

$\angle CAB = 60°$ [3]



Required to construct accurately, the triangle ABC, where

$AC = 6\ cm, \angle ACB = 60°$ and $\angle CAB = 60°$.

The diagram is shown as follows:

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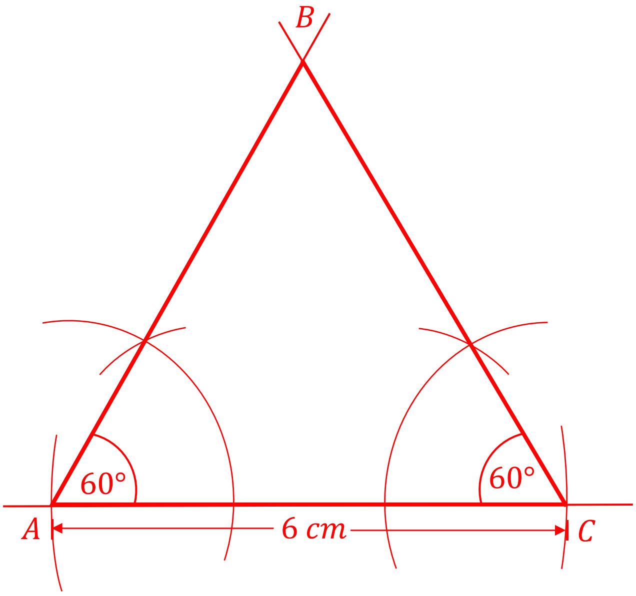
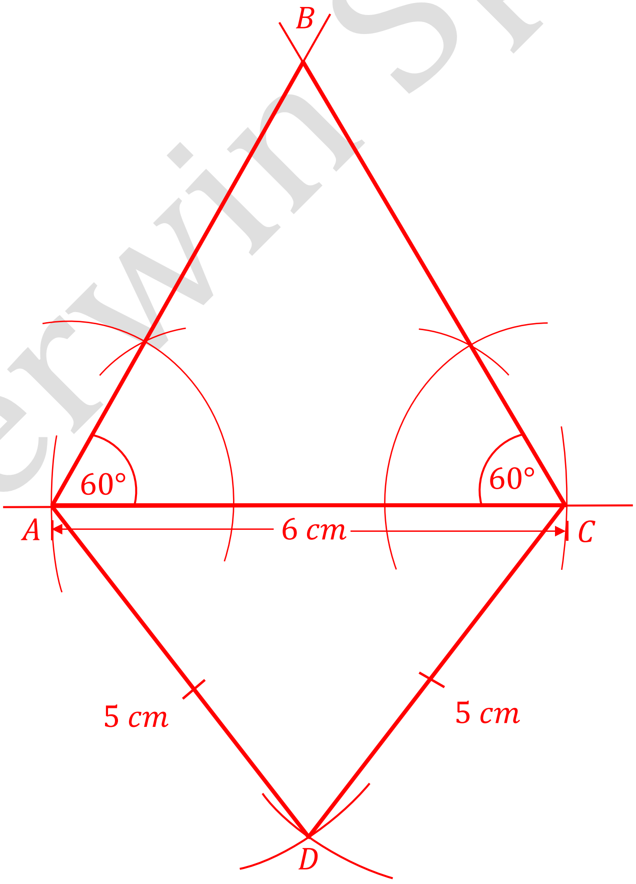


(ii) Complete the diagram to show the kite, $ABCD$, in which

$AD = 5\ cm$. [2]



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(iii) Measure and state the size of $\angle DAC$. [1]

By measurement, $D\hat{A}C = 53°$.

Total: 12 marks

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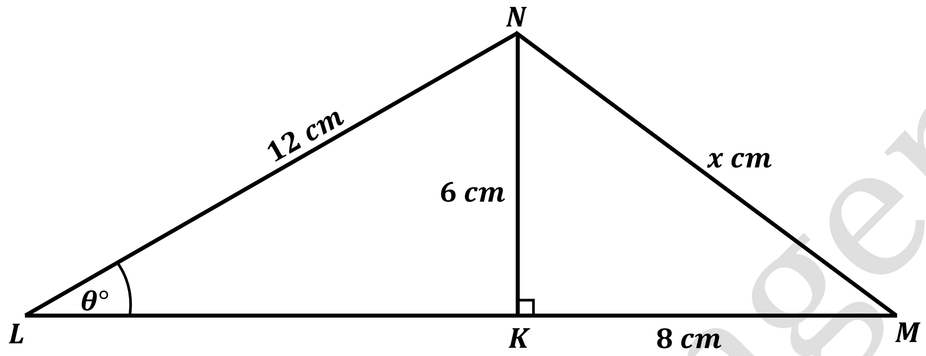
4. (a) The diagram below, **not drawn to scale**, shows a triangle LMN with $LN = 12$ cm, $NM = x\ cm$, and $\angle NLM = \theta°$. The point K on LM is such that NK is perpendicular to LM, $NK = 6\ cm$, and $KM = 8\ cm$.



Calculate the value of

(i) x [2]

Consider triangle NKM.

By Pythagoras' Theorem,

$c^2 = a^2 + b^2$

$x^2 = (6)^2 + (8)^2$

$x^2 = 36 + 64$

$x^2 = 100$

$x = \sqrt{100}$

$x = 10\ cm$

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(ii) θ [3]

Consider triangle NLK.

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{6}{12}$

$\sin\theta = \frac{1}{2}$

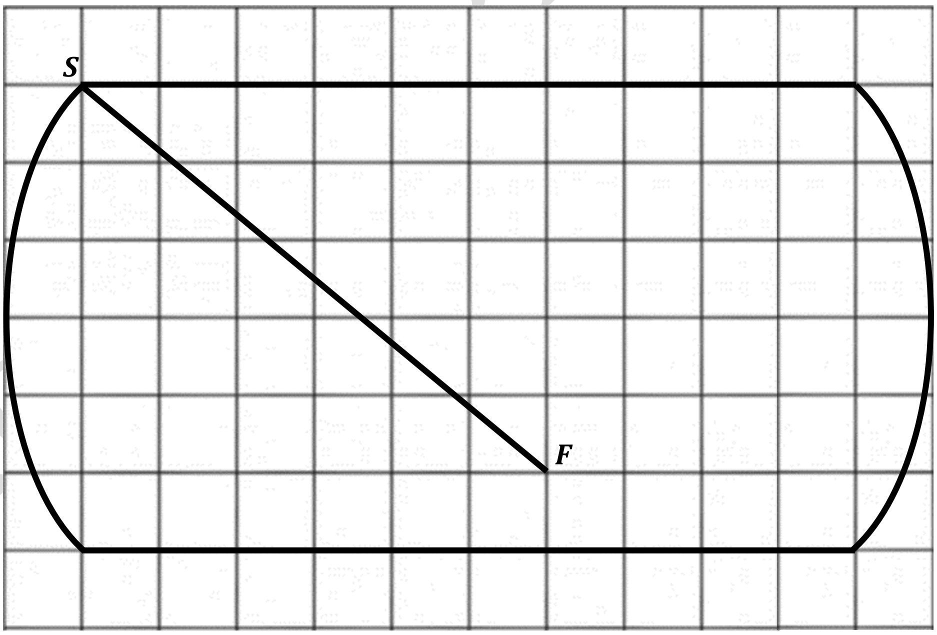
$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = 30°$

(b) The diagram below shows a map of a playing field drawn on a grid of

1 cm squares.

The scale of the map is 1 : 1 250.



(i) Measure and state, in centimetres, the distance from S to F on the map. [1]

By measurement, $SF = 7.8\ cm$.

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(ii) Calculate the distance, in metres, from S to F on the ACTUAL playing field. [2]

The scale is 1 : 1 250.

1 cm on the map represents 1250 cm on the field.

7.8 cm on the map $= 7.8 \times 1250$

$= 9750\ cm$ on the field.

Now,

$100\ cm = 1\ m$

$1\ cm = \frac{1}{100}$

$9750\ cm = \frac{1}{100} \times 9750$

$= 97.5\ m$

$\therefore$ The distance from S to F on the actual playing field is 97.5 m.

(iii) Daniel ran the distance from S to F in 9.72 seconds. Calculate his average speed in

(a) m/s

$$\text{Average speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

$$= \frac{97.5}{9.72}$$

$$= 10.0\ m/s \quad \text{(to 3 significant figures)}$$

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(b) km/h

$\text{Distance} = 97.5\ m$

$= 97.5 \div 1000\ km$

$= 0.0975\ km$

Now,

$1 \text{ hour} = 60 \times 60$

$1 \text{ hour} = 3600 \text{ seconds}$

$3600 \text{ seconds} = 1 \text{ hour}$

$1 \text{ second} = \frac{1}{3600}$

$9.72 \text{ seconds} = 0.0027 \text{ hours}$

$\text{Time} = 9.72 \text{ seconds}$

$= 0.0027 \text{ hours}$

$\text{Average speed} = \frac{\textit{Distance covered}}{\textit{Time taken}}$

$= \frac{0.0975}{0.0027}$

$= 36.1\ km/h$ (to 3 significant figures)

giving your answer correct to 3 significant figures. [3]

Total: 11 marks

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5. (a) A straight line passes through the point $T(4, 1)$ and has a gradient of $\frac{3}{5}$.

