THE STUDENT HUB

CSEC Mathematics

June 2010 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Determine the EXACT value of:

(i) $$\frac{1\frac{1}{2} - \frac{2}{5}}{4\frac{2}{5} \times \frac{3}{4}}$$ [3]

We first work out the numerator and denominator separately.

$$\text{Numerator} = 1\frac{1}{2} - \frac{2}{5}$$

$$= \frac{3}{2} - \frac{2}{5}$$

$$= \frac{15-4}{10}$$

$$= \frac{11}{10}$$

$$\text{Denominator} = 4\frac{2}{5} \times \frac{3}{4}$$

$$= \frac{22}{5} \times \frac{3}{4}$$

$$= \frac{66}{20}$$

$$= \frac{33}{10}$$

Now,

$$\text{Numerator} \div \text{Denominator} = \frac{11}{10} \div \frac{33}{10}$$

$$= \frac{11}{10} \times \frac{10}{33}$$

$$= \frac{11}{33}$$

$$= \frac{1}{3}$$

$$\therefore \frac{1\frac{1}{2} - \frac{2}{5}}{4\frac{2}{5} \times \frac{3}{4}} = \frac{1}{3}$$

(ii) $2.5^2 - \frac{2.89}{17}$

giving your answer to 2 significant figures [3]

Using a calculator,

$$2.5^2 - \frac{2.89}{17} = 6.25 - 0.17$$

$$= 6.08$$

$$= 6.1 \text{ (to 2 significant figures)}$$

(b) Mrs. Jack bought 150 T-shirts for $1 920 from a factory.

(i) Calculate the cost of ONE T-shirt. [1]

150 T-shirts = $1920

$1 \text{ T-shirt} = \frac{\$1920}{150}$

$= \$12.80$

∴ The cost of 1 T-shirt is $12.80.

The T-shirts are sold at $19.99 each.

Calculate

(ii) the amount of money Mrs. Jack received after selling ALL of the T-shirts. [1]

1 T-shirt = $19.99

150 T-shirts = $19.99 × 150

150 T-shirts = $2998.50

∴ Mrs. Jack will receive $2998.50 after selling all of the T-shirts.

(iii) the TOTAL profit made [1]

Profit = Selling Price – Cost Price

$= \$2998.50 - \$1920$

$= \$1078.50$

∴ The total profit made is \$1078.50.

(iv) the profit made as a percentage of the cost price, giving your answer correct to the nearest whole number. [2]

$$\text{Percentage profit} = \frac{Profit}{Cost\ Price} \times 100\%$$

$$= \frac{1078.50}{1920} \times 100\%$$

$$\text{Percentage profit} = 56.2\%$$

$$\text{Percentage profit} = 56\% \qquad \text{(to the nearest whole number)}$$

∴ The profit made as a percentage of the cost price is 56%.

Total: 11 marks

2. (a) Given that $a = -1, b = 2$ and $c = -3$, find the value of:

(i) $a + b + c$ [1]

$$a + b + c = (-1) + 2 + (-3)$$
$$= -1 + 2 - 3$$
$$= 2 - 4$$
$$= -2$$

(ii) $b^2 - c^2$ [1]

$$b^2 - c^2 = (2)^2 - (-3)^2$$
$$= 4 - 9$$
$$= -5$$

(b) Write the following phrases as algebraic expressions:

(i) seven times the sum of x and y [1]

Algebraic expression: $7(x + y)$

(ii) the product of TWO consecutive numbers when the smaller number is y [1]

If the smaller number is y, then the next larger, consecutive number is $y + 1$.

Algebraic expression: $y(y + 1)$

(c) Solve the pair of simultaneous equations: [3]

$2x + y = 7$

$x - 2y = 1$

$2x + y = 7 \quad \rightarrow$ Equation 1

$x - 2y = 1 \quad \rightarrow$ Equation 2

Multiplying Equation 2 by 2 gives:

$2x - 4y = 2 \quad \rightarrow$ Equation 3

Equation 1 – Equation 3 gives:

$5y = 5$

$y = \frac{5}{5}$

$y = 1$

Substituting $y = 1$ into Equation 2 gives:

$x - 2(1) = 1$

$x - 2 = 1$

$x = 1 + 2$

$x = 3$

$\therefore x = 3$ and $y = 1$

(d) Factorise completely:

(i) $4y^2 - z^2$ [1]

