THE STUDENT HUB

CSEC Mathematics

January 2022 – Paper 2

Solutions

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, find

(i) the EXACT value of $\frac{8.9 + 31.6}{0.75 \times 5.4}$ [1]

Using a calculator,

$\frac{8.9 + 31.6}{0.75 \times 5.4} = 10$

(ii) the value of $3.9\tan(18°)$ correct to 1 decimal place. [1]

Using a calculator,

$3.9\tan(18°) = 1.3$ (to 1 decimal place)

(b) (i) Ria is paid at a rate of $13.50 per hour. During a certain week she worked 40 hours. How much did she earn that week? [1]

She earned = Number of hours worked × Hourly Rate

$= 40 \times \$13.50$

$= \$540$

(ii) Ria worked 4 weeks in the month of August and her gross earnings was \$2 463.75. Her regular week comprised 40 hours and overtime was paid at $1\frac{1}{2}$ times the hourly rate.

Show that Ria worked 15 hours overtime in August. [2]

4 weeks $= 4 \times \$540$

$= \$2160$

Overtime wages $= \$2463.75 - \$2160$

$= \$303.75$

Overtime rate per hour $= 1.5 \times \$13.50$

$= \$20.25$

Number of overtime hours $= \frac{303.75}{20.25}$

$= 15$ hours overtime

(iii) In August, 20% of Ria's gross earnings was deducted as tax. How much money does she have left after the deduction? [2]

Percentage left = $(100 - 20)\%$

$= 80\%$

Amount of money she had left = 80% of \$2463.75

$= \$1971$

(iv) Ria invested \$219 of her earnings for 3 years at a rate of 4.5% per annum simple interest. How much interest does she receive after 3 years? [2]

$SI = \frac{P \times R \times T}{100}$

$= \frac{219 \times 4.5 \times 3}{100}$

$= \$29.57$

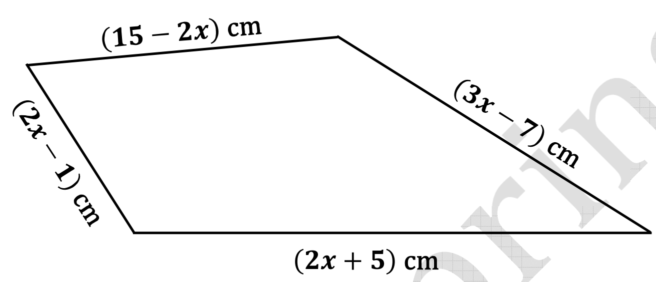
Total 9 marks

2. (a) Factorize completely

$$3n^2 + 15np \qquad [2]$$

$3n^2 + 15np = 3n(n + 5p)$

(b) The diagram below shows a quadrilateral with the length of its sides written in terms of x.



(i) Write an expression, in terms of x, for the perimeter of the quadrilateral.

Express your answer in its simplest form. [2]

Perimeter $= 15 - 2x + 3x - 7 + 2x - 1 + 2x + 5$

$= 5x + 12$

(ii) The perimeter of the quadrilateral is 32 cm.

Find the longest side of the quadrilateral. [2]

$5x + 12 = 32$

$5x = 32 - 12$

$5x = 20$

$x = \frac{20}{5}$

$x = 4$ cm

Longest side $= 2x + 5$

$= 2(4) + 5$

$= 8 + 5$

$= 13$ cm

(c) Determine ALL the **integer values** of x which satisfy the inequality

$$-1 < \frac{2-4x}{3} < 5$$ [3]

$$-1 < \frac{2-4x}{3} < 5$$

$$-3 < 2 - 4x < 15$$

$$-3 - 2 < -4x < 15 - 2$$

$$-5 < -4x < 13$$

$$\frac{-5}{-4} > x > \frac{13}{-4}$$

$$\frac{5}{4} > x > -\frac{13}{4}$$

$$-\frac{13}{4} < x < \frac{5}{4}$$

$$-3.25 < x < 1.25$$

The integer values which satisfy this inequality are $\{-3, -2, -1, 0, 1\}$.

Total 9 marks

3. (a) The box below contains the names of 5 quadrilaterals.

Trapezium	Rhombus	
Kite	Square	Rectangle

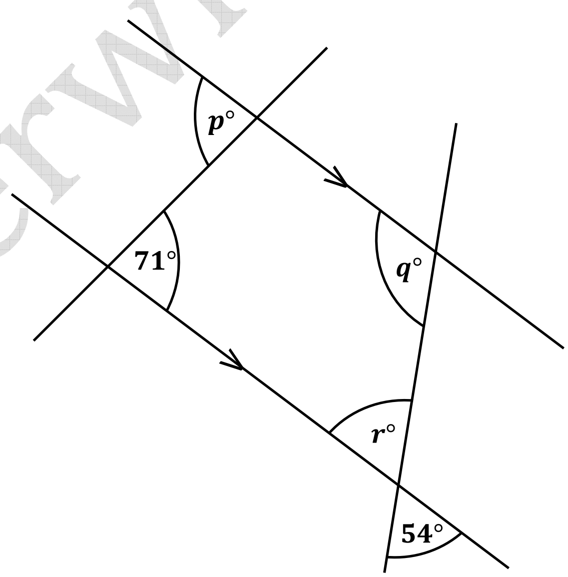
Choose the name of **one** quadrilateral from the box that BEST completes **each** statement. [3]

(i) A ………………… trapezium …………………. has no lines of symmetry and has rotational symmetry of order one.