Determine the equation of this line. [3]

Substituting $m = \frac{3}{5}$ and point $T(4, 1)$ into $y - y_1 = m(x - x_1)$ gives:

$$y - 1 = \frac{3}{5}(x - 4)$$

$$y - 1 = \frac{3}{5}x - \frac{12}{5}$$

$$y = \frac{3}{5}x - \frac{12}{5} + 1$$

$$y = \frac{3}{5}x - \frac{7}{5}$$

$$5y = 3x - 7$$

∴ The equation of the line is: $5y = 3x - 7$

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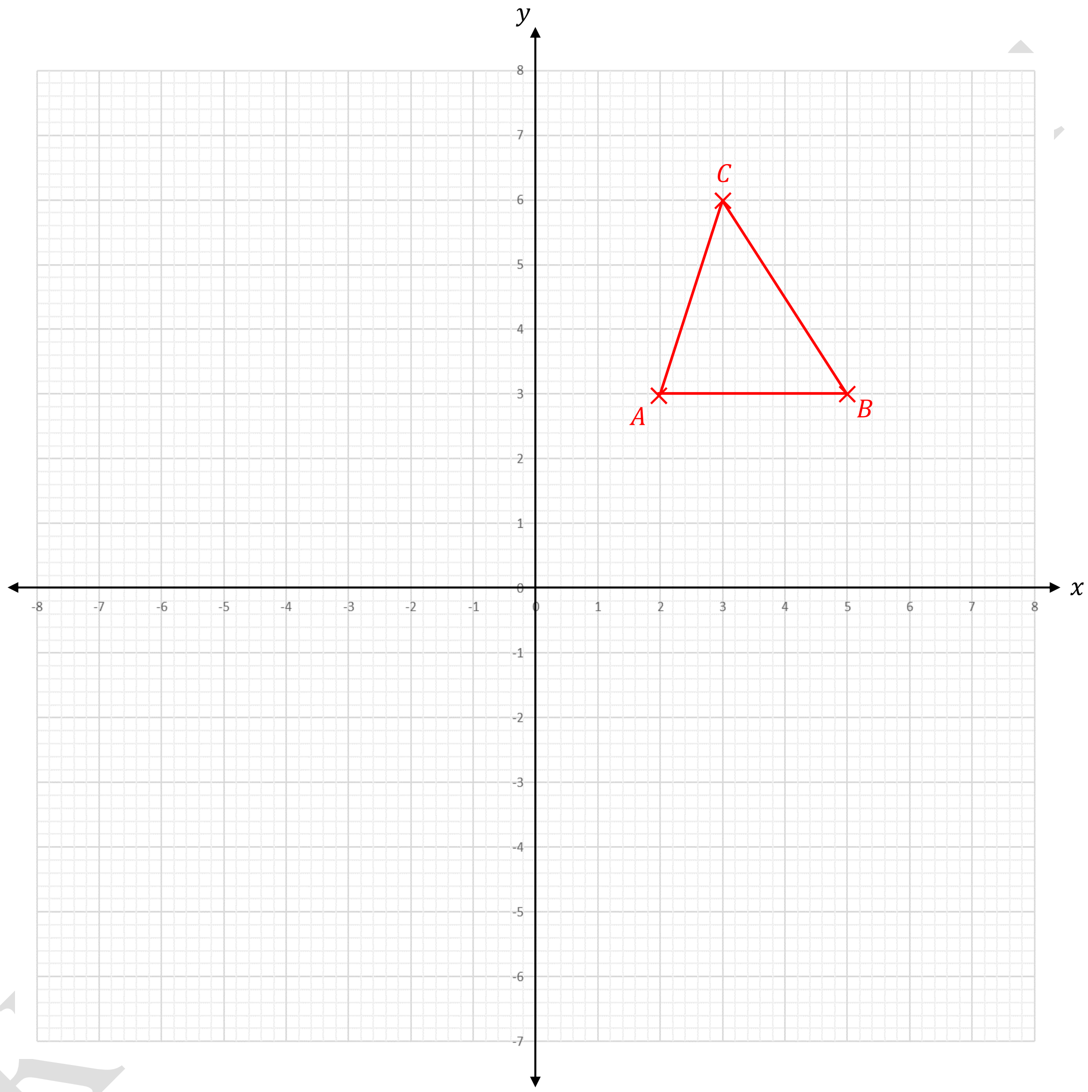
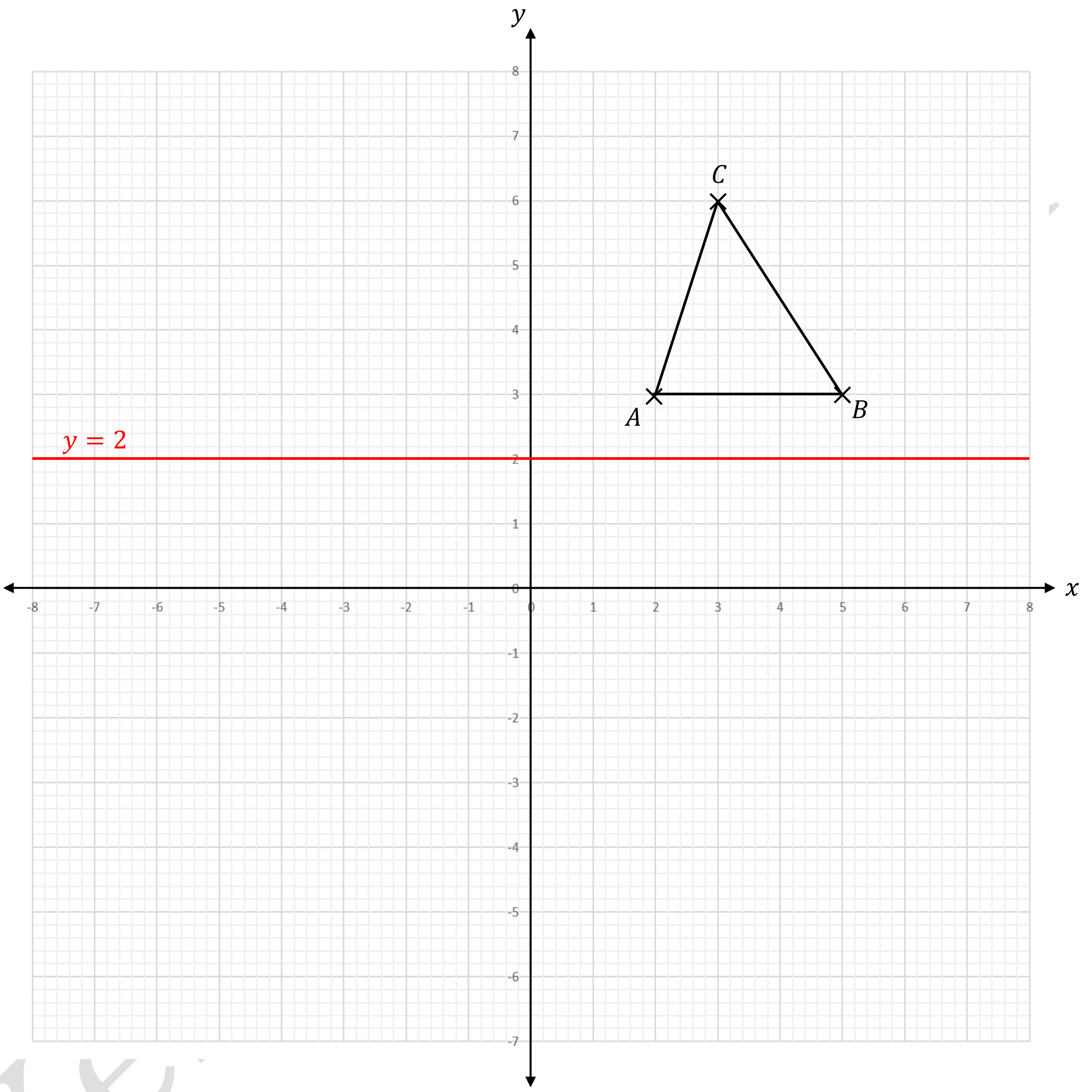
(b) (i) **Using a scale of 1 cm to represent 1 unit on both axes**, draw the triangle ABC with vertices $A(2,3)$, $B(5,3)$ and $C(3,6)$. [3]

The graph is shown below.



