CSEC Mathematics

January 2024 – Paper 2

Solutions

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Express, as a single fraction in its **simplest** form,

$$1-\left(\frac{1}{30}+\frac{4}{15}\right)$$ [2]

$$1-\left(\frac{1}{30}+\frac{4}{15}\right)$$

$$=1-\left(\frac{1}{30}+\frac{8}{30}\right)$$

$$=1-\frac{9}{30}$$

$$=\frac{30}{30}-\frac{9}{30}$$

$$=\frac{21}{30}$$

$$=\frac{7}{10}$$

(b) A two-storey car park has a total of 1 020 parking spaces. At 06:30 hours one morning, $\frac{1}{30}$ of the 1 020 spaces are filled. During the next hour, no cars left the car park but another $\frac{4}{15}$ of the 1 020 spaces filled. Determine the number of parking spaces that are NOT filled at 07:30 hours. [1]

Fraction of spaces that are filled $=\frac{1}{30}+\frac{4}{15}$

$$=\frac{1}{30}+\frac{8}{30}$$

$$=\frac{9}{30}$$

Fraction of spaces that are not filled $= 1 - \frac{9}{30}$

$$= \frac{30}{30} - \frac{9}{30}$$

$$= \frac{21}{30}$$

$$= \frac{7}{10}$$

Number of parking spaces that are not filled $= \frac{7}{10} \times 1\ 020$

$= 714$ spaces

$\therefore$ The number of parking spaces that are not filled at 07:30 hours is 714 spaces.

(c) Of the 1 020 parking spaces, 20% are on the top level. How many parking spaces are on the top level? [1]

Number of parking spaces on the top level $= 20\%$ of $1\ 020$

$= \frac{20}{100} \times 1020$

$= 204$ spaces

$\therefore$ The number of parking spaces that are on the top level are 204 spaces.

(d) Some of the spaces are reserved for monthly paying customers. The ratio of reserved spaces to non-reserved spaces is 5: 7.

Calculate the number of **non-reserved** parking spaces. [1]

Reserved : Non-reserved

5 : 7

Total parts $= 5 + 7$

$= 12$ parts

Fraction of non-reserved parking spaces $= \frac{7}{12}$

Number of non-reserved parking spaces $= \frac{7}{12} \times 1020$

$= 595$ spaces

$\therefore$ The number of non-reserved parking spaces is 595 spaces.

(e) The cost for parking at the car park is shown in the table below.

Length of visit	**Cost ($)**
Under 30 minutes	Free
More than 30 minutes and up to 2 hours	$2.25
More than 2 hours and up to 4 hours	$5.50
More than 4 hours and up to 8 hours	$9.25
More than 8 hours and up to 24 hours	$15.00
One-week ticket	$40.00

(i) Mikayla leaves the car park at 18:30 hours and pays \$9.25. Determine the **earliest** time she could have arrived at the car park. [1]

If she paid \$9.25, then her length of visit at the car park was more than 4 hours and up to 8 hours.

Now,

$$\text{Time arrived} = 18:30 - 8:00$$
$$= 10:30$$

$\therefore$ The earliest time she could have arrived at the car park is 10:30 hours.

(ii) Dhanraj bought a weekly parking ticket for \$40. That week, he visited the car park five **different** times. The length of time he parked his car on each occasion is given below.

25 minutes $\quad 7\frac{1}{2}$ hours $\quad$ 11 hours $\quad 8\frac{3}{4}$ hours $\quad$ 8 hours

Show that Dhanraj saved \$8.50 by buying the weekly ticket. [3]

For 25 minutes, cost = \$0.

For $7\frac{1}{2}$ hours, cost = \$9.25.

For 11 hours, cost = \$15.00.

For $8\frac{3}{4}$ hours, cost = \$15.00.

For 8 hours, cost = \$9.25.

Total cost of each occasion $= 0 + 9.25 + 15 + 15 + 9.25$

$= \$48.50$

Cost of weekly parking ticket $= \$40$

Amount saved by buying the weekly ticket $= \$48.50 - \$40$

$= \$8.50$

$\therefore$ Dhanraj saved \$8.50 by buying the weekly ticket.

Q.E.D.

Total: 9 marks

2. (a) Simplify

$\frac{x^2+7x}{x^2-49}$ [2]

$\frac{x^2+7x}{x^2-49} = \frac{x(x+7)}{(x-7)(x+7)}$

$= \frac{x}{x-7}$

(b) Find the value of

(i) r, when $x^2 \times x^6 = x^r$ [1]

$x^2 \times x^6 = x^r$

$x^{2+6} = x^r$

$x^8 = x^r$

Since the bases are the same, we can equate the powers.

$8 = r$

$\therefore$ The value of $r = 8$.

(ii) s, when $s^3 = 8$ [1]

$s^3 = 8$

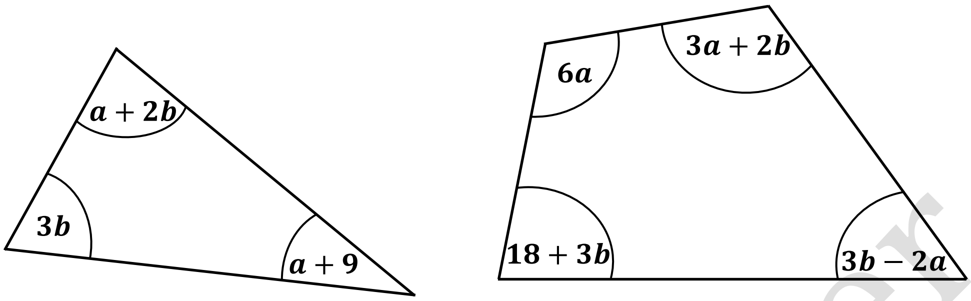
$s^3 = 2^3$

Since the powers are the same, we can equate the bases.

$s = 2$.

$\therefore$ The value of $s = 2$.

(c) The diagram below shows a triangle and a quadrilateral. All angles are in degrees and are written in terms of a and/or b.



(i) For the triangle, show that $2a + 5b = 171$. [1]

All angles in a triangle add up to 180°.

$(a + 2b) + (3b) + (a + 9) = 180$

$2a + 5b + 9 = 180$

$2a + 5b = 180 - 9$

$2a + 5b = 171$

$\therefore 2a + 5b = 171$

Q.E.D.

(ii) For the quadrilateral, show that $7a + 8b = 342$. [1]

All angles in a quadrilateral add up to 360°.

$(6a) + (3a + 2b) + (3b - 2a) + (18 + 3b) = 360$

$7a + 8b + 18 = 360$

$7a + 8b = 360 - 18$

$7a + 8b = 342$

$\therefore 7a + 8b = 342$

Q.E.D.