(ii) A ………………… rectangle ……………………… has EXACTLY **two** lines of symmetry and 4 right angles.

(iii) A …………………….. kite ……………………… has one line of symmetry but no rotational symmetry.

(b) The diagram below shows 4 straight lines, 2 of which are parallel.



(i) Determine the values of q and r.

q [1]

Angles on a straight line add up to 180°.

$q = 180° - 54°$

$= 126°$

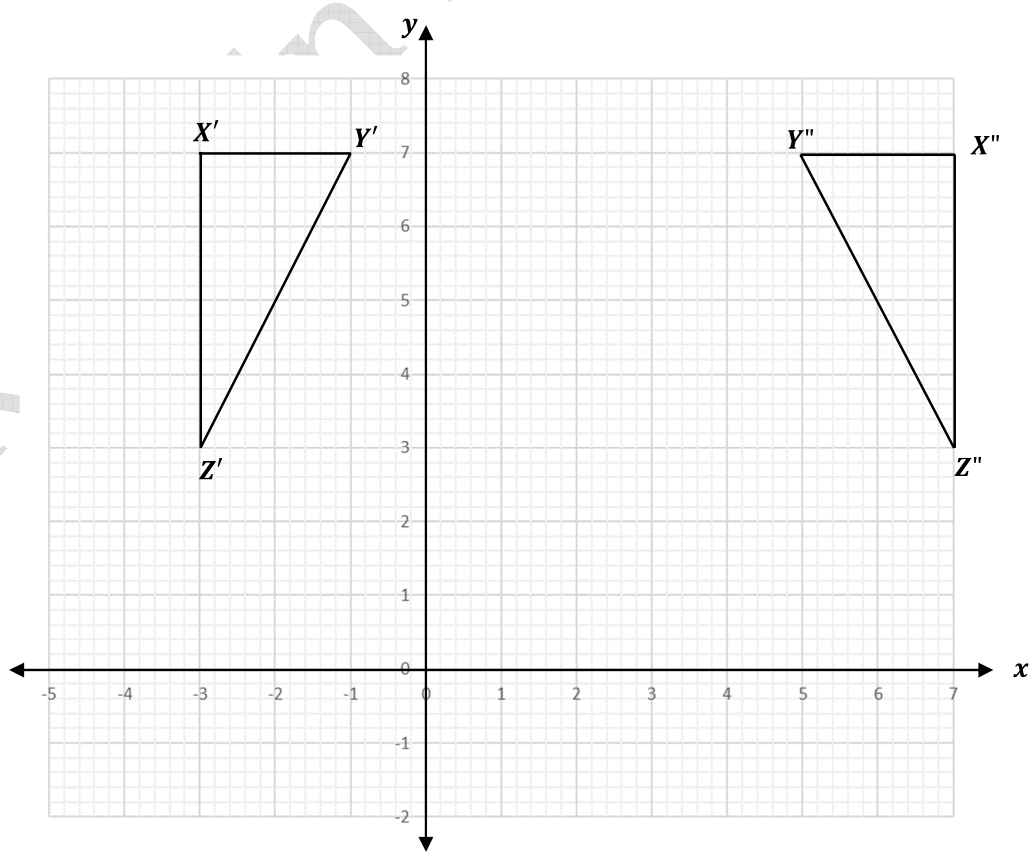
r [1]

Since vertically opposite angles are equal, $r = 54°$.

(ii) Give a geometrical reason why $\angle p = 71°$. [1]

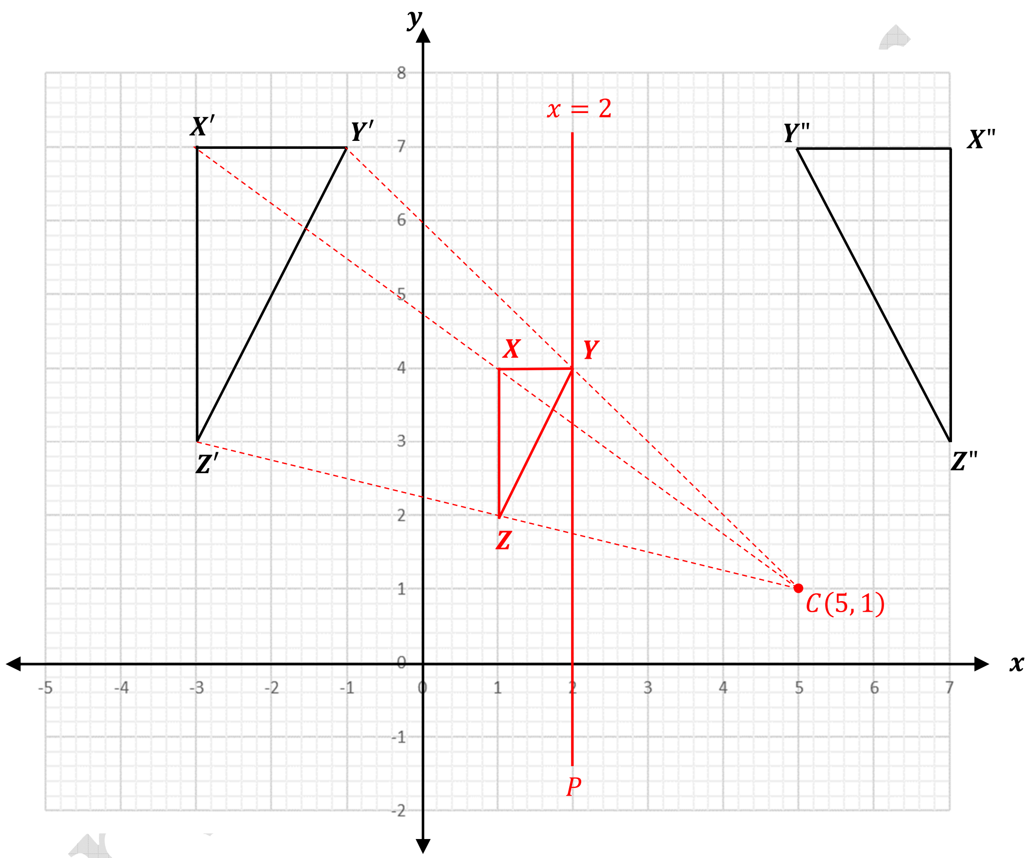
Since alternate angles are equal, then $\angle p = 71°$.