(ii) On the same axes used in (b)(i), draw and label the line $y = 2$. [1]

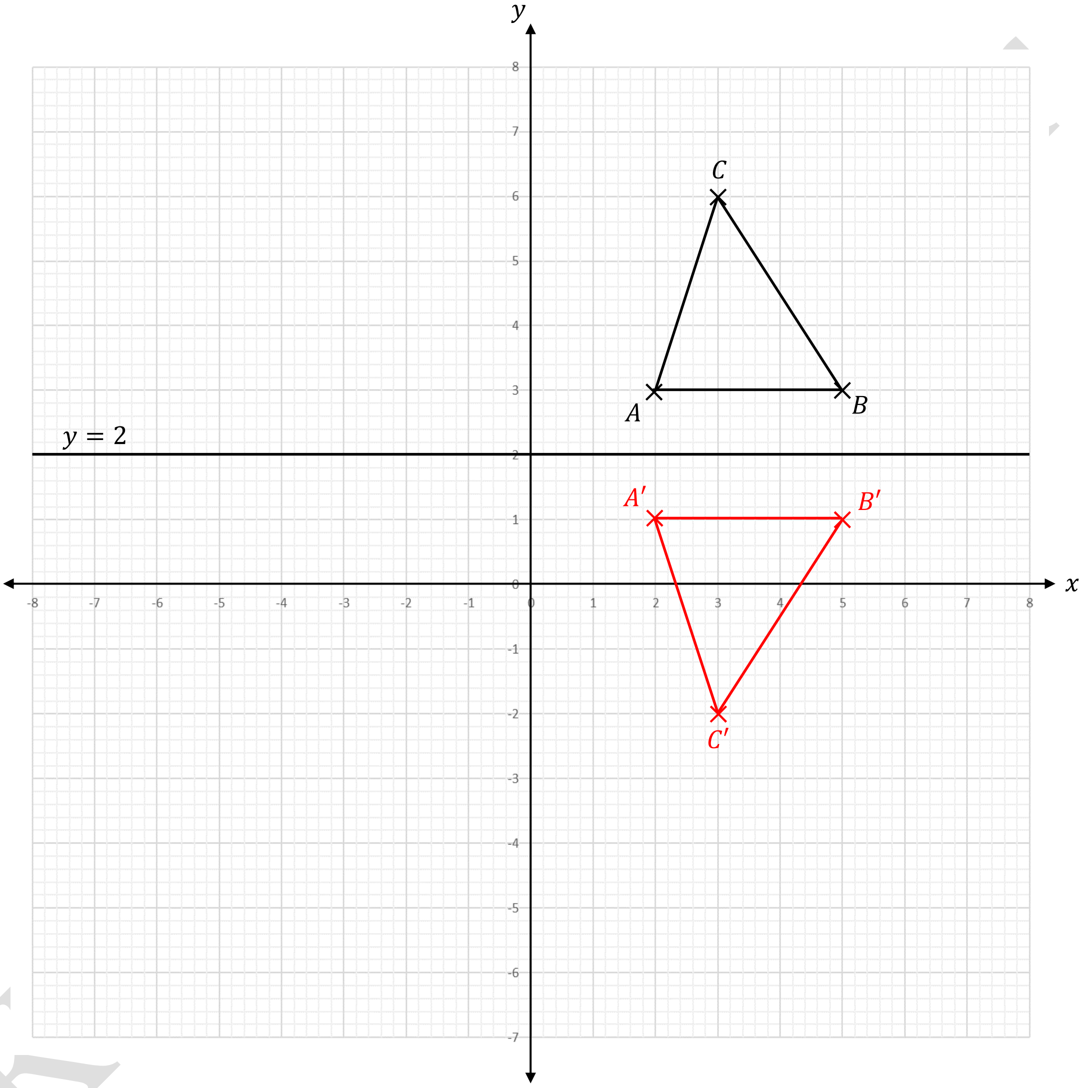
The graph is shown below.



(iii) Draw the image of triangle ABC under a reflection in the line $y = 2$.

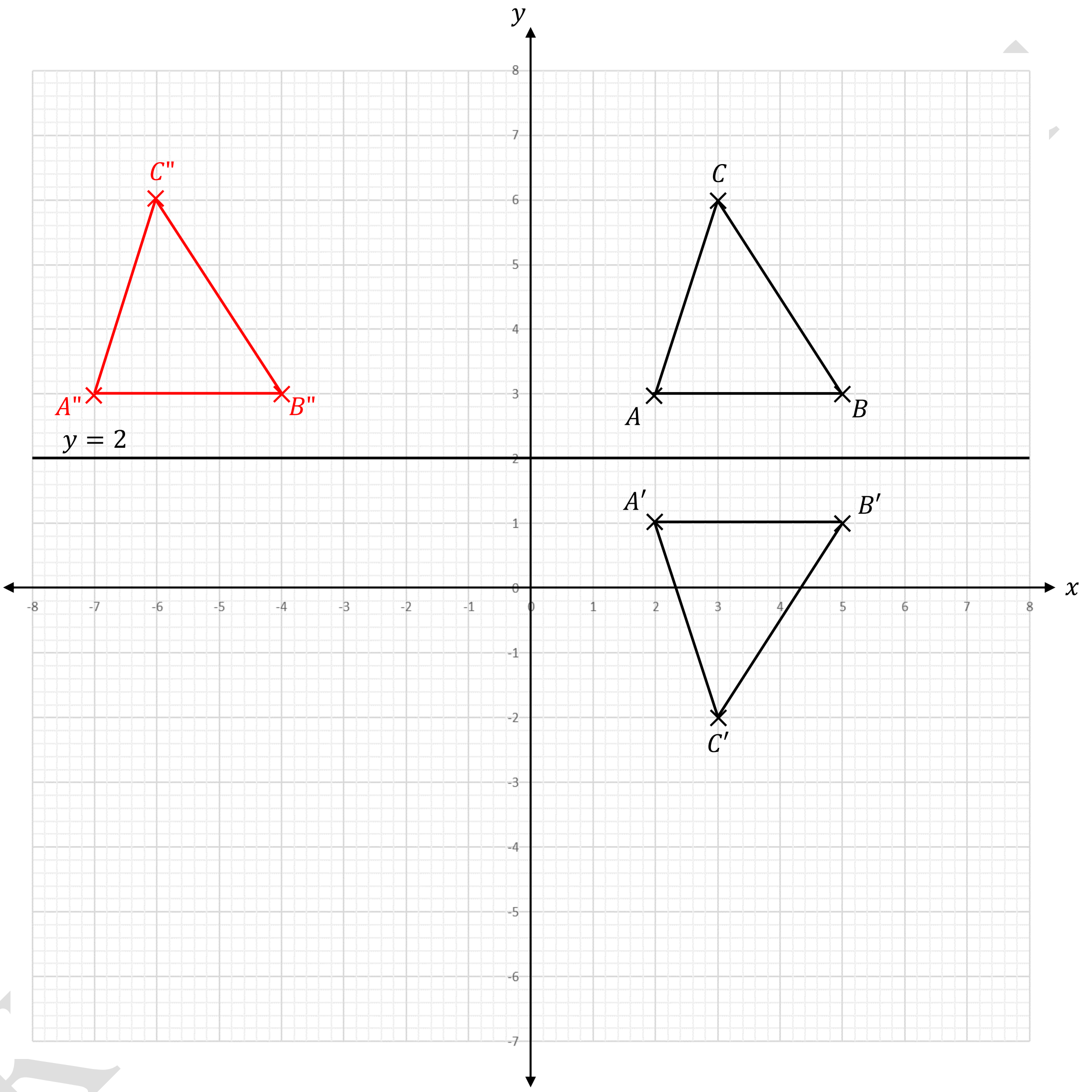
Label the image $A'B'C'$. [2]

The graph is shown below.



(iv) Draw a new triangle $A"B"C"$ with vertices $A"(-7, 4)$, $B"(-4, 4)$ and $C"(-6, 7)$. [1]

The graph is shown below.



(v) Name and describe the single transformation that maps triangle ABC onto triangle $A"B"C"$. [2]

The transformation is a translation.

Consider points $A(2, 3)$ and $A"(-7, 4)$.

The point A is mapped onto $A"$ by a horizontal shift of 9 units to the left and a vertical shift of 1 unit upwards.

The translation is represented by $T = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$.

$\therefore$ ΔABC is mapped into $\Delta A"B"C"$ by a translation, $T = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$.