(iii) Solve the pair of simultaneous equations in (i) and (ii) to find the values of a and b. **Show all working.** [3]

$2a + 5b = 171$ $\rightarrow$ Equation 1

$7a + 8b = 342$ $\rightarrow$ Equation 2

Multiplying Equation 1 by 7 gives:

$14a + 35b = 1197$ $\rightarrow$ Equation 3

Multiplying Equation 2 by 2 gives:

$14a + 16b = 684$ $\rightarrow$ Equation 4

Equation 3 – Equation 4 gives:

$19b = 513$

$b = \frac{513}{19}$

$b = 27$

Substituting $b = 27$ into Equation 1 gives:

$2a + 5(27) = 171$

$2a + 135 = 171$

$2a = 171 - 135$

$2a = 36$

$a = \frac{36}{2}$

$a = 18$

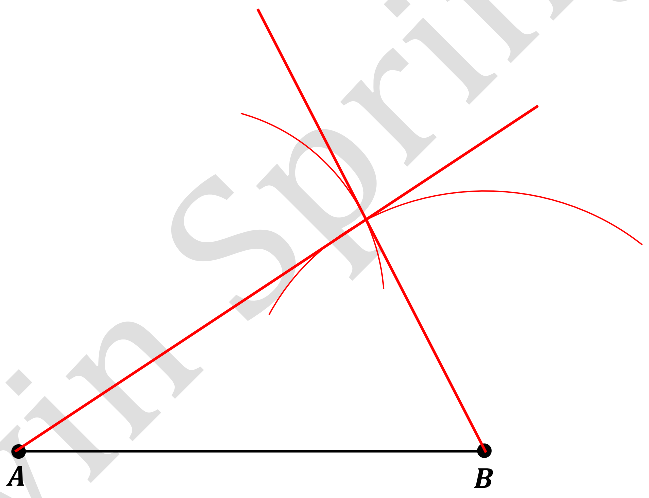
$\therefore$ The values are $a = 18$ and $b = 27$.

Total: 9 marks

3. (a) In triangle ABC, $AC = 8\ cm$ and $BC = 5\ cm$.

(i) Using a ruler and compasses only, construct triangle ABC. The line AB has been drawn for you. [2]

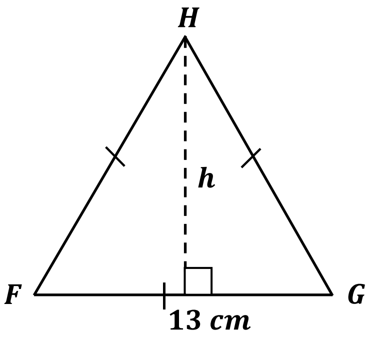
The triangle ABC is constructed below:



(ii) Measure and state the value of Angle BAC. [1]

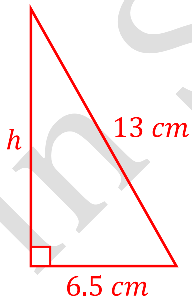
The value of Angle $BAC = 84°$

(b) The diagram below shows an equilateral triangle, FGH, whose base is $13\ cm$ and its height, h.



Calculate the value of h. [2]

Consider the right-angled triangle below:



Using Pythagoras' Theorem,

$$c^2 = a^2 + b^2$$

$$(13)^2 = (6.5)^2 + h^2$$

$$169 = 42.25 + h^2$$

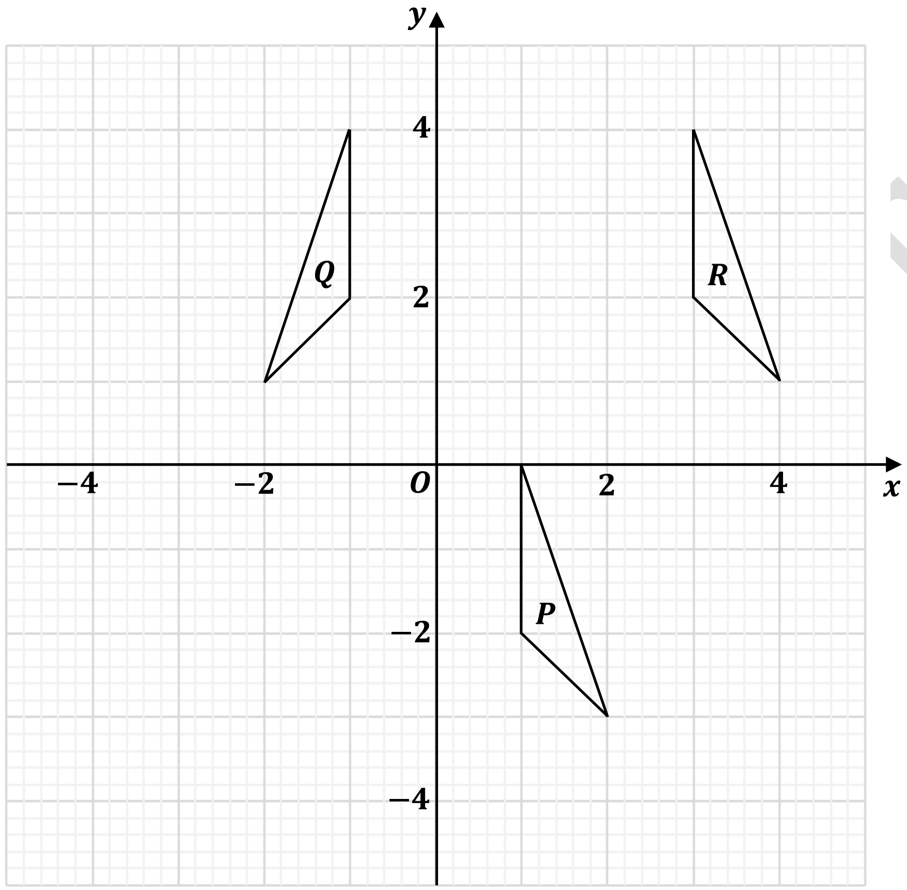
$$h^2 = 169 - 42.25$$

$$h^2 = 126.75$$

$$h = \sqrt{126.75}$$

$h = 11.3\ cm$ (to 3 significant figures)

(c) The diagram below shows 3 triangles, P, Q and R. Triangles Q and R are images of Triangle P after it undergoes a double transformation.



Describe fully the single transformation that maps Triangle

(i) P onto Triangle R [2]

The transformation that maps Triangle P onto Triangle R is a translation by a vector $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

(ii) R onto Triangle Q [2]

The transformation that maps Triangle R onto Triangle Q is a reflection in the line $x = 1$.