$4y^2 - z^2 = (2y + z)(2y - z)$ → difference of two squares

(ii) $2ax - 2ay - bx + by$ [2]

$$\begin{aligned} & 2ax - 2ay - bx + by \\ &= 2a(x - y) - b(x - y) \\ &= (2a - b)(x - y) \end{aligned}$$

(iii) $3x^2 + 10x - 8$ [2]

$$\begin{aligned} & 3x^2 + 10x - 8 \\ &= 3x^2 + 12x - 2x - 8 \\ &= 3x(x + 4) - 2(x + 4) \\ &= (3x - 2)(x + 4) \end{aligned}$$

Total: 12 marks

3. (a) A survey was conducted among 40 tourists. The results were:

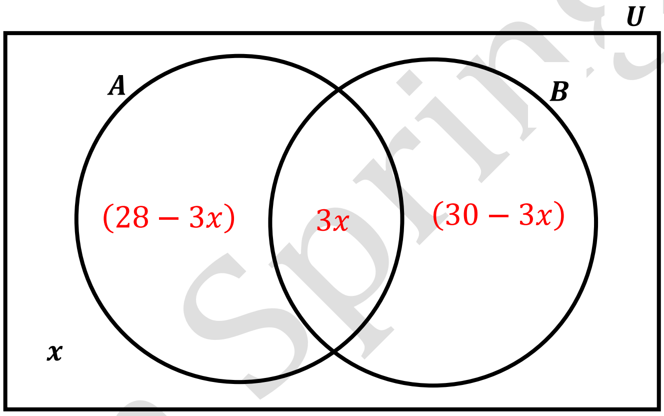
28 visited Antigua (A)

30 visited Barbados (B)

$3x$ visited both Antigua and Barbados

x visited neither Antigua nor Barbados

(i) Copy and complete the Venn diagram below to represent the given information above. [2]



(ii) Write an expression, in x, to represent the TOTAL number of tourists in the survey. [2]

$$\text{Total number of tourists} = (28 - 3x) + 3x + (30 - 3x) + x$$
$$= 28 - 3x + 3x + 30 - 3x + x$$
$$= 58 - 2x$$

$\therefore$ An expression to represent the total number of tourists in the survey is $58 - 2x$.

(iii) Calculate the value of x. [2]

The survey was conducted among 40 tourists.

$58 - 2x = 40$

$2x = 58 - 40$

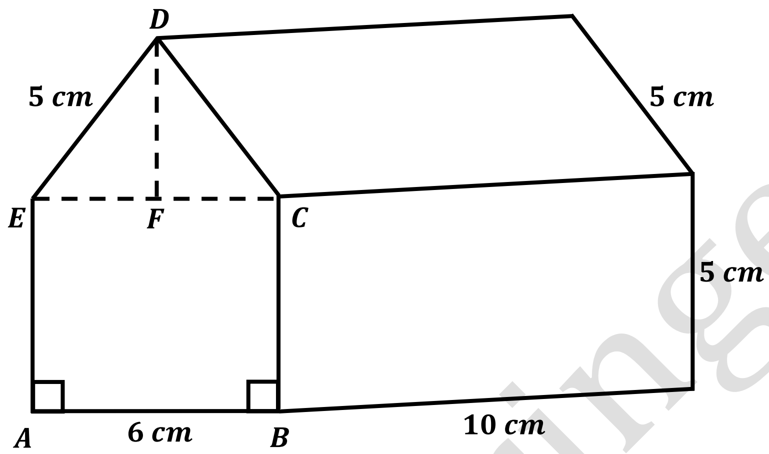
$2x = 18$

$x = \frac{18}{2}$

$x = 9$

$\therefore$ The value of $x = 9$.

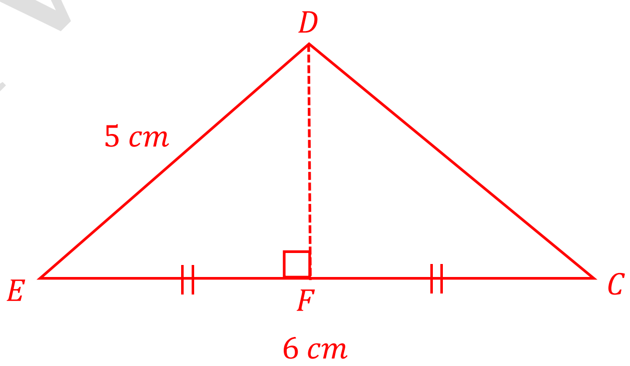
(b) The diagram below, **not drawn to scale**, shows a wooden toy in the shape of a prism, with cross section $ABCDE$. F is the midpoint of EC, and $\angle BAE = \angle CBA = 90°$.