Total: 12 marks

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6. A class of 26 students each recorded the distance travelled to school. The distance, to the nearest km, is recorded below:

21	11	3	22	6	32	22	18	28
26	16	17	34	12	25	8	19	14
39	17	22	24	30	18	13	23	

(a) Copy and complete the frequency table to represent this data. [2]

Distance in kilometres	**Frequency**
1 – 5	1
6 – 10	2
11 – 15	4
16 – 20	6
21 – 25	7
26 – 30	3
31 – 35	2
36 – 40	1

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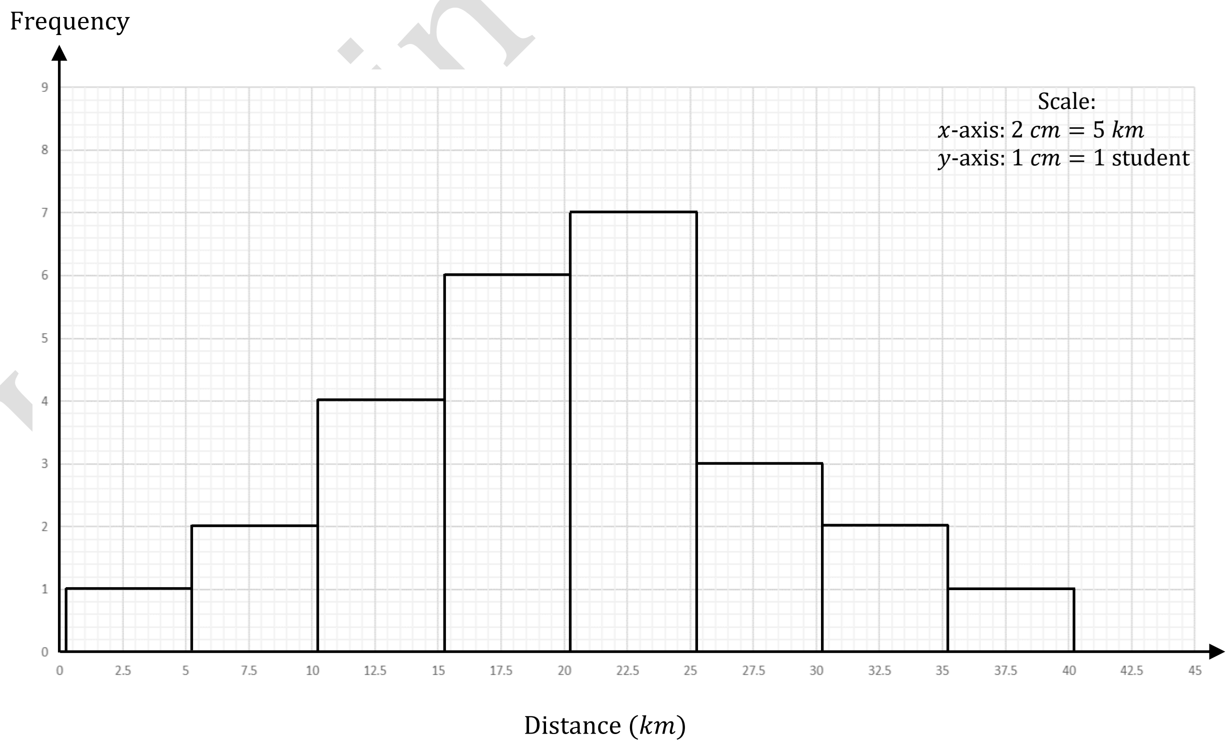
(b) **Using a scale of 2 cm to represent 5 km** on the horizontal axis and **a scale of 1 cm to represent 1 student** on the vertical axis, draw a histogram to represent the data. [5]

Distance in kilometres	LCB	UCB	Frequency
1 – 5	0.5	5.5	1
6 – 10	5.5	10.5	2
11 – 15	10.5	15.5	4
16 – 20	15.5	20.5	6
21 – 25	20.5	25.5	7
26 – 30	25.5	30.5	3
31 – 35	30.5	35.5	2
36 – 40	35.5	40.5	1

Title: Graph showing a histogram representing the data given.



(c) Calculate the probability that a student chosen at random from this class recorded the distance travelled to school as 26 km or **more**. [2]

$$P(student\ travelled \geq 26\ km) = \frac{\text{Number of students travelling} \geq 26 \text{ km}}{\text{Total number of students}}$$

$$P(student\ travelled \geq 26\ km) = \frac{3+2+1}{26}$$

$$P(student\ travelled \geq 26\ km) = \frac{6}{26}$$

$$P(student\ travelled \geq 26\ km) = \frac{3}{13}$$

(d) The P.T.A. plans to set up a transportation service for the school. Which average, mean, more or median, is MOST appropriate for estimating the cost of the service? Give a reason for your answer. [2]

The mean is most appropriate for estimating the cost of transportation because it takes into account each student and the actual distance that is covered.

Total: 11 marks

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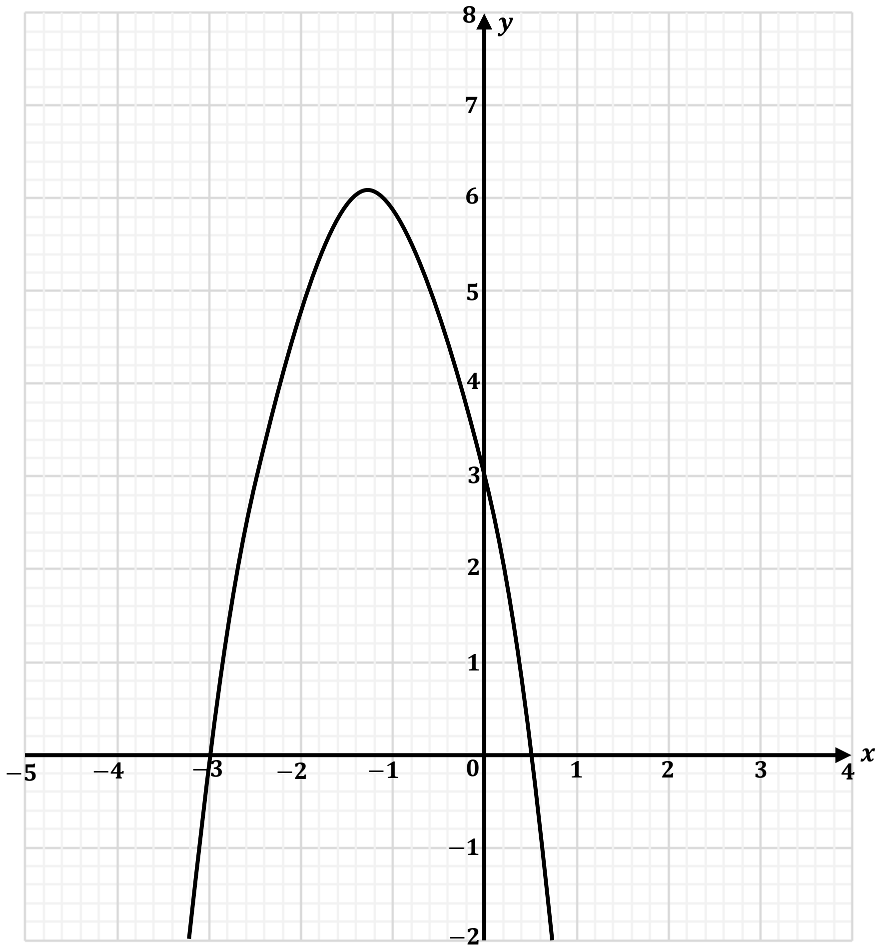
7. The graph shown below represents a function of the form: $f(x) = ax^2 + bx + c$



Using the graph above, determine

(i) the value of $f(x)$ when $x = 0$ [1]

From graph, when $x = 0, f(x) = 3$.

(ii) the values of x when $f(x) = 0$ [2]

From graph, when $f(x) = 0, x = -3$ and $x = \frac{1}{2}$.

These are the points where the curve cuts the x-axis.

(iii) the coordinates of the maximum point [2]

The coordinates of the maximum point are $(-1.3, 6.1)$.

(iv) the equation of the axis of symmetry [2]

The equation of the axis of symmetry is: $x = -1.3$.

(v) the values of x when $f(x) = 5$ [2]

From graph, when $f(x) = 5, x = -2$ and $x = -\frac{1}{2}$.

(vi) the interval within which x lies when $f(x) > 5$. Write your answer in the form $a < x < b$. [2]

The range of values of x for which $f(x)$ lies above the line $y = 5$ is $\{x: -2 < x < -0.5\}$.

Total: 11 marks

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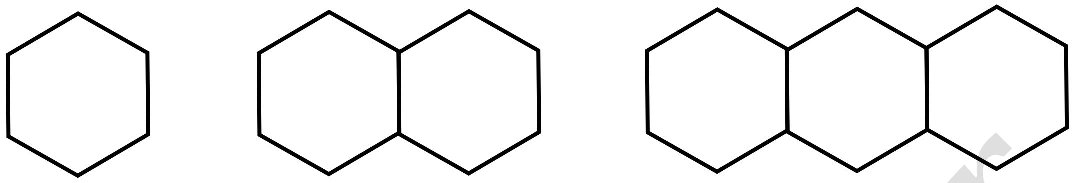
8. Bianca makes hexagons using sticks of equal length. She then creates patterns by joining the hexagons togather. Patterns 1, 2 and 3 are shown below.