(c) The diagram below shows triangles $X'Y'Z'$ and $X"Y"Z"$ drawn on a square grid.



(i) Triangle $X'Y'Z'$ is the image of Triangle XYZ after an enlargement of scale factor 2, with centre $(5, 1)$.

Draw triangle XYZ, the **OBJECT** for Triangle $X'Y'Z'$, on the grid above. [2]



(ii) Triangle $X'Y'Z'$ is mapped onto Triangle $X"Y"Z"$ by a reflection in the line P. State the equation of the mirror line, P. [1]

The equation of the mirror line, P, is $x = 2$.

Total 9 marks

4. Three functions f, g and h are defined as

$$f(x) = 2x - 1\ ;\ g(x) = 3x + 2 \text{ and } h(x) = 5^x.$$

(a) Find the value of

(i) $f\left(\frac{1}{2}\right)$ [1]

$f(x) = 2x - 1$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) - 1$

$= 1 - 1$

$= 0$

(ii) $h(0)$ [1]

$h(x) = 5^x$

$h(0) = 5^0$

$= 1$

(iii) $g^2(-3)$ [2]

$g(x) = 3x + 2$

$g(-3) = 3(-3) + 2$

$= -9 + 2$

$= -7$

Now,

$$g^2(-3) = g[g(-3)]$$

$$= g(-7)$$

$$= 3(-7) + 2$$

$$= -21 + 2$$

$$= -19$$

(b) Find $gf(x)$, giving your answer in its simplest form. [2]

$$gf(x) = g[f(x)]$$

$$= g(2x - 1)$$

$$= 3(2x - 1) + 2$$

$$= 6x - 3 + 2$$

$$= 6x - 1$$

(c) (i) Find $g^{-1}(x)$. [2]

Let $y = g(x)$.

$y = 3x + 2$

Interchanging variables x and y.

$x = 3y + 2$

Making y the subject of the formula.

$x - 2 = 3y$

$\frac{x-2}{3} = y$

$\therefore g^{-1}(x) = \frac{x-2}{3}$

(ii) Hence, or otherwise, determine the value of x when $g^{-1}(x) = 4$. [1]