Total: 9 marks

4. Consider the following functions.

$$f(x) = 2 - 5x \text{ and } h(x) = 5^x.$$

(a) Calculate the value of

(i) $f(4)$ [1]

$f(x) = 2 - 5x$

$f(4) = 2 - 5(4)$

$= 2 - 20$

$= -18$

(ii) $h(0)$ [1]

$h(x) = 5^x$

$h(0) = 5^0$

$= 1$

(iii) $fh(-2)$ [2]

$h(x) = 5^x$

$h(-2) = 5^{-2}$

$= \frac{1}{5^2}$

$= \frac{1}{25}$

$$f(x) = 2 - 5x$$

$$fh(-2) = f\left(\frac{1}{25}\right)$$

$$= 2 - 5\left(\frac{1}{25}\right)$$

$$= 2 - \frac{1}{5}$$

$$= 1\frac{4}{5}$$

$$= \frac{9}{5}$$

(b) Find $f^{-1}(x)$. [2]

$f(x) = 2 - 5x$

Let $y = f(x)$.

$y = 2 - 5x$

Interchange the variables x and y.

$x = 2 - 5y$

Making y the subject of the formula.

$5y = 2 - x$

$y = \frac{2-x}{5}$

$\therefore f^{-1}(x) = \frac{2-x}{5}$

(c) Given that $ff(x) = a + bx$, determine the values of a and b. [3]

$$f(x) = 2 - 5x$$

$$ff(x) = f(2 - 5x)$$

$$= 2 - 5(2 - 5x)$$

$$= 2 - 10 + 25x$$

$$= -8 + 25x$$ which is in the form $ff(x) = a + bx$,

where $a = -8$ and $b = 25$.

∴ The values of a and b are $a = -8$ and $b = 25$.

Total: 9 marks

5. The mass, m, in kilograms, of 120 newborns at a hospital is recorded in the table below.

Mass (m kg)	**Frequency (f)**
$2.6 < m \leq 3.5$	7
$3.5 < m \leq 4.4$	18
$4.4 < m \leq 5.3$	30
$5.3 < m \leq 6.2$	29
$6.2 < m \leq 7.1$	28
$7.1 < m \leq 8.0$	8

(a) (i) State the modal class. [1]

The modal class is $4.4 < m \leq 5.3$.

(ii) Complete the table below and calculate an estimate of the mean mass of the 120 newborns. [3]

Mass (m kg)	**Midpoint (x)**	**Frequency (f)**	**Frequency × Midpoint (fx)**
$2.6 < m \leq 3.5$	3.05	7	21.35
$3.5 < m \leq 4.4$	3.95	18	71.1
$4.4 < m \leq 5.3$	4.85	30	145.5
$5.3 < m \leq 6.2$	5.75	29	166.75
$6.2 < m \leq 7.1$	6.65	28	186.2
$7.1 < m \leq 8.0$	7.55	8	60.4

Consider the interval $5.3 < m \leq 6.2$.

$$\text{Midpoint, } x = \frac{5.3+6.2}{2}$$

$$= \frac{11.5}{2}$$

$$= 5.75$$

$$fx = 29 \times 5.75$$

$$= 166.75$$

Consider the interval $7.1 < m \leq 8.0$.

$$\text{Midpoint, } x = \frac{7.1+8.0}{2}$$

$$= \frac{15.1}{2}$$

$$= 7.55$$

$$fx = 8 \times 7.55$$

$$= 60.4$$

Now,

$$\text{Mean, } \overline{x} = \frac{\sum fx}{\sum x}$$

$$= \frac{651.3}{120}$$

$$= 5.43\ kg \qquad \text{(to 3 significant figures)}$$

$\therefore$ The mean mass of the 120 newborns is $5.43\ kg$.

(iii) One newborn is chosen at random from the hospital. Find the probability that the newborn has a mass greater than 5.3 kg. [1]

Number of newborns that has a mass greater than 5.3 kg $= 29 + 28 + 8$

$= 65$ newborns

Probability $= \frac{Number\ of\ favourable\ outcomes}{Total\ outcomes}$

$= \frac{65}{120}$

$= \frac{13}{24}$

$\therefore$ The probability that the newborn has a mass greater than 5.3 kg is $\frac{13}{24}$.

(b) (i) Complete the cumulative frequency table shown below. [1]

Mass ($m\ kg$)	Cumulative Frequency
$m \leq 3.5$	7
$m \leq 4.4$	25
$m \leq 5.3$	55
$m \leq 6.2$	84
$m \leq 7.1$	112
$m \leq 8.0$	120

For $m \leq 6.2$, Cumulative Frequency $= 55 + 29$

$= 84$

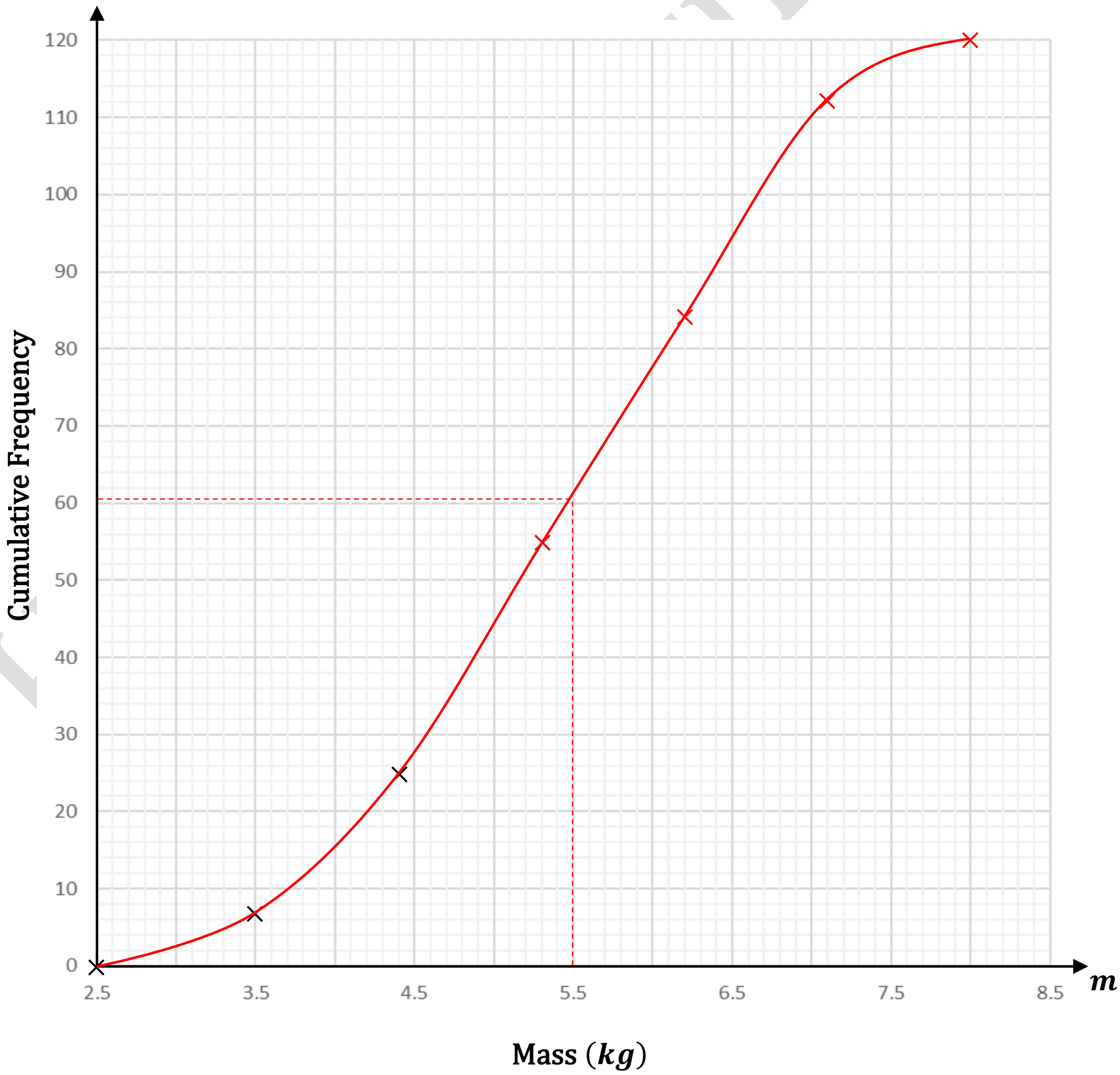
For $m \leq 7.1$, Cumulative Frequency $= 84 + 28$