Pattern 1 | **Pattern 2** | **Pattern 3**



1 Hexagon | **2 Hexagons** | **3 Hexagons**

The table below shows the number of hexagons in EACH pattern created and the number of sticks used to make EACH pattern.

Number of hexagons in the pattern	1	2	3	4	5	20	$\boldsymbol{n}$
Number of sticks used for the pattern	6	11	16	$\boldsymbol{x}$	$\boldsymbol{y}$	$\boldsymbol{z}$	$\boldsymbol{S}$

(a) Determine the values of

(i) x [2]

For n hexagons in the pattern, the number of sticks used is $S = 5n + 1$.

When $n = 4$ and $S = x$,

$x = 5(4) + 1$

$= 20 + 1$

$= 21$

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(ii) y [2]

When $n = 5$ and $S = y$,

$y = 5(5) + 1$

$= 25 + 1$

$= 26$

(iii) z [2]

When $n = 20$ and $S = z$,

$z = 5(20) + 1$

$= 100 + 1$

$= 101$

(b) Write down an expression for $\boldsymbol{S}$ in terms of $\boldsymbol{n}$, where $\boldsymbol{S}$ represents the number of sticks used to make a pattern of $\boldsymbol{n}$ hexagons. [2]

S represents the number of sticks used to make a pattern of n hexagons.

$\therefore S = 5n + 1$

(c) Bianca used a total of 76 sticks to make a pattern of $\boldsymbol{h}$ hexagons. Determine the value of $\boldsymbol{h}$. [2]

Bianca used a total of 76 sticks to make a pattern of h hexagons.

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When $n = h$ and $S = 76$,

$$76 = 5(h) + 1$$

$$76 - 1 = 5h$$

$$75 = 5h$$

$$\frac{75}{5} = h$$

$$15 = h$$

Total: 10 marks

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SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The relationship between kinetic energy, E, mass, m, and velocity, v, for a moving particle is

$$E = \frac{1}{2}mv^2$$

(i) Express v in terms of E and m. [3]

$$E = \frac{1}{2}mv^2$$

$$2E = mv^2$$

$$\frac{2E}{m} = v^2$$

$$\sqrt{\frac{2E}{m}} = v$$

$$\therefore v = \sqrt{\frac{2E}{m}}$$

(ii) Determine the value of v when $E = 45$ and $m = 13$. [2]

Substituting $E = 45$ and $m = 13$ into $v = \sqrt{\frac{2E}{m}}$ gives:

$$v = \sqrt{\frac{2(45)}{13}}$$

$$= \sqrt{\frac{90}{13}}$$

$$= 2.63 \quad \text{(to 2 decimal places)}$$

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(b) Given $g(x) = 3x^2 - 8x + 2$,

(i) write $g(x)$ in the form $a(x+b)^2 + c$, where a, b and $c \in \mathbb{R}$ [3]

$g(x) = 3x^2 - 8x + 2$ is in the form $ax^2 + bx + c$,

where $a = 3, b = -8$ and $c = 2$.

$$h = \frac{b}{2a} = \frac{-8}{2(3)} = \frac{-8}{6} = -\frac{4}{3}$$

$$k = \frac{4ac - b^2}{4a} = \frac{4(3)(2) - (-8)^2}{4(3)} = \frac{24 - 64}{12} = \frac{-40}{12} = -\frac{10}{3}$$

So, we have, $g(x) = 3\left(x - \frac{4}{3}\right)^2 - \frac{10}{3}$.

Alternatively,

$$g(x) = 3x^2 - 8x + 2$$
$$= 3\left(x^2 - \frac{8}{3}x\right) + 2$$
$$= 3\left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) + 2 - 3\left(\frac{16}{9}\right)$$
$$= 3\left(x - \frac{4}{3}\right)^2 + 2 - \frac{16}{3}$$
$$= 3\left(x - \frac{4}{3}\right)^2 - \frac{10}{3}$$

$\therefore g(x) = 3\left(x - \frac{4}{3}\right)^2 - \frac{10}{3}$ which is in the form $a(x+b)^2 + c$,

where $a = 3, b = -\frac{4}{3}$ and $c = -\frac{10}{3}$.

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(ii) solve the equation $g(x) = 0$, writing your answer(s) correct to 2 decimal places. [4]

$$g(x) = 3\left(x - \frac{4}{3}\right)^2 - \frac{10}{3}$$

Let $g(x) = 0$.

$$3\left(x - \frac{4}{3}\right)^2 - \frac{10}{3} = 0$$

$$3\left(x - \frac{4}{3}\right)^2 = \frac{10}{3}$$

$$\left(x - \frac{4}{3}\right)^2 = \frac{10}{9}$$

$$x - \frac{4}{3} = \pm\sqrt{\frac{10}{9}}$$

$$x = \frac{4}{3} \pm \sqrt{\frac{10}{9}}$$

Either $x = \frac{4}{3} - \sqrt{\frac{10}{9}}$ or $x = \frac{4}{3} + \sqrt{\frac{10}{9}}$

$x = 0.28$ (to 2 d.p.) $x = 2.39$ (to 2 d.p.)

$\therefore x = 0.28$ or $x = 2.39$

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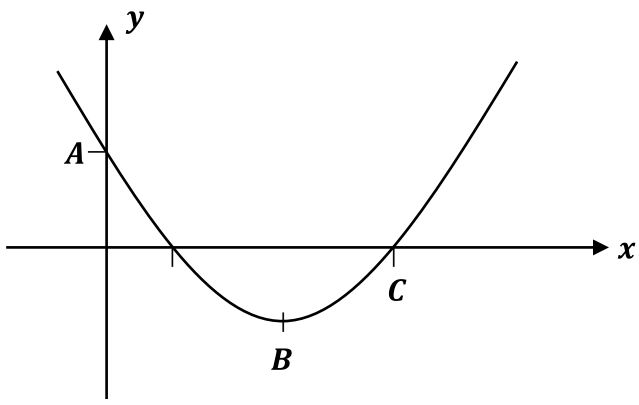
(iii) A sketch of the graph of $g(x)$ is shown below.



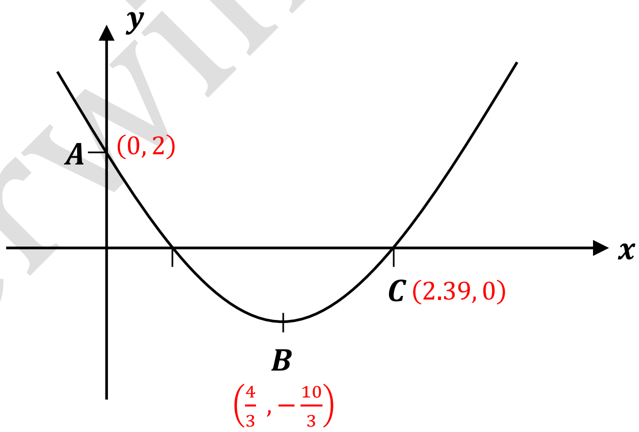
Copy the sketch and state

(a) the y-coordinate of A

(b) the x-coordinate of C

(c) the x and y-coordinates of B [3]

The sketch is shown below:



Total: 15 marks

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10. (a) The manager of a pizza shop wishes to make x small pizzas and y large pizzas. His oven holds no more than 20 pizzas.

(i) Write an inequality to represent the given condition. [2]

Condition: The manager of a pizza shop wishes to make x small pizzas and y large pizzas. His oven holds no more than 20 pizzas.

Inequality: $x + y \leq 20$

(ii) The ingredients for each small pizza cost $15 and for each large pizza $30. The manager plans to spend no more than $450 on ingredients. Write an inequality to represent this condition. [2]

Condition: The ingredients for each small pizza cost $15 and for each large pizza $30. The manager plans to spend no more than $450 on ingredients.

Inequality:

$15x + 30y \leq 450$

$(\div 15)$

$x + 2y \leq 30$

(b) (i) **Using a scale of 2 cm on the x-axis to represent 5 small pizzas and 2 cm on the y-axis to represent 5 large pizzas**, draw the graphs of the lines associated with the inequalities at (a)(i) and (a)(ii) above. [4]

Consider $x + y \leq 20$ and $x + 2y \leq 30$.

The equations are $x = y = 20$ and $x + 2y = 30$ respectively.

Consider $x + y = 20$.

When $x = 0, y = 20$.

When $y = 0, x = 20$.

Points to be plotted are $(0, 20)$ and $(20, 0)$.

Consider $x + 2y = 30$.

When $x = 0$,

$0 + 2y = 30$

$2y = 30$

$y = \frac{30}{2}$

$y = 15$

When $y = 0$,

$x + 2(0) = 30$

$x + 0 = 30$

$x = 30$

Points to be plotted are $(0, 15)$ and $(30, 0)$.