$g^{-1}(x) = \frac{x-2}{3}$

Hence,

$4 = \frac{x-2}{3}$

$12 = x - 2$

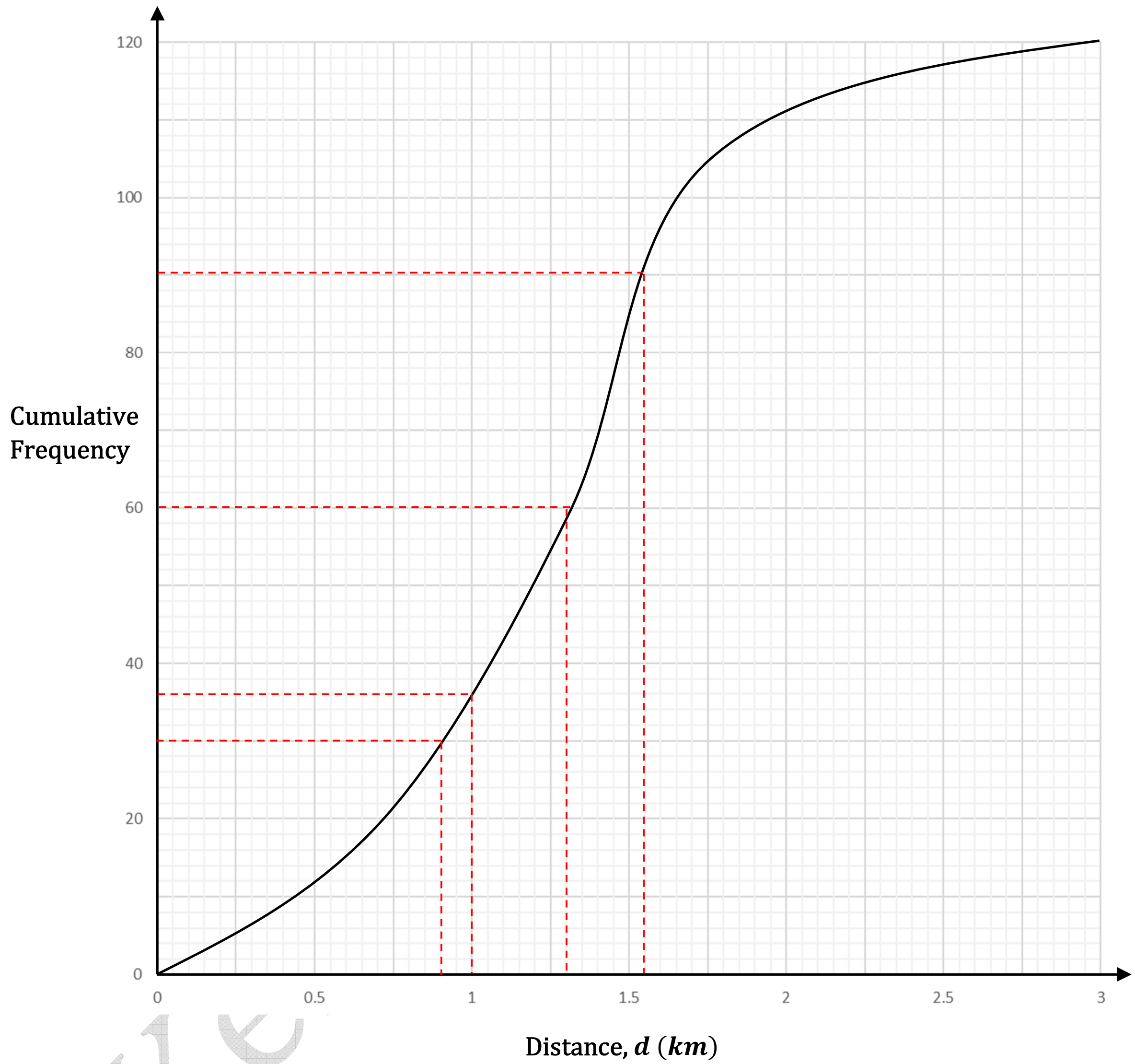
$12 + 2 = x$

$14 = x$

$\therefore$ The value of x when $g^{-1}(x) = 4$ is $x = 14$.

Total 9 marks

5. The cumulative frequency diagram below shows information about the distance, d km, that each of 120 students walks to school on a particular day.



(a) How many students walked AT MOST 1 km to school on that day? [1]

The number of students who walked at most 1 km to school on that day is 36 students.

(b) Using the cumulative frequency diagram, determine an estimate of

(i) the median [1]

The median occurs at $\frac{120}{2} = 60^{\text{th}}$ value.

From the graph, the median $= 1.3$ km

(ii) the lower quartile [1]

The lower quartile occurs at $\frac{120}{4} = 30^{\text{th}}$ value.

From the graph, the lower quartile $= 0.9$ km

(iii) the interquartile range [1]

The upper quartile occurs at $\frac{3(120)}{4} = 90^{\text{th}}$ value.

From the graph, the upper quartile $= 1.55$ km

Interquartile range = Upper quartile – Lower quartile

$= 1.55 - 0.9$

$= 0.65$ km

(c) A student is chosen at random. What is the probability that the student walked for **more than** 1.5 km to school that day? [2]

Number of students who walked less than 1.5 km $= 82$ (from graph)

Number of students who walked more than 1.5 km $= 120 - 82$

$= 38$

$\therefore P(\text{student walked more than 1.5 km}) = \frac{\text{Number of students who walked more than 1.5 km}}{\text{Total number of students}}$

$= \frac{38}{120}$

$= \frac{19}{60}$

(d) Complete the table below and use the information to calculate an estimate of the mean distance walked by the students on that day. [3]

Distance, d (km)	Midpoint (x)	Number of students (f)	$f \times x$
$0 < d \leq 0.5$	0.25	12	3.0
$0.5 < d \leq 1.0$	0.75	24	18
$1.0 < d \leq 1.5$	1.25	46	57.5
$1.5 < d \leq 2.0$	1.75	30	52.5
$2.0 < d \leq 2.5$	2.25	6	13.5
$2.5 < d \leq 3.0$	2.75	2	5.5

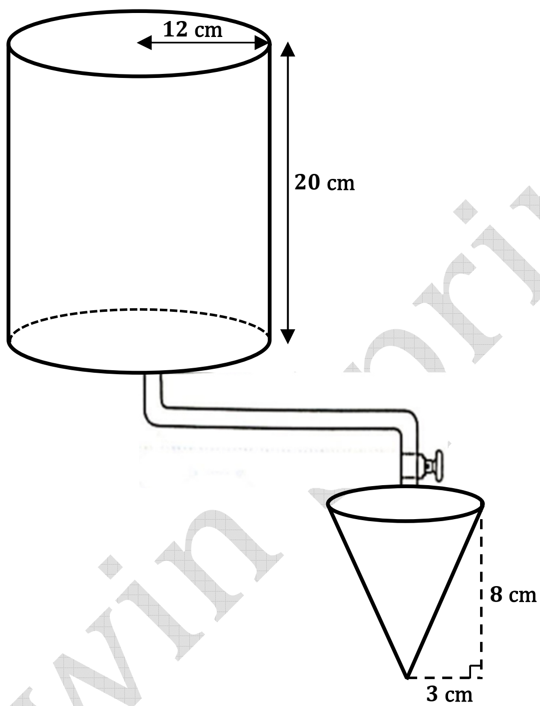
Mean $= \frac{\sum fx}{\sum f}$

$= \frac{150}{120}$

$= 1.25\ km$

Total 9 marks

6. At a track meet, a cylindrical container, fitted with a pipe as shown in the diagram below, is used to serve water to athletes. The cylindrical container of radius 12 cm and height 20 cm is **completely** filled with water and the pipe fitted at the bottom dispenses water into cone-shaped cups. The cone-shaped cups have a radius of 3 cm and a height of 8 cm.