$= 112$

For $m \leq 8.0$, Cumulative Frequency $= 112 + 8$

$= 120$

(ii) On the grid below, draw a cumulative frequency curve to show the information in the table (b)(i) **on page 16.** [1]



(iii) Use your diagram to find an estimate for the median mass of the newborns. [1]

The median value occurs at $\frac{n+1}{2} = \frac{120+1}{2} = \frac{121}{2} = 60.5^{\text{th}}$ value.

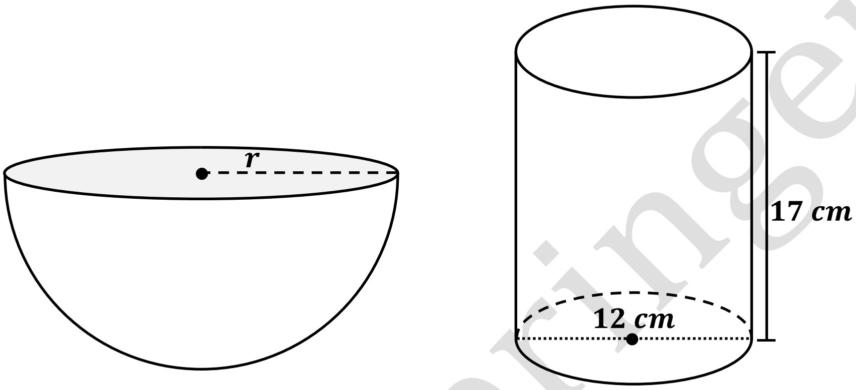
At the 60.5^{th} value, mass $= 5.5\ kg$.

$\therefore$ The median mass of the newborns is $5.5\ kg$.

Total: 9 marks

6. [In this question, use $\pi = \frac{22}{7}$.]

The diagram below shows a hemispherical bowl and a cylindrical tin. The diameter of the cylindrical tin is 12 cm, the height is 17 cm and the radius of the bowl is r.



(a) (i) Show that the volume of the cylindrical tin is 1 923 cm^3, correct to 4 significant figures. [2]

Radius of cylindrical tin $= \frac{Diameter}{2}$

$= \frac{12}{2}$

$= 6\ cm$

Volume of the cylindrical tin $= \pi r^2 h$

$= \frac{22}{7} \times (6)^2 \times 17$

$= 1923\ cm^3$ (to 4 significant figures)

$\therefore$ The volume of cylindrical tin is 1 923 cm^3, correct to 4 significant figures.

Q.E.D.

(ii) The bowl is completely filled with soup. When all the soup is poured into the empty cylindrical tin, 90% of the volume of the tin is filled. Calculate the radius of the bowl.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

Volume of the bowl = 90% of Volume of the cylindrical tin

$$= \frac{90}{100} \times 1923$$

$$= 1730.7\ cm^3$$

Volume of the bowl $= \frac{1}{2} \times$ Volume of a sphere

$$1730.7 = \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$1730.7 = \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$1730.7 = \frac{44}{21} \times r^3$$

$$r^3 = 1730.7 \times \frac{21}{44}$$

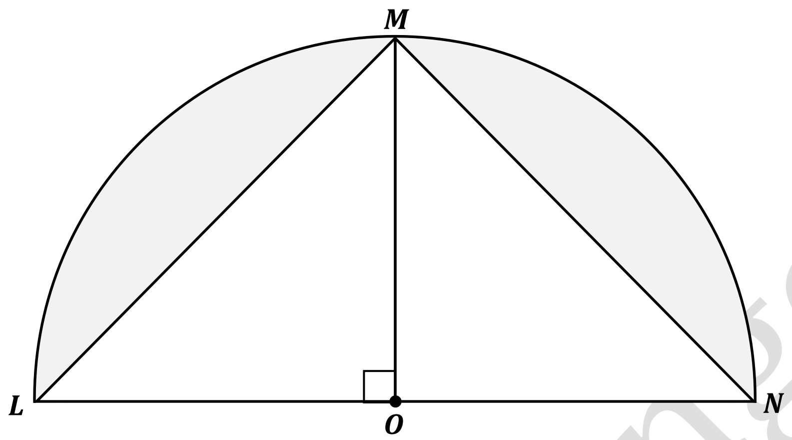
$$r^3 = 826.0159091$$

$$r = \sqrt[3]{826.0159091}$$

$$r = 9.38\ cm \qquad \text{(to 3 significant figures)}$$

$\therefore$ The radius of the bowl is 9.38 cm.

(b) In the diagram below, points L, M and N are on the circumference of a semicircle, with centre O, and a radius of 18 cm.