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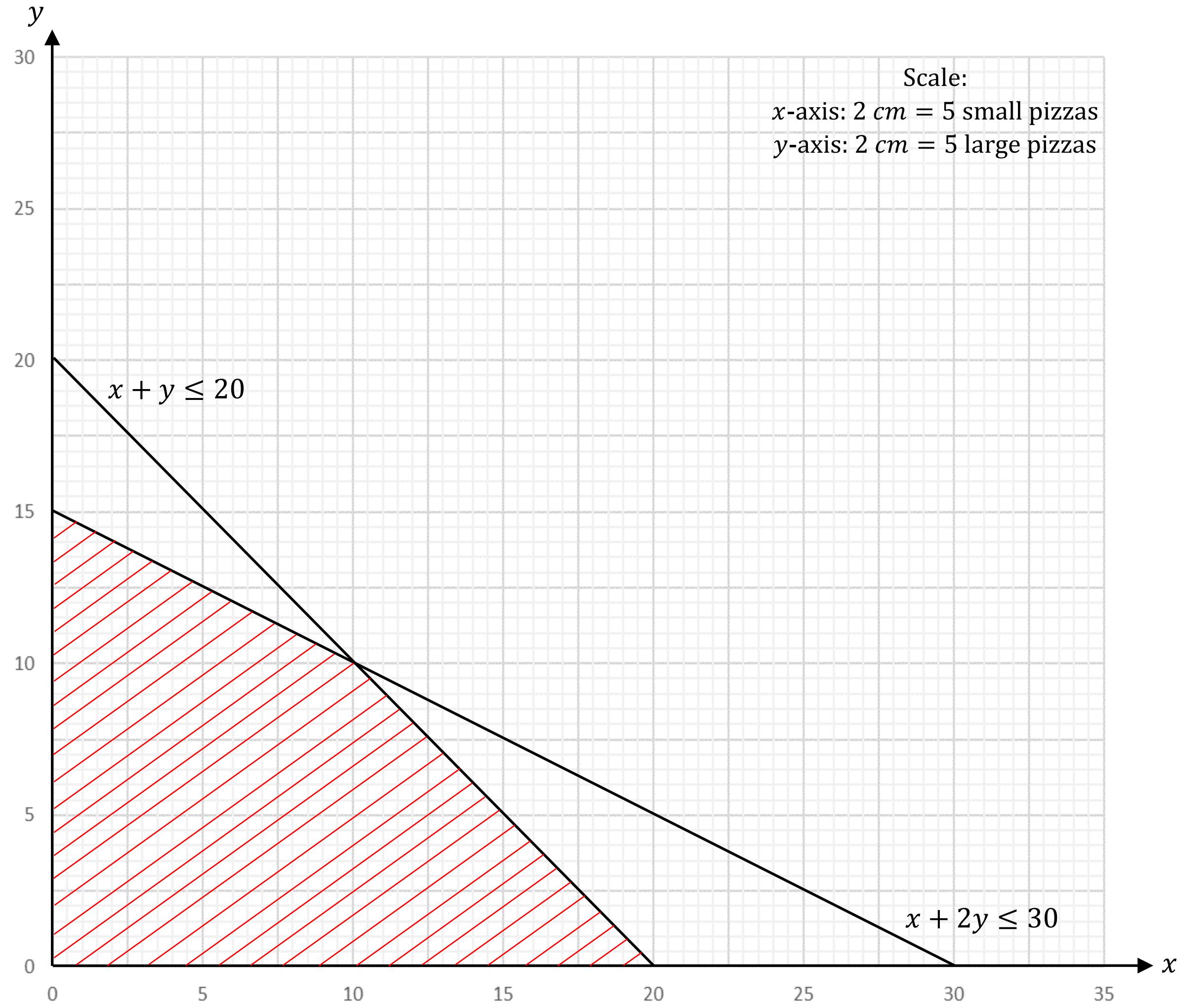
The graph is shown below:



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(ii) Shade the region which is defined by ALL of the following combined:

- the inequalities written at (a)(i) and (a)(ii)
- the inequalities $x \geq 0$ and $y \geq 0$ [1]

See graph above.

(iii) Using your graph, state the coordinates of the vertices of the shaded region. [2]

From the graph, the coordinates of the vertices are:

$(0, 0)$, $(0, 15)$, $(10, 10)$ and $(20, 0)$

(c) The pizza shop makes a profit of \$8 on the sale of EACH small pizza and \$20 on the sale of EACH large pizza. All the pizzas that were made were sold.

(i) Write an expression in x and y for the TOTAL profit made in the sale of the pizzas. [1]

Let the total profit be P.

The profit made on each small pizza is \$8.

The profit made on each large pizza is \$20.

$\therefore P = 8x + 20y$

(ii) Use the coordinates of the vertices given at (b)(iii) to determine the MAXIMUM profit. [3]

The coordinates of the vertices are $(0, 0)$, $(0, 15)$, $(10, 10)$ and $(20, 0)$.

When $x = 0$ and $y = 15$,

$P = 8(0) + 20(15)$

$= 0 + 300$

$= \$300$

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When $x = 10$ and $y = 10$,

$P = 8(10) + 20(10)$

$= 80 + 200$

$= \$280$

When $x = 20$ and $y = 0$,

$P = 8(20) + 20(0)$

$= 160 + 0$

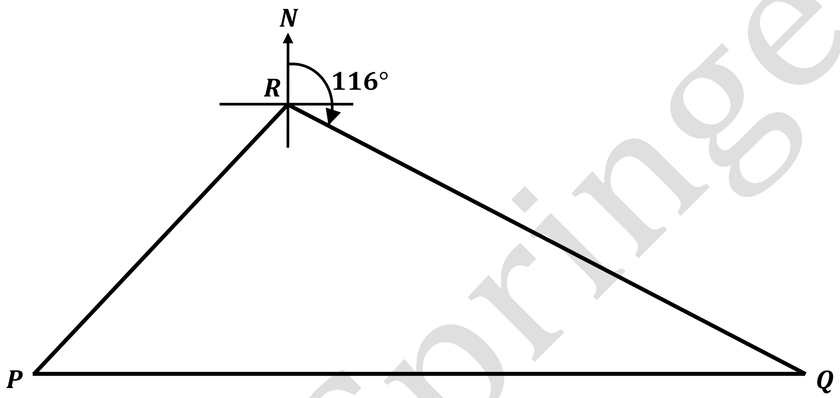
$= \$160$

$\therefore$ The maximum profit occurs when the shop sells 15 large pizzas.

Total: 15 marks

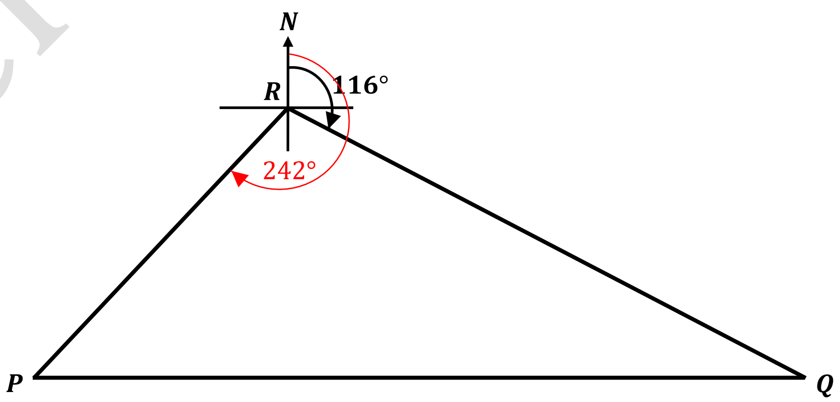
Kerwin Springer

11. (a) The diagram below, **not drawn to scale**, shows three stations P, Q and R, such that the bearing of Q from R is $116°$ and the bearing of P from R is $242°$. The vertical line at R shows the North direction.



(i) Show that angle $PRQ = 126°$. [2]

As stated in the question, $242°$ was the bearing of P from R as shown below.



$242°$ was the bearing of P from R as shown above.

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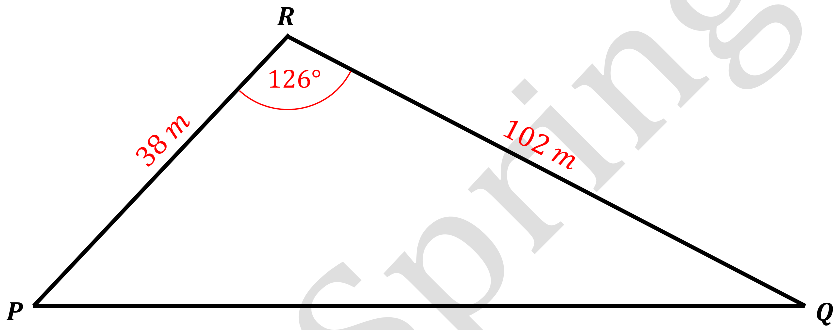
$\therefore P\hat{R}Q = 242° - 116°$

$= 126°$

(ii) Given that $PR = 38$ metres and $QR = 102$ metres, calculate the distance PQ, giving your answer to the nearest metre. [3]

Consider the diagram below:



Using the cosine rule,

$(PQ)^2 = (PR)^2 + (RQ)^2 - 2(PR)(RQ)\cos\hat{R}$

$(PQ)^2 = (38)^2 + (102)^2 - 2(38)(102)\cos 126°$

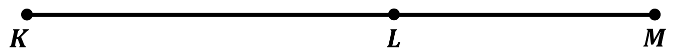
$(PQ)^2 = 16404.51128$

$PQ = \sqrt{16404.51128}$

$PQ = 128\ m$ (to the nearest metre)

$\therefore$ The distance PQ is $128\ m$.