(a) Calculate the volume of water in the cylindrical container, **in litres**. Write your answer correct to 2 decimal places. [1 000 $\text{cm}^3 = 1$ litre] [3]

Volume $= \pi r^2 h$

$= \pi(12)^2(20)$

$= 9047.79\ cm^3$

[π was used from calculator where $\pi = 3.141592654$]

Now,

$1\ 000\ \text{cm}^3 = 1 \text{ litre}$

$1\ \text{cm}^3 = \frac{1}{1000} \text{ litre}$

$9047.79\ \text{cm}^3 = \frac{1}{1000} \times 9047.79$

9.05 litres (to 2 decimal places)

(b) Water flows from the cylindrical container along the pipe into the cone-shaped cups at a rate of 7.8 ml per second.

Calculate the time taken to fill ONE of the empty cone-shaped cups. Give your answer correct to the nearest second.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [3]

$1\ ml = 1\ \text{cm}^3$

$7.8\ ml = 7.8 \times 1$

$= 7.8\ \text{cm}^3$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3)^2(8)$

$= 24\pi\ \text{cm}^3$

Number of seconds $= \frac{Volume\ of\ cone}{7.8}$

$= \frac{24\pi}{7.8}$

$= 10$ seconds (to the nearest second)

(c) Determine the number of empty cone-shaped cups in (b) that can be **completely** filled from the cylindrical container. [3]

$$\text{Number of cups} = \frac{2880\,\pi}{24\,\pi}$$

$$= 120 \text{ cups}$$

Total 9 marks

7. A sequence of figures is made from lines of unit length and dots. The lines form a series of octagons and squares. The dots are placed at each vertex.

The first 3 figures in the sequence are shown below.

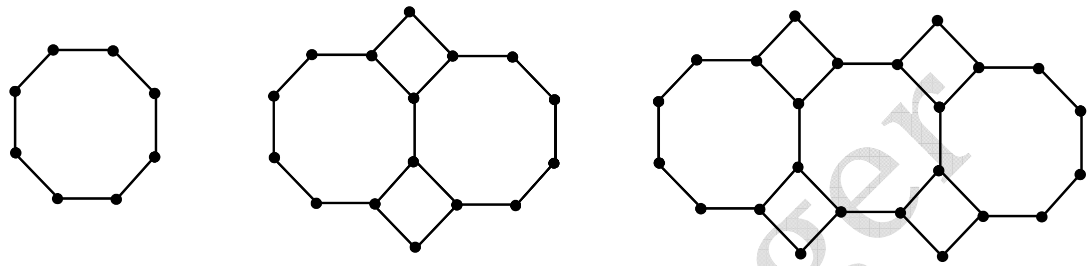
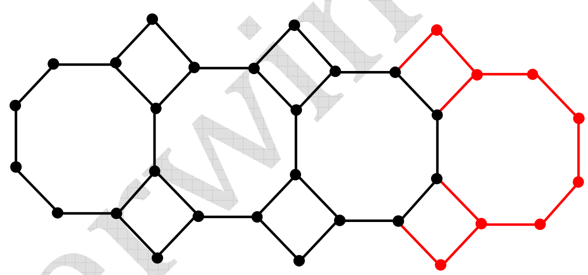


Figure 1 **Figure 2** **Figure 3**

(a) Figure 3 of the sequence is shown by itself below. Add more lines of unit length and dots to Figure 3 to correctly represent Figure 4. [2]



(b) The number of dots, D, and the number of unit lines that form the perimeter of the shape, P, form a pattern. The values for D and P for the first 3 figures are written in the table below.

Complete the rows numbered (i), (ii) and (iii).

	Figure Number	Number of Dots (D)	Perimeter of Figure (P)	
	1	8	8	
	2	16	14	
	3	24	20	
(i)	4	32	26	[2]
	⋮	⋮	⋮	
(ii)	14	112	86	[2]
	⋮	⋮	⋮	
(iii)	n	$8n$	$6n+2$	[2]

(c) For any figure, $n > 1$, the number of dots, D, is greater than its perimeter, P. Determine the value of n for a figure in which the difference between D and P is 36. [2]

$$D - P = 36$$

$$8n - (6n + 2) = 36$$

$$8n - 6n - 2 = 36$$

$$2n - 2 = 36$$

$$2n = 36 + 2$$

$$2n = 38$$

$$n = \frac{38}{2}$$

$$n = 19$$

Total 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. The function $f: x \to 3 + 5x - x^2$.

(a) (i) Complete the table of values for $f(x) = 3 + 5x - x^2$. [2]

x	-1	0	1	2	3	4	5	6
$f(x)$	-3	3	7	9	9	7	3	-3

Let $y = 3 + 5x - x^2$.