Calculate the TOTAL area of the shaded sections in the diagram. [4]

Area of a semicircle $= \frac{1}{2} \times$ Area of a circle

$$= \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (18)^2$$

$$= \frac{3564}{7}$$

$= 509.14\ cm^2$ (to 2 decimal places)

Area of triangle $LMN = \frac{1}{2} bh$

$$= \frac{1}{2} \times (2 \times 18) \times 18$$

$$= 324\ cm^2$$

Now,

Area of the shaded sections $=$ Area of a semicircle $-$ Area of triangle LMN

$$= \frac{3564}{7} - 324$$

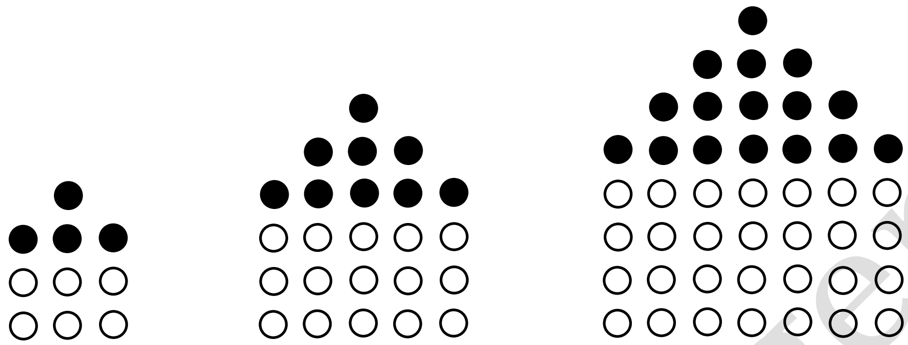
$$= 185.14\ cm^2 \qquad \text{(to 2 decimal places)}$$

$\therefore$ The total area of the shaded sections in the diagram is $185.14\ cm^2$.

Total: 9 marks

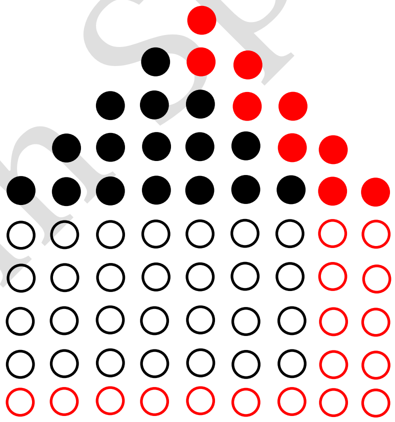
7. The diagram below shows the first 3 shapes in a sequence, which forms a pattern.

Each shape is made using a set of small white counters and black counters.



Shape 1 Shape 2 Shape 3

(a) Complete the diagram below to represent Shape 4. [2]



(b) The number of white counters, W, the number of black counters, B, and the total number of counters, T, that form each shape follow a pattern. The values for W, B and T for the first 3 shapes are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.

Complete the rows marked (i), (ii) and (iii) in the table below.

	Shape Number (S)	Number of White Counters (W)	Number of Black Counters (B)	Total Number of Counters (T)	
	1	$(1+1)[2(1)+1]=6$	$(1+1)^2=4$	10	
	2	$(2+1)[2(2)+1]=15$	$(2+1)^2=9$	24	
	3	$(3+1)[2(3)+1]=28$	$(3+1)^2=16$	44	
(i)	4	**45**	$(4+1)^2=25$	70	[2]
	⋮	⋮	⋮	⋮	
(ii)	11	$(11+1)[2(11)+1]=276$	**144**	**420**	[2]
	⋮	⋮	⋮	⋮	
(iii)	n	$(n+1)[2n+1]$	$(n+1)^2$	$3n^2+5n+2$	[2]

For the nth term,

$W=(n+1)(2n+1)$

$B=(n+1)^2$

When $B=144$.

$(n+1)^2=144$

$n+1=12$

$n=11$

When $n = 11$,

$W = (11 + 1)[2(11) + 1]$

$= (12)(23)$

$= 276$

(c) The expression for the total number of counters, $T = W + B$, in Shape S is given by $T = aS^2 + bS + 2$, where a and b are both positive integers.

By substituting suitable values for S, show that the **total** number of counters in Shape 1 and Shape 3, in terms of a and b, is represented by the equations

$$a + b = 8 \text{ and}$$

$$3a + b = 14 \text{ respectively.}$$ [2]

Total number of counters, $T = 3n^2 + 5n + 2$.

Comparing it with $T = aS^2 + bS + 2$, then $a = 3$ and $b = 5$.

For Shape 1, let $S = 1$.

From the table, $T = 10$.

Then, we have,

$T = a(1)^2 + b(1) + 2$

$10 = a + b + 2$

$10 - 2 = a + b$

$8 = a + b$

$a + b = 8$

For Shape 3, let $S = 3$.

From the table, $T = 44$.

Then, we have,

$$T = a(3)^2 + b(3) + 2$$

$$44 = 9a + 3b + 2$$

$$44 - 2 = 9a + 3b$$

$$42 = 9a + 3b$$

$(\div 3)$

$$14 = 3a + b$$

$$3a + b = 14$$

$\therefore$ The total number of counters in Shape 1 and Shape 3, in terms of a and b, is represented by the equations $a + b = 8$ and $3a + b = 14$ respectively.

Q.E.D.

Total: 10 marks

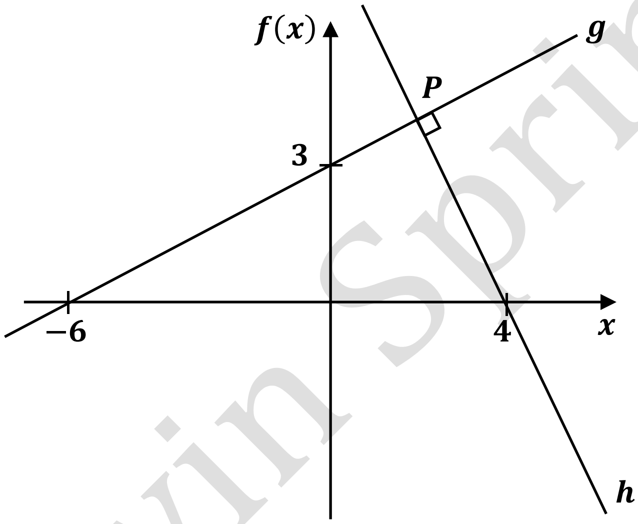
SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The diagram below shows the graphs of two functions on the same pair of axes.

The lines g and h are perpendicular.



Determine the

(i) equation that represents the line g [2]

Points on line g are: $(-6, 0)$ and $(0, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-0}{0-(-6)}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

The gradient, $m = \frac{1}{2}$ and the y-intercept is $c = 3$.

$\therefore$ The equation of the line g is $y = \frac{1}{2}x + 3$.

(ii) equation that represents the line h [2]

The line g is perpendicular to the line h.

Gradient of line $g = \frac{1}{2}$

Gradient of line $h = -2$

Substituting $m = -2$ and point $(4, 0)$ into $y - y_1 = m(x - x_1)$ gives:

$y - 0 = -2(x - 4)$

$y = -2x + 8$

$\therefore$ The equation of the line h is $y = -2x + 8$.