Calculate

(i) the length of EF [1]

Consider the triangle DEC.



F is the midpoint of EC.

$\therefore EF = \frac{6}{2}$

$= 3\ cm$

(ii) the length of DF [2]

By Pythagoras' Theorem,

$$(EF)^2 + (DF)^2 = (DE)^2$$

$$(3)^2 + (DF)^2 = (5)^2$$

$$9 + (DF)^2 = 25$$

$$(DF)^2 = 25 - 9$$

$$(DF)^2 = 16$$

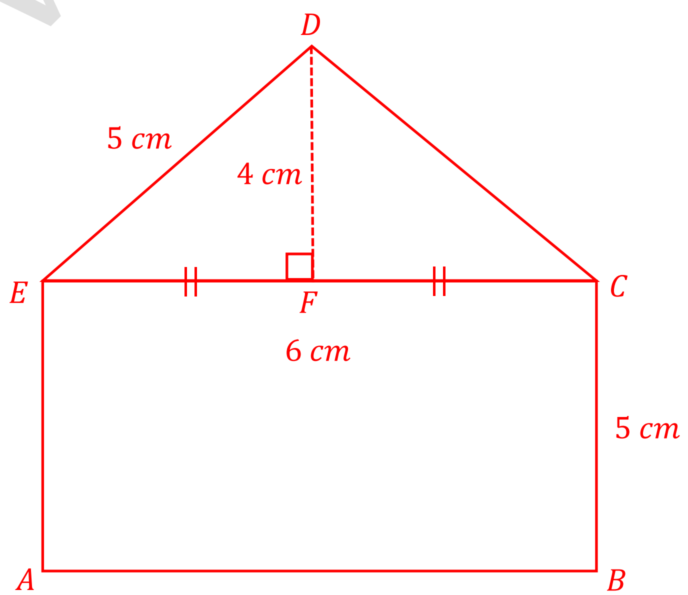
$$DF = \sqrt{16}$$

$$DF = 4\ cm$$

$\therefore$ The length of DF is $4\ cm$.

(iii) the area of the face $ABCDE$ [3]

Consider the diagram below of face $ABCDE$.



Area of $\Delta DEC = \frac{b \times h}{2}$

$= \frac{6 \times 4}{2}$

$= \frac{24}{2}$

$= 12\ cm^2$

Area of rectangle $ABCE = l \times b$

$= 6 \times 5$

$= 30\ cm^2$

Area of the entire face $ABCDE = 12 + 30$

$= 42\ cm^2$

$\therefore$ The area of the face $ABCDE$ is $42\ cm^2$.

Total: 12 marks

4. (a) When y varies directly as the square of x, the variation equation is written

$y = kx^2$, where k is a constant.

(i) Given that $y = 50$ when $x = 10$, find the value of k. [2]

$y = kx^2$

Substituting $y = 50$ and $x = 10$ into $y = kx^2$ gives:

$50 = k(10)^2$

$50 = 100k$

$k = \frac{50}{100}$

$k = \frac{1}{2}$

$\therefore$ The value of $k = \frac{1}{2}$.

(ii) Calculate the value of y when $x = 30$. [2]

$y = \frac{1}{2}x^2$

When $x = 30$,

$y = \frac{1}{2}(30)^2$

$= \frac{1}{2}(900)$

$= 450$

$\therefore$ When $x = 30$, the value of $y = 450$.

(b) (i) **Using a a ruler, a pencil and a pair of compasses**, construct triangle EFG