When $x = 0$,

$y = 3 + 5(0) - (0)^2$

$= 3 + 0 - 0$

$= 3$

When $x = 0$,

$y = 3 + 5(1) - (1)^2$

$= 3 + 5 - 1$

$= 7$

When $x = 4$,

$y = 3 + 5(4) - (4)^2$

$= 3 + 20 - 16$

$= 7$

When $x = 6$,

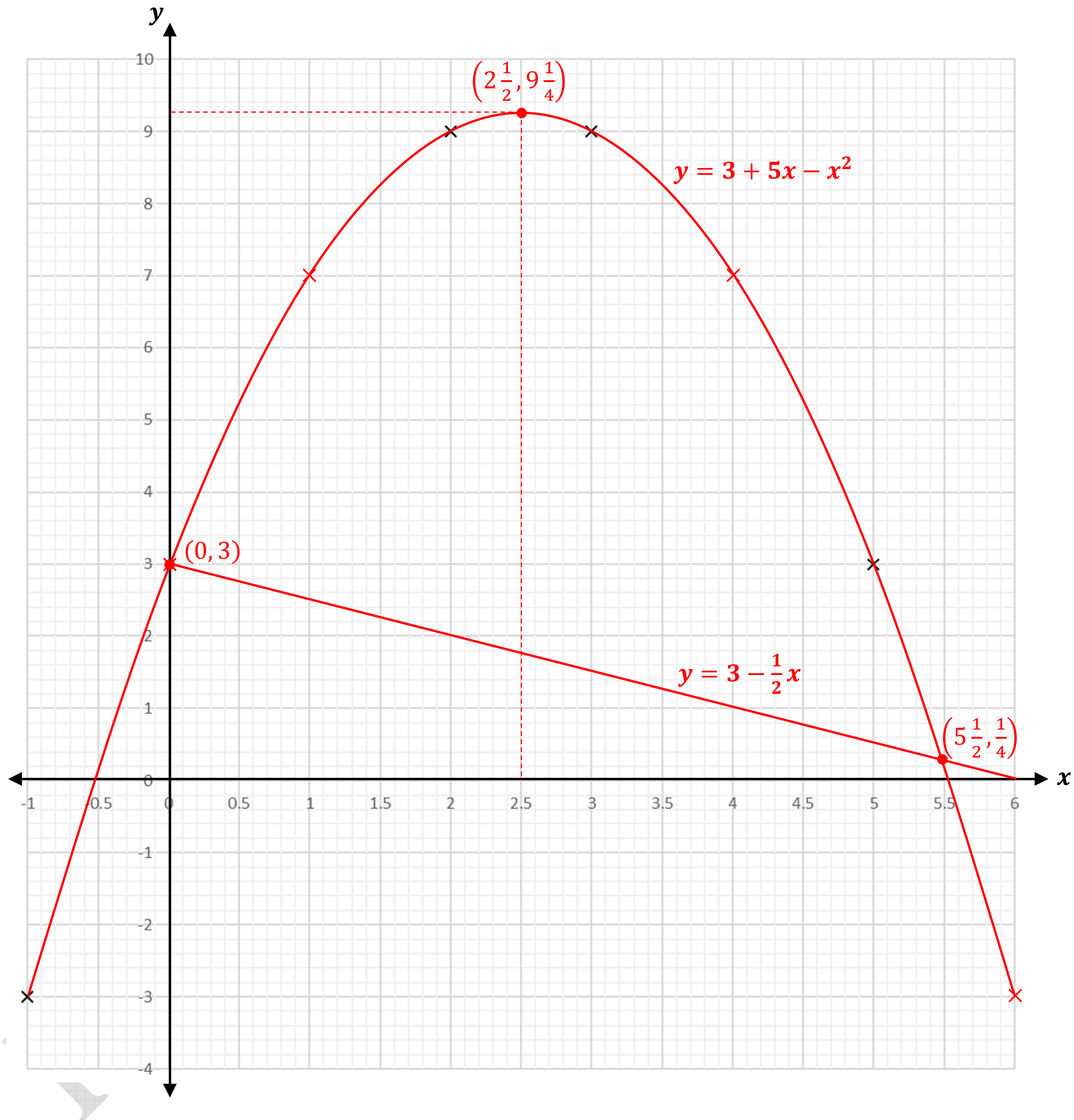
$y = 3 + 5(6) - (6)^2$

$= 3 + 30 - 36$

$= -3$

(ii) On the grid below, complete the graph of $f(x) = 3 + 5x - x^2$ for

$-1 \leq x \leq 6$. [3]



(b) (i) Write down the equation of the axis of symmetry of the graph of

$f(x) = 3 + 5x - x^2$. [1]

From the graph, the axis of symmetry is $x = 2\frac{1}{2}$.

(ii) State the maximum value of the function. [1]

The maximum value of $f(x) = 9\frac{1}{4}$.

(c) Write down the co-ordinates of the point where the line $y = 3 - \frac{1}{2}x$

(i) crosses the x-axis [1]

$y = 3 - \frac{1}{2}x$

When $y = 0$,

$0 = 3 - \frac{1}{2}x$

$\frac{1}{2}x = 3$

$x = 3 \times 2$

$x = 6$

$\therefore$ The line crosses the x-axis at the point $(6, 0)$.