(iii) coordinates of the point P. **Show all working.** [2]

The coordinates of the point P can be found the solving the following pair of equations simultaneously.

$y = \frac{1}{2}x + 3 \quad \rightarrow$ Equation 1

$y = -2x + 8 \quad \rightarrow$ Equation 2

Equating both equations gives:

$\frac{1}{2}x + 3 = -2x + 8$

$2x + \frac{1}{2}x = 8 - 3$

$\frac{5}{2}x = 5$

$x = 5 \div \frac{5}{2}$

$x = 5 \times \frac{2}{5}$

$x = 2$

Substituting $x = 2$ into Equation 2 gives:

$y = -2(2) + 8$

$= -4 + 8$

$= 4$

$\therefore$ The coordinates of the point P are $(2, 4)$.

(b) (i) Write $4x^2 - 24x + 31$ in the form $a(x + h)^2 + k$. [2]

$4x^2 - 24x + 31$ is of the form $ax^2 + bc + c$,

where $a = 4, b = -24$ and $c = 31$.

$h = \frac{b}{2a}$

$= \frac{-24}{2(4)}$

$= \frac{-24}{8}$

$= -3$

$$k = \frac{4ac - b^2}{4a}$$

$$= \frac{4(4)(31) - (-24)^2}{4(4)}$$

$$= \frac{496 - 576}{86}$$

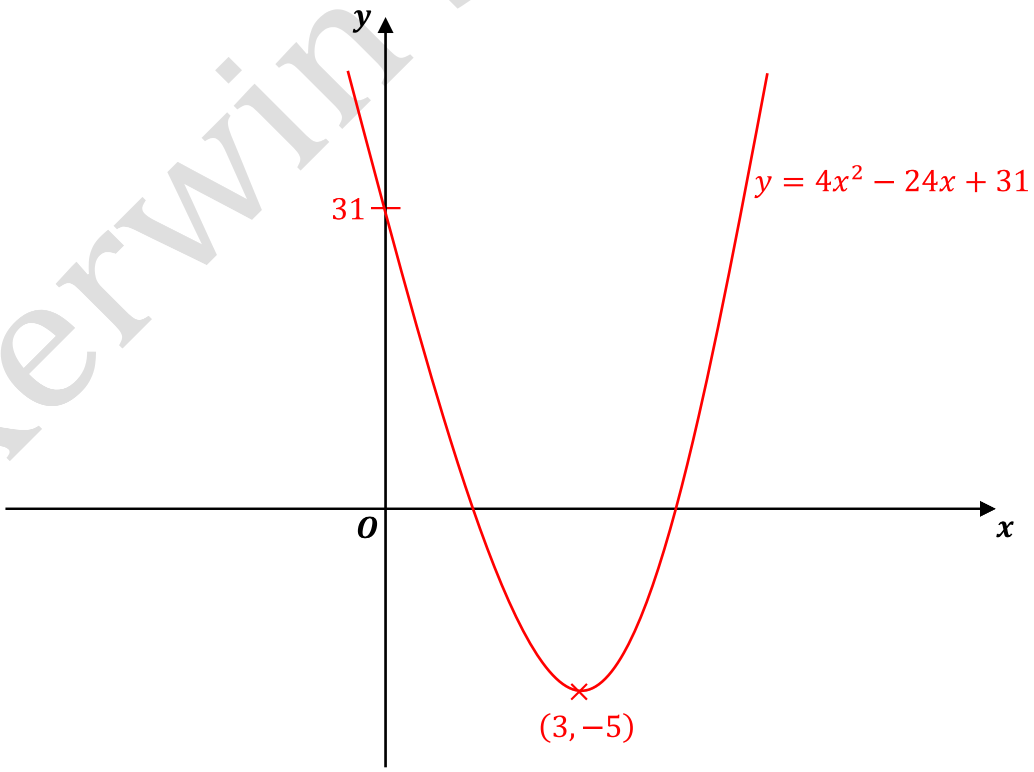
$$= \frac{-80}{16}$$

$$= -5$$

$\therefore 4x^2 - 24x + 31 = 4(x-3)^2 - 5$ which is in the form $a(x+h)^2 + k$,

where $a = 4, h = -3$ and $k = -5$.

(ii) On the axes below, sketch the graph of $4x^2 - 24x + 31$, indicating the coordinates of the maximum/minimum point and the y-intercept. [3]



The y-intercept of $4x^2 - 24x + 31$ is $c = 31$.

The minimum point of $4(x-3)^2 - 5$ is $(3, -5)$.

(iii) State the equation of the axis of symmetry. [1]

$x = -h$

$x = -(-3)$

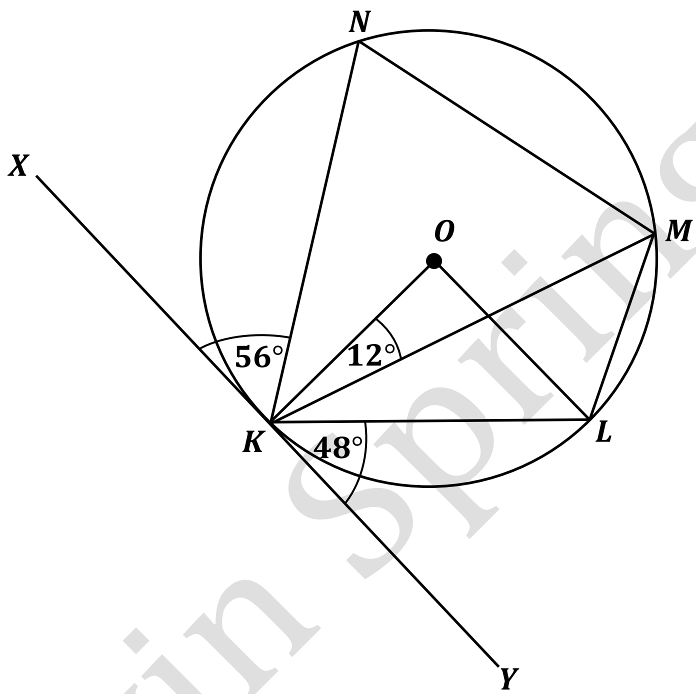
$x = 3$

$\therefore$ The equation of the axis of symmetry is $x = 3$.

Total: 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) K, L, M and N are points on the circumference of a circle with centre O. XY is a tangent to the circle at K. Angle $LKY = 48°$, $MKO = 12°$ and Angle $XKN = 56°$.



(i) Find the value of Angle KOL, giving reasons for EACH step of your work. [2]

The tangent at XY makes a right angle at K with the radius KO.