(b) K, L and M are points along a straight line on a horizontal plane, as shown below.

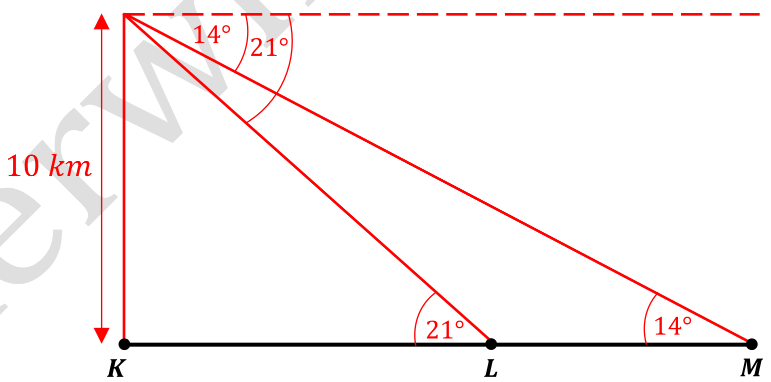


A vertical pole, SK, is positioned such that the angles of elevation of the top of the pole S from L and M are 21° and 14° respectively.

The height of the pole, SK, is 10 metres.

(i) Copy and complete the diagram to show the pole SK and the angles of elevation of S from L to M. [4]

The completed diagram is shown below:



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(ii) Calculate, correct to ONE decimal place,

(a) the length of KL

$$\tan 21° = \frac{10}{KL}$$

$$KL = \frac{10}{\tan 21°}$$

$$KL = 26.1\ m \text{ (to 1 decimal place)}$$

$\therefore$ The length of KL is $26.1\ m$.

(b) the length of LM [6]

$$\tan 14° = \frac{10}{KM}$$

$$KM = \frac{10}{\tan 14°}$$

$$KM = 40.10780934 \text{ m}$$

$$LM = KM - Kl$$

$$= 40.10780934 - 26.1$$

$$= 14.0\ m \text{ (to 1 decimal place)}$$

$\therefore$ The length of LM is $14.0\ m$.

Total: 15 marks

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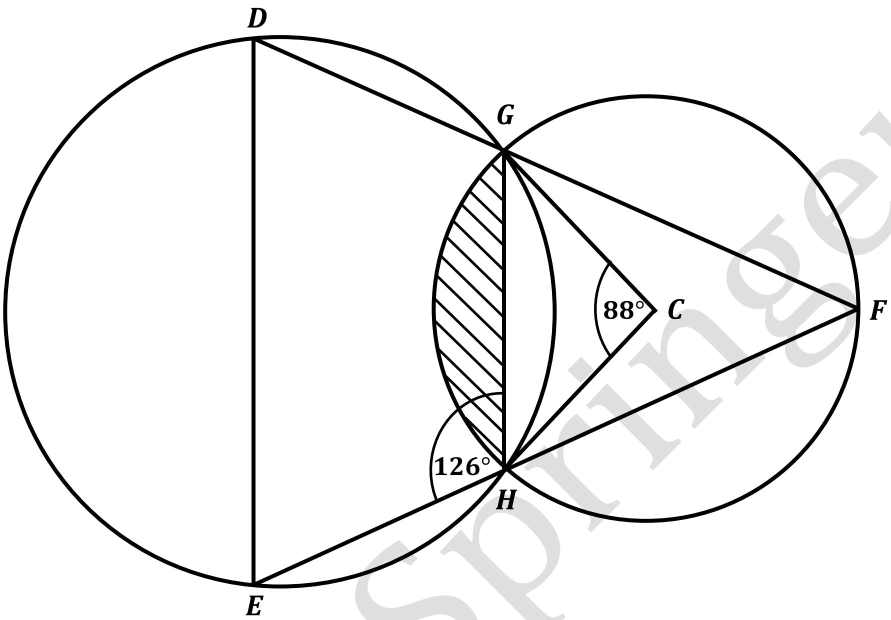
12. (a) The diagram below, **not drawn to scale**, shows two circles. C is the centre of the smaller circle, GH is a common chord and DEF is a triangle.

Angle $GCH = 88°$ and angle $GHE = 126°$



Calculate, giving reasons for your answer, the measure of angle

(i) GFH [2]

The angle subtended by a chord GH at the centre of the circle, C, is twice the angle at the circumference.

$$\text{Angle } GFH = \frac{1}{2} \times \text{Angle } GCH$$

$$= \frac{1}{2}(88°)$$

$$= 44°$$

$\therefore$ Angle $GFH = 44°$

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(ii) GDE [3]

The opposite angles of a cyclic quadrilateral are supplementary.

$$\text{Angle } GDE = 180° - \text{Angle } GHE$$
$$= 180° - 126°$$
$$= 54°$$

$\therefore$ Angle $GDE = 54°$

(iii) DEF [2]

All angles in a triangle add up to 180°.

$$\text{Angle } DEF = 180° - \text{Angle } GDE - \text{Angle } GFH$$
$$= 180° - 54° - 44°$$
$$= 82°$$

$\therefore$ Angle $DEF = 82°$

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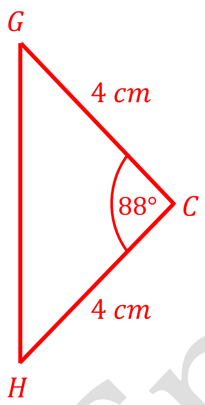
(b) **Use $\pi = 3.14$ in this part of the question.**

Given that $GC = 4\ cm$, calculate the area of

(i) triangle GCH [3]

Since GC and HC are both radii of the same circle, then $GC = HC$.

Consider the diagram below:



Now,

$$\text{Area of triangle } GCH = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(4)(4)\sin 88°$$

$$= 7.995\ cm^2 \qquad \text{(to 3 decimal places)}$$

∴ The area of triangle GCH is $7.995\ cm^2$.

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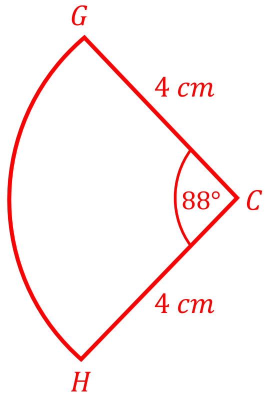
(ii) the minor sector bounded by arc GH and radii GC and HC [3]

Consider the diagram below:



Now,

$$\text{Area of minor sector} = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{88°}{360°} \times 3.14 \times (4)^2$$

$$= 12.3\ cm^2 \quad \text{(to 3 significant figures)}$$

$\therefore$ The area of the minor sector is 12.3 cm^2.

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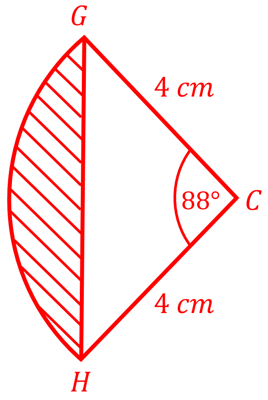
(iii) the shaded segment [2]

Consider the diagram below:



Now,

Area of shaded segment = Area of sector GCH − Area of triangle GCH

$$= \left[\frac{88°}{360°} \times 3.14 \times (4)^2\right] - \left[\frac{1}{2}(4)(4)\sin 88°\right]$$

$$= 4.29\ cm^2 \quad \text{(to 3 significant figures)}$$

∴ The area of the shaded segment is 4.29 cm^2.

Total: 15 marks

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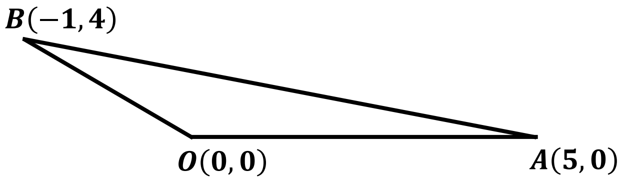
13. (a) The figure below, **not drawn to scale**, shows the points $O(0,0)$, $A(5,0)$ and $B(-1,4)$ which are the vertices of a triangle OAB.



(i) Express in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors

(a) $\overrightarrow{OB}$

The coordinates of B are $(-1,4)$.