(ii) crosses the y-axis [1]

$y = 3 - \frac{1}{2}x$

When $x = 0$,

$y = 3 - \frac{1}{2}(0)$

$= 3 - 0$

$= 3$

$\therefore$ The line crosses the y-axis at the point $(0, 3)$.

(d) On the grid **on page 22**, draw the line $y = 3 - \frac{1}{2}x$. [1]

See graph above.

(e) Using your graph, determine the solution to the equations

$y = 3 + 5x - x^2$

$y = 3 - \frac{1}{2}x$ [2]

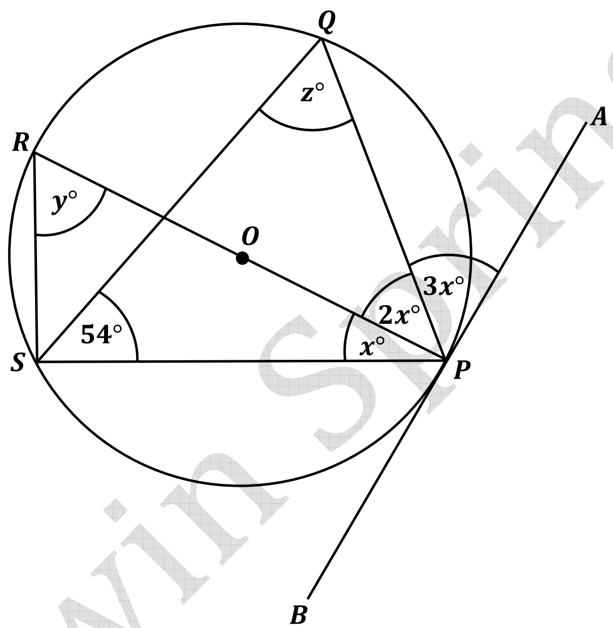
The solution of the equations will be at the points of intersections of both graphs.

Points of intersection are $(0, 3)$ and $\left(5\frac{1}{2}, \frac{1}{4}\right)$.

$\therefore x = 0$ and $y = 3$ or $x = 5\frac{1}{2}$ and $y = \frac{1}{4}$

Total 12 marks

9. (a) The diagram below shows a circle, with the points P, Q, R and S lying on its circumference and its centre marked O. RP is a diameter of the circle and AB is a tangent to the circle at P. Angle $APQ = 3x°$, angle $QPR = 2x°$, angle $RPS = x°$ and angle $QSP = 54°$.



Determine the value of EACH of the following angles. Show detailed working where possible and give a **reason** for your answer.

(i) x [2]

Angle made by a tangent (PA) and a radius (OP) at the point of contact (P) is 90°.

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$O\hat{P}A = 2x° + 3x°$

$= 5x°$

Hence,

$5x° = 90°$

$x = \frac{90°}{5}$

(ii) y [2]

The angle formed in a semi-circle is a right angle.

$R\hat{S}P = 90°$

Consider ΔRSP.

Angles in a triangle add up to 180°.

$y + 90° + 18° = 180°$

$y = 180° - 90° - 18°$

$y = 72°$

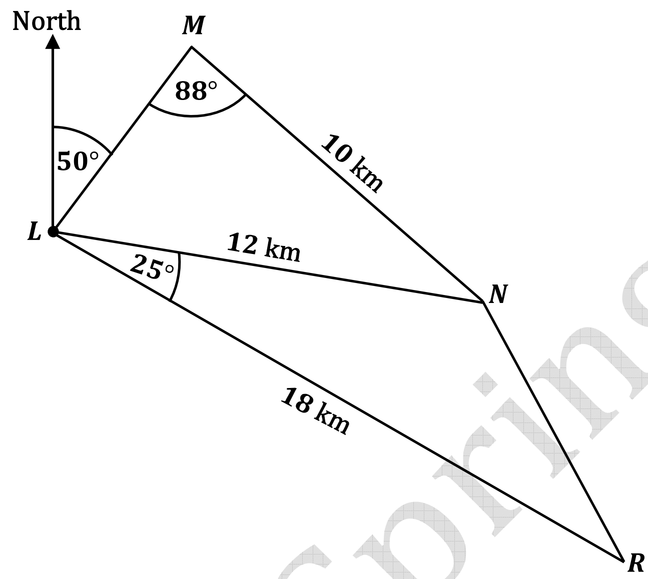
(iii) z [2]

Angles subtended by a chord, SP, at the circumference of a circle and standing on the same arc $(S\hat{R}P$ and $S\hat{Q}P)$ are equal.

$\therefore z = 72°$

(b) The diagram below shows straight roads connecting the towns L, M, N and R.

$LR = 18$ km, $LN = 12$ km and $MN = 10$ km. Angle $RLN = 25°$ and angle $LMN = 88°$.



(i) Calculate angle MLN. [3]

Consider ΔMLN.

Using the sine rule,

$$\frac{MN}{\sin M\hat{L}N} = \frac{LN}{\sin L\hat{M}N}$$

$$\frac{10}{\sin M\hat{L}N} = \frac{12}{\sin 88°}$$

$$\sin M\hat{L}N = \frac{10 \times \sin 88°}{12}$$

$$\sin M\hat{L}N = 0.8328$$

$$M\hat{L}N = \sin^{-1}(0.8328)$$

$$M\hat{L}N = 56.4° \qquad \text{(to 1 decimal place)}$$

(ii) Calculate the distance NR. [2]

Consider the ΔNLR.