Angle $MKL = 90° - 12° - 48°$

$= 90° - 60°$

$= 30°$

Angle $OKL = 12° + 30°$

$= 42°$

Triangle OKL is an isosceles triangle since OK and OL are both radii of the same circle and are equal.

So, Angle $OLK = 42°$.

All angles in a triangle add up to 180°.

Angle OKL + Angle OLK + Angle $KOL = 180°$

$42° + 42°$ + Angle $KOL = 180°$

Angle $KOL = 180° - 42° - 42°$

$= 96°$

∴ The value of Angle $KOL = 96°$.

(ii) Find the value of EACH of the following angles.

(a) Angle LMN [1]

The tangent at XY makes a right angle at K with the radius KO.

Angle $OKN = 90° - 56°$

$= 34°$

Opposite angles in a cyclic quadrilateral, $LMNK$, sum to 180°.

$34° + 12° + 40°$ + Angle $LMN = 180°$

Angle $LMN = 180° - 34° - 12° - 40°$

$= 104°$

∴ The value of Angle $LMN = 104°$.

(b) Angle KLO [1]

Triangle OKL is an isosceles triangle since OK and OL are both radii of the same circle and are equal.

So, Angle KLO = Angle OKL

$= 42°$

$\therefore$ The value of Angle $KLO = 42°$.

(c) Angle MLK [1]

The angle at the centre of a circle is twice the angle at the circumference of the circle from the same chord.

Angle $KML = \frac{1}{2} \times$ Angle KOL

$= \frac{1}{2}(96°)$

$= 48°$

Angles in a triangle add up to 180°.

$30° + 48° +$ Angle $MLK = 180°$

Angle $MLK = 180° - 30° - 48°$

$= 102°$

$\therefore$ The value of Angle $MLK = 102°$.

(d) Angle KNM [1]

Opposite angles in a cyclic quadrilateral, $LMNK$, sum to $180°$.

$$\text{Angle } KNM + \text{Angle } KLM = 180°$$

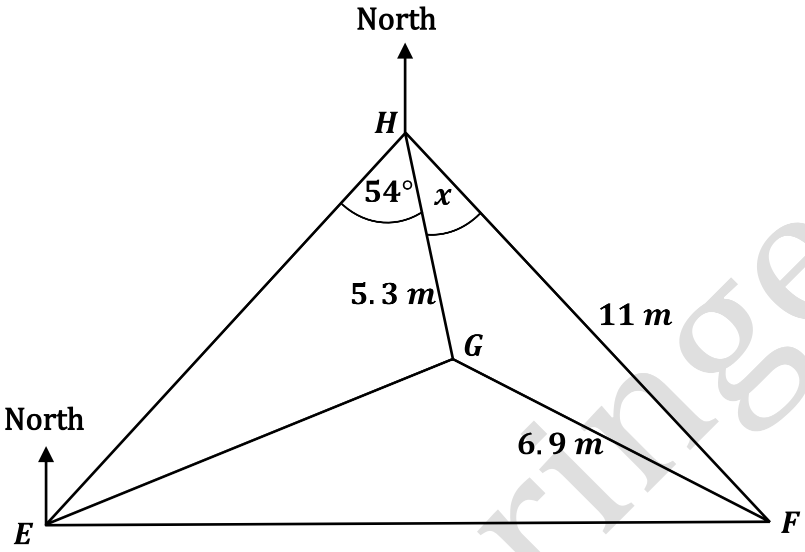
$$\text{Angle } KNM + 102° = 180°$$

$$\text{Angle } KNM = 180° - 102°$$

$$= 78°$$

$\therefore$ The value of Angle $KNM = 78°$.

(b) E, F, G and H are 4 points on level ground. The diagram below gives information on the distances and angles between the points.



(i) Show that the value of x is 29.5°, correct to 1 decimal place. [2]

Using the cosine rule,

$$(GF)^2 = (GH)^2 + (HF)^2 - 2(GH)(HF)\cos x$$

$$(6.9)^2 = (5.3)^2 + (11)^2 - 2(5.3)(11)\cos x$$

$$47.61 = 149.09 - 116.6\cos x$$

$$47.61 - 149.09 = -116.6\cos x$$

$$-101.48 = -116.6\cos x$$

$$\frac{-101.48}{-116.6} = \cos x$$

$$0.870 = \cos x$$

$$x = \cos^{-1}(0.870)$$

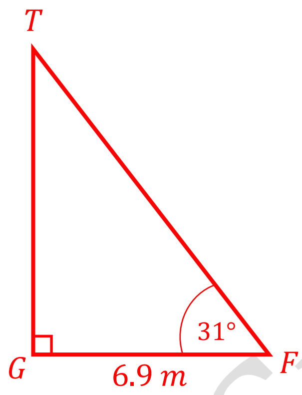
$$x = 29.5° \text{ (to 1 decimal place)}$$

$\therefore$ The value of x is 29.5°, correct to 1 decimal place. Q.E.D.

(ii) A vertical tower, GT, is constructed at the point G and is pivoted to the ground at the points E, F and H using pieces of wire. The angle of elevation of the top of the tower, T, from the point F is 31°.

What length of wire was used to secure Point T to Point F? [2]

Consider the diagram below:



$$\cos\theta = \frac{adj}{hyp}$$

$$\cos 31° = \frac{6.9}{TF}$$

$$TF = \frac{6.9}{\cos 31°}$$

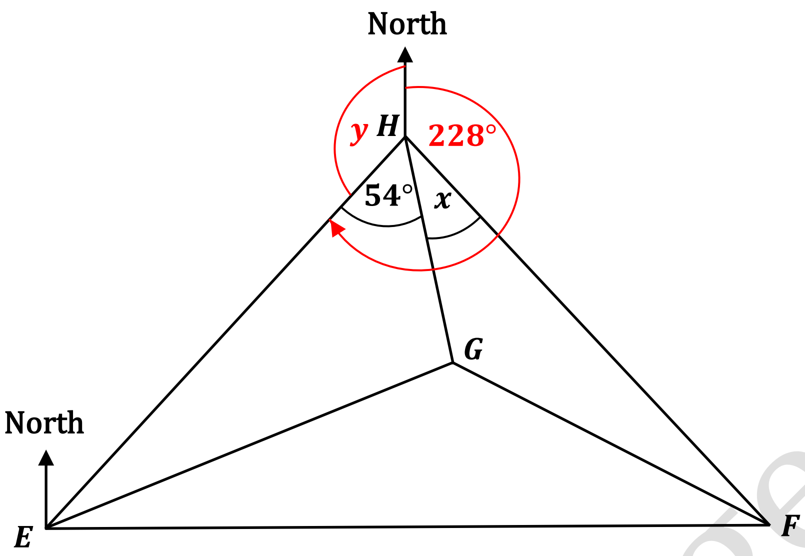
$$TF = 8.05\ m \qquad \text{(to 3 significant figures)}$$

∴ The length of wire used to secure Point T to Point F is $8.05\ m$.