So the vector $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

(b) $\overrightarrow{OA} + \overrightarrow{OB}$ [3]

The coordinates of A are $(5,0)$.

So the vector $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$.

Now,

$$\overrightarrow{OA} + \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 + (-1) \\ 0 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

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(ii) If $M(x, y)$ is the midpoint of AB, determine the values of x and y. [2]

Points are $A(5, 0)$ and $B(-1, 4)$.

Since $M(x, y)$ is the midpoint of AB, then,

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{5+(-1)}{2}, \frac{0+4}{2}\right)$$

$$= \left(\frac{4}{2}, \frac{4}{2}\right)$$

$= (2, 2)$ which is in the form (x, y),

where $x = 2$ and $y = 2$.

$\therefore$ The values of x and y are $x = 2$ and $y = 2$.

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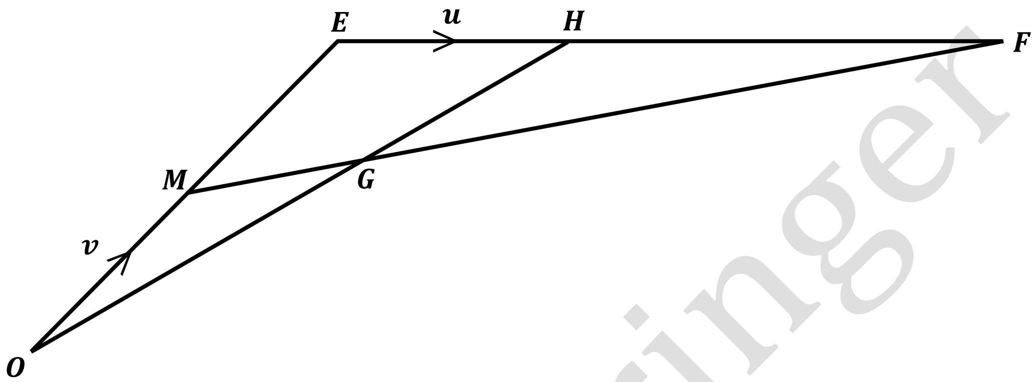
(b) In the figure below, **not drawn to scale**, OE, EF and MF are straight lines. The point H is such that $EF = 3EH$. The point G is such that $MF = 5MG$. M is the midpoint of OE.

The vector $\overrightarrow{OM} = \boldsymbol{v}$ and $\overrightarrow{EH} = \boldsymbol{u}$.



(i) Write in terms of $\boldsymbol{u}$ and/or $\boldsymbol{v}$, an expression for:

(a) $\overrightarrow{HF}$ [1]

We are given that,

$\overrightarrow{EF} = 3\overrightarrow{EH}$

$= 3\boldsymbol{u}$

Now,

Using the triangle law,

$\overrightarrow{EF} = \overrightarrow{EH} + \overrightarrow{HF}$

$\overrightarrow{HF} = \overrightarrow{EF} - \overrightarrow{EH}$

$= 3\boldsymbol{u} - \boldsymbol{u}$

$= 2\boldsymbol{u}$

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(b) $\overrightarrow{MF}$ [2]

Since M is the midpoint of $\overrightarrow{OE}$, then

$\overrightarrow{ME} = \overrightarrow{OM}$

$= \boldsymbol{v}$

Now,

Using the triangle law,

$\overrightarrow{MF} = \overrightarrow{ME} + \overrightarrow{EF}$

$= \boldsymbol{v} + 3\boldsymbol{u}$

(c) $\overrightarrow{OH}$ [2]

Using the triangle law,

$\overrightarrow{OE} = \overrightarrow{OM} + \overrightarrow{ME}$

$= \boldsymbol{v} + \boldsymbol{v}$

$= 2\boldsymbol{v}$

Now,

Using the triangle law,

$\overrightarrow{OH} = \overrightarrow{OE} + \overrightarrow{EH}$

$= 2\boldsymbol{v} + \boldsymbol{u}$

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(ii) Show that $\overrightarrow{OG} = \frac{3}{5}(2v + u)$ [2]

We are given that,

$\overrightarrow{MF} = 5\overrightarrow{MG}$

$= \frac{1}{5}\overrightarrow{MF}$

$= \frac{1}{5}(\boldsymbol{v} + 3\boldsymbol{u})$

Now,

Using the triangle law,

$\overrightarrow{OG} = \overrightarrow{OM} + \overrightarrow{MG}$

$= \boldsymbol{v} + \frac{1}{5}(\boldsymbol{v} + 3\boldsymbol{u})$

$= \boldsymbol{v} + \frac{1}{5}\boldsymbol{v} + \frac{3}{5}\boldsymbol{u}$

$= \frac{6}{5}\boldsymbol{v} + \frac{3}{5}\boldsymbol{u}$

$= \frac{3}{5}(2\boldsymbol{v} + \boldsymbol{u})$

$\therefore \overrightarrow{OG} = \frac{3}{5}(2\boldsymbol{v} + \boldsymbol{u})$

Q.E.D.

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(iii) Hence, prove that O, G and H lie on a straight line. [3]

$\overrightarrow{OH} = 2\boldsymbol{v} + \boldsymbol{u}$ and $\overrightarrow{OG} = \frac{3}{5}(2\boldsymbol{v} + \boldsymbol{u})$

So, $\overrightarrow{OG} = \frac{3}{5}\overrightarrow{OH}$

Since $\overrightarrow{OH}$ is a scalar multiple of $\overrightarrow{OG}$, then $\overrightarrow{OG}$ and $\overrightarrow{OH}$ are parallel.

Since O is a common point on both lines, then G lies on $\overrightarrow{OH}$ (as shown on the diagram).

$\therefore O, G$ and H all lie on a straight line.

Q.E.D.

Total: 15 marks

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14. (a) L and N are two matrices where

$$L = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \text{ and } N = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}.$$

Evaluate $L - N^2$. [3]

$$N^2 = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (-1 \times -1) + (3 \times 0) & (-1 \times 3) + (3 \times 2) \\ (0 \times -1) + (2 \times 0) & (0 \times 3) + (2 \times 2) \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 0 & -3 + 6 \\ 0 + 0 & 0 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$$

Now,

$$L - N^2 = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

(b) The matrix, M, is given as $M = \begin{pmatrix} x & 12 \\ 3 & x \end{pmatrix}$. Calculate the values of x for which M is singular. [2]

$$M = \begin{pmatrix} x & 12 \\ 3 & x \end{pmatrix}$$

$$\det(M) = ad - bc$$

$$= (x)(x) - (12)(3)$$

$$= x^2 - 36$$

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If M is singular, then the $\det(M) = 0$.

So, we have,

$$x^2 - 36 = 0$$

$$(x+6)(x-6) = 0$$

Either $x + 6 = 0$ or $x - 6 = 0$

$x = -6$ $\qquad$ $x = 6$

$\therefore$ The values of x for which M is singular are $x = -6$ and $x = 6$.

(c) A geometric transformation, R, maps the point $(2, 1)$ onto $(-1, 2)$.

Given that $R = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$, calculate the values of p and q. [3]

$$\begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} (0 \times 2) + (p \times 1) \\ (q \times 2) + (0 \times 1) \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 + p \\ 2q + 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} p \\ 2q \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Comparing corresponding entries gives:

$p = -1$ and $2q = 2$

$$q = \frac{2}{2}$$

$$q = 1$$

$\therefore p = -1$ and $q = 1$

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(d) A translation, $T = \begin{pmatrix} r \\ s \end{pmatrix}$ maps the point $(5, 3)$ onto $(1, 1)$. Determine the values of r and s. [3]

$$\begin{pmatrix} r \\ s \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} r+5 \\ s+3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Comparing corresponding entries gives:

$r + 5 = 1$	and	$s + 3 = 1$
$r = 1 - 5$		$s = 1 - 3$
$r = -4$		$s = -2$

$\therefore r = -4$ and $s = -2$

(e) Determine the coordinates of the image of $(8, 5)$ under the combined transformation, R followed by T. [4]

$R = \begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}$

Since $p = -1$ and $q = 1$, then $R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Firstly, we transform under R.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 8 \\ 5 \end{pmatrix} = \begin{pmatrix} (0 \times 8) + (-1 \times 5) \\ (1 \times 8) + (0 \times 5) \end{pmatrix}$$

$$= \begin{pmatrix} 0 + (-5) \\ 8 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ 8 \end{pmatrix}$$

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$T = \binom{r}{s}$

Since $r = -4$ and $s = -2$, then $T = \binom{-4}{-2}$.

Then, we transform under T.

$$\binom{-5}{8} + \binom{-4}{-2} = \binom{-5 + (-4)}{8 + (-2)}$$

$$= \binom{-5 - 4}{8 - 2}$$

$$= \binom{-9}{6}$$

$\therefore$ The image of $(8, 5)$ under the combined transformation, R followed by T is $(-9, 6)$.

Total: 15 marks

END OF TEST

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