Using the cosine rule,

$$(NR)^2 = (LN)^2 + (LR)^2 - 2(LN)(LR)\cos N\hat{L}R$$

$$= (12)^2 + (18)^2 - 2(12)(18)\cos 25°$$

$$= 76.475036$$

$$NR = \sqrt{76.475036}$$

$$= 8.75\ km \qquad \text{(to 2 decimal places)}$$

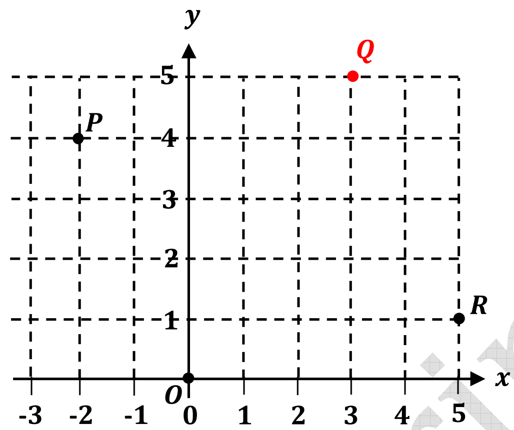
(iii) Determine the bearing of Town R from Town L. [1]

Bearing of Town R from Town $L = 50° + 56.4° + 25°$

$= 131.4°$

Total 12 marks

10. (a) Three points, O, P and R, are shown on the grid below. O is the origin.



(i) Write the position vector of R, $\overrightarrow{OR}$, in the form $\begin{pmatrix} a \\ b \end{pmatrix}$. [1]

$R = (5, 1)$

$\therefore \overrightarrow{OR} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ which is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = 5$ and $b = 1$.

(ii) Another point, Q, is located in such a way that $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

Using this information, plot the point Q on the graph. [1]

Let $Q = (x, y)$

So, $\overrightarrow{OQ} = \begin{pmatrix} x \\ y \end{pmatrix}$

Now,

$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$$\begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 - x \\ 1 - y \end{pmatrix}$$

Comparing and equating corresponding entries gives:

$2 = 5 - x$ and $-4 = 1 - y$

$x = 5 - 2$ $\quad$ $y = 1 + 4$

$x = 3$ $\quad$ $y = 5$

Hence, $\overrightarrow{OQ} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and $Q = (3, 5)$

See grid above.

(iii) Determine $|\overrightarrow{QR}|$, the magnitude of $\overrightarrow{QR}$. [2]

$\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$$|\overrightarrow{QR}| = \sqrt{(2)^2 + (-4)^2}$$

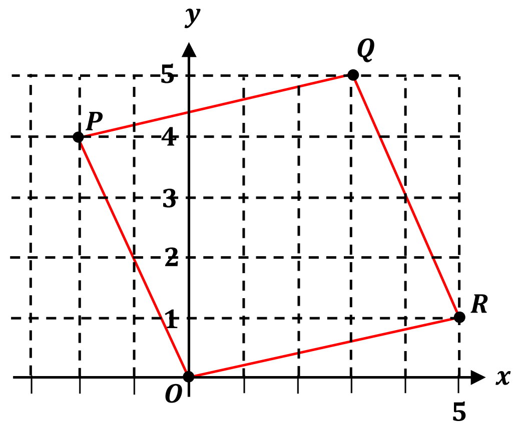
$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

$\therefore$ The magnitude of $\overrightarrow{QR}$ is $|\overrightarrow{QR}| = 2\sqrt{5}$ units.

(iv) Show, **by calculation**, that $OPQR$ is a parallelogram. [3]



The coordinates of $P = (-2, 4)$.

So, $\overrightarrow{OP} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

$\overrightarrow{PO} = -\overrightarrow{OP}$

$= -\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

And $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

Hence, $|\overrightarrow{QR}| = |\overrightarrow{PO}|$ and $\overrightarrow{QR}$ is parallel to $\overrightarrow{PO}$.

If one pair of opposite sides of a quadrilateral is both parallel and equal, then the quadrilateral is a parallelogram.

Q.E.D.

(b) Calculate the value of x and the value of y in the matrix equation below.

$$\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$ [3]

$$\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

$$\begin{pmatrix} (1 \times -4) + (5 \times 2) & (1 \times 1) + (5 \times 9) \\ (2 \times -4) + (y \times 2) & (2 \times 1) + (y \times 9) \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

$$\begin{pmatrix} -4 + 10 & 1 + 45 \\ -8 + 2y & 2 + 9y \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 46 \\ -8 + 2y & 2 + 9y \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$$

Comparing and equating corresponding entries gives:

$6 = x$ and $2 + 9y = 65$

$9y = 65 - 2$

$9y = 63$

$y = \frac{63}{9}$

$y = 7$

$\therefore x = 6$ and $y = 7$

(c) A transformation, T, is represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, maps $S(2, 5)$ onto $S'(5, 2)$. Describe fully the **single** transformation T. [2]

$T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

T maps $S(2, 5)$ onto $S'(5, 2)$.

Therefore, T represents a reflection in the line $y = x$.

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.