(iii) The bearing of E from H is 228°. Find the bearing of

(a) H from E [1]

Consider the diagram below:



Angle $y = 360° - 228°$

$= 132°$

The bearing of H from E and Angle y are co-interior angles.

So, we have,

Bearing of H from E $= 180° - 132°$

$= 48°$

(b) G from H [1]

Bearing of G from H $= 360° - (132° + 54°)$

$= 360° - 186°$

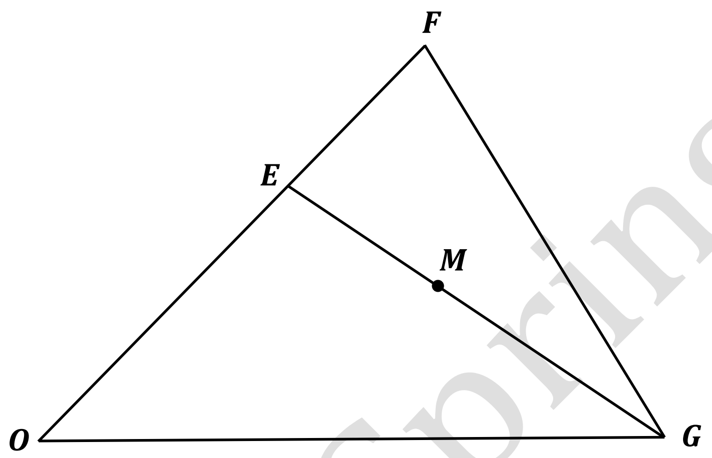
$= 174°$

Total: 12 marks

VECTORS AND MATRICES

10. (a) In the diagram below, O is the origin, $OE = 2EF$ and M is the midpoint of EG.

$\overrightarrow{OG} = \boldsymbol{c}$ and $\overrightarrow{OF} = \boldsymbol{d}$.



Find in terms of $\boldsymbol{c}$ and $\boldsymbol{d}$, in its simplest form,

(i) $\overrightarrow{FG}$ [1]

Using the triangle law,

$\overrightarrow{FG} = \overrightarrow{OG} - \overrightarrow{OF}$

$= \boldsymbol{c} - \boldsymbol{d}$

(ii) $\overrightarrow{EG}$ [2]

$\overrightarrow{OE} + \overrightarrow{EF} = \overrightarrow{OF}$

Since $\overrightarrow{OE} = 2\overrightarrow{EF}$, then

$$2\overrightarrow{EF} + \overrightarrow{EF} = \overrightarrow{OF}$$

$$3\overrightarrow{EF} = \overrightarrow{OF}$$

$$\overrightarrow{EF} = \frac{1}{3}\overrightarrow{OF}$$

Using triangle law,

$$\overrightarrow{EG} = \overrightarrow{EF} + \overrightarrow{FG}$$

$$= \frac{1}{3}\overrightarrow{OF} + \overrightarrow{FG}$$

$$= \frac{1}{3}\boldsymbol{d} + (\boldsymbol{c} - \boldsymbol{d})$$

$$= \frac{1}{3}\boldsymbol{d} + \boldsymbol{c} - \boldsymbol{d}$$

$$= \boldsymbol{c} - \frac{2}{3}\boldsymbol{d}$$

(iii) $\overrightarrow{OM}$ [2]

Since M is the midpoint of $\overrightarrow{EG}$, then

$$\overrightarrow{MG} = \frac{1}{2}\overrightarrow{EG}$$

Now,

$$\overrightarrow{GM} = -\overrightarrow{MG}$$

$$= -\frac{1}{2}\overrightarrow{EG}$$

$$= -\frac{1}{2}\left(\boldsymbol{c} - \frac{2}{3}\boldsymbol{d}\right)$$

$$= -\frac{1}{2}\boldsymbol{c} + \frac{1}{3}\boldsymbol{d}$$

Using triangle law,

$$\overrightarrow{OM} = \overrightarrow{OG} + \overrightarrow{GM}$$

$$= \boldsymbol{c} - \frac{1}{2}\boldsymbol{c} + \frac{1}{3}\boldsymbol{d}$$

$$= \frac{1}{2}\boldsymbol{c} + \frac{1}{3}\boldsymbol{d}$$

(b) The matrices P, Q and R are given below, in terms of the scalar constants a, b and c, as

$$P = \begin{pmatrix} 3 & -9 \\ a & 7 \end{pmatrix}, \qquad Q = \begin{pmatrix} -1 & b \\ -4 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}$$

Given that $P + Q = R$, find the value of a, b and c. [3]

$$P + Q = R$$

$$\begin{pmatrix} 3 & -9 \\ a & 7 \end{pmatrix} + \begin{pmatrix} -1 & b \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 3-1 & -9+b \\ a-4 & 7+1 \end{pmatrix} = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -9+b \\ a-4 & 8 \end{pmatrix} = \begin{pmatrix} c & -3 \\ -4 & 8 \end{pmatrix}$$

Comparing and equating corresponding entries gives:

$2 = c$ $\qquad -9 + b = -3$ $\qquad a - 4 = -4$

$c = 2$ $\qquad b = -3 + 9$ $\qquad a = -4 + 4$

$b = 6$ $\qquad a = 0$

$\therefore a = 0, b = 6$ and $c = 2$

(c) Solve the following pair of simultaneous equations using a matrix method.

$$5x - 2y = 44$$

$$2x + 3y = 10$$

[4]

$5x - 2y = 44$

$2x + 3y = 10$

In matrix form,

$$\begin{pmatrix} 5 & -2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 44 \\ 10 \end{pmatrix}$$

which is in the form $AX = B$

$$X = A^{-1}B$$

Let $A = \begin{pmatrix} 5 & -2 \\ 2 & 3 \end{pmatrix}$.

$\det(A) = ad - bc$

$= (5)(3) - (-2)(2)$

$= 15 - (-4)$

$= 15 + 4$

$= 19$

$adj(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$adj(A) = \begin{pmatrix} 3 & 2 \\ -2 & 5 \end{pmatrix}$

$A^{-1} = \frac{1}{\det(A)} \times adj(A)$

$= \frac{1}{14}\begin{pmatrix} 3 & 2 \\ -2 & 5 \end{pmatrix}$

Now,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 3 & 2 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 44 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19}\begin{pmatrix} (3 \times 44) + (2 \times 10) \\ (-2 \times 44) + (5 \times 10) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 132 + 20 \\ -88 + 50 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 152 \\ -38 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$$

$\therefore x = 8$ and $y = -2$

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.