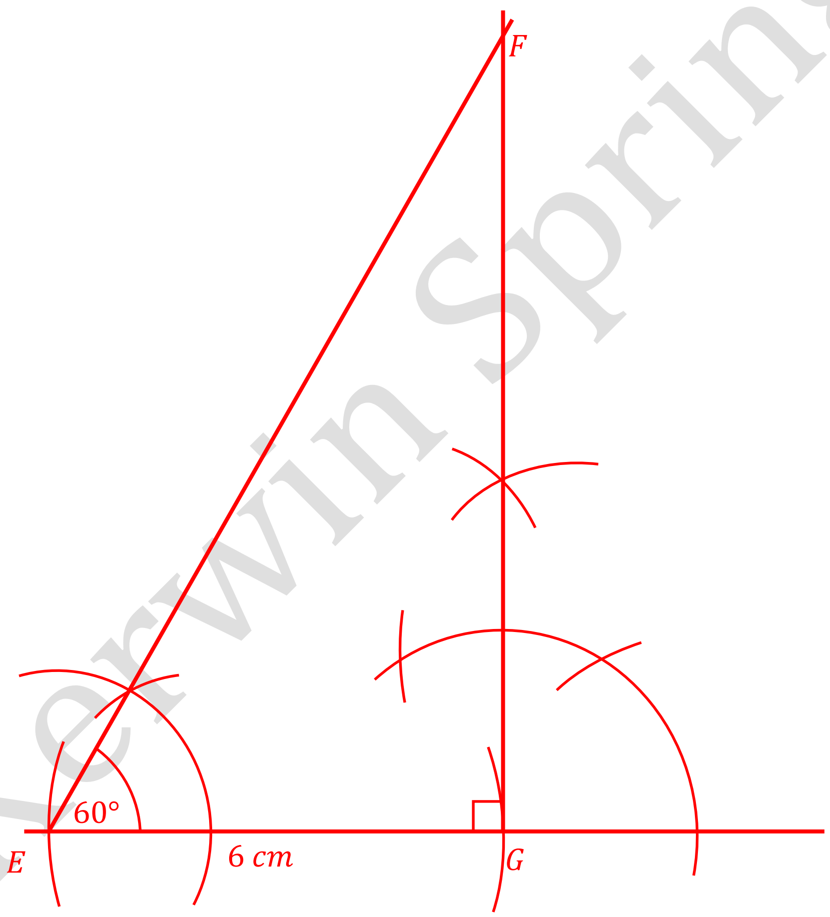
with

$EG = 6$ cm

$\angle FEG = 60°$

and $\angle EGF = 90°$. [5]

The construction is shown below:



(ii) Measure and state

(a) the length of EF

By measurement, $EF = 12.0\ cm$.

(b) the size of $\angle EFG$ [2]

By measurement, $E\hat{F}G = 30°$.

Total: 11 marks

5. (a) The functions f and g are defined as $f(x) = 2x - 5$ and $g(x) = x^2 + 3$.

(i) Calculate the value of

(a) $f(4)$ [1]

$f(x) = 2x - 5$

$f(4) = 2(4) - 5$

$= 8 - 5$

$= 3$

$\therefore$ The value of $f(4) = 3$.

(b) $gf(4)$ [2]

$gf(4) = g[f(4)]$

$= g(3)$

$= (3)^2 + 3$

$= 9 + 3$

$= 12$

$\therefore$ The value of $gf(4) = 12$.

(ii) Find $f^{-1}(x)$. [2]

$f(x) = 2x - 5$

Let $y = f(x)$.

$y = 2x - 5$

Interchanging variables x and y gives:

$x = 2y - 5$

Making y the subject of the formula gives:

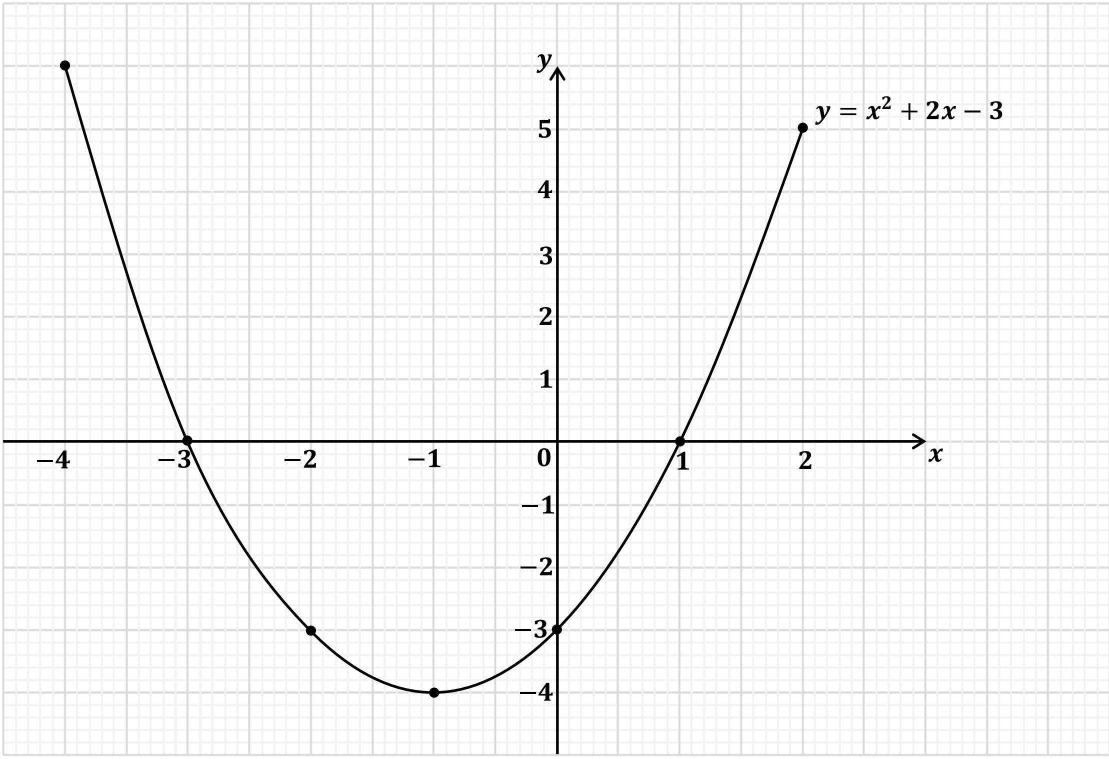
$x + 5 = 2y$

$\frac{x+5}{2} = y$

$\therefore f^{-1}(x) = \frac{x+5}{2}$

(b) The diagram below shows the graph of $y = x^2 + 2x - 3$ for the domain

$-4 \leq x \leq 2$.



Use the graph above to determine

(i) the scale used on the x-axis [1]

On the x-axis, the scale used is $2\ cm = 1$ unit or $1\ cm = 0.5$ unit.

(ii) the value of y for which $x = -1.5$ [2]

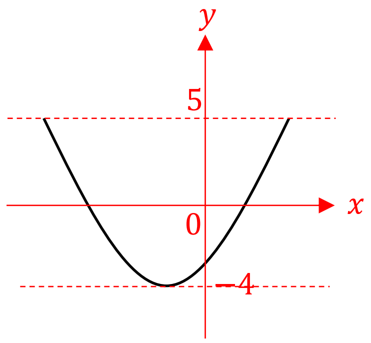
When $x = -1.5, y = -3.8$. (by read-off)

(iii) the values of x for which $y = 0$ [2]

When $y = 0, x = -3$ and $x = 1$. (by read-off)

(iv) the range of values of y, giving your answer in the form $a \leq y \leq b$, where a and b are real numbers [2]

Consider the sketch below:



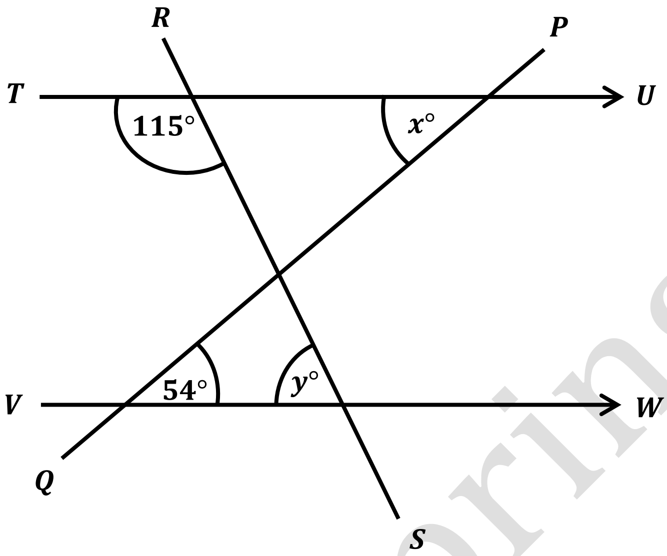
The range of values of y is: $-4 \leq y \leq 5$

which is of the form $a \leq y \leq b$

Total: 12 marks

6. **An answer sheet is provided for this question.**

(a) The diagram below, not drawn to scale, shows two straight lines, PQ and RS, intersecting a pair of parallel lines, TU and VW.



Determine, giving a reason for EACH of your answers, the value of

(i) x [2]

Since alternate angles, $T\hat{U}V$ and $P\hat{V}W$, are equal, then

$x = 54°$

(ii) y [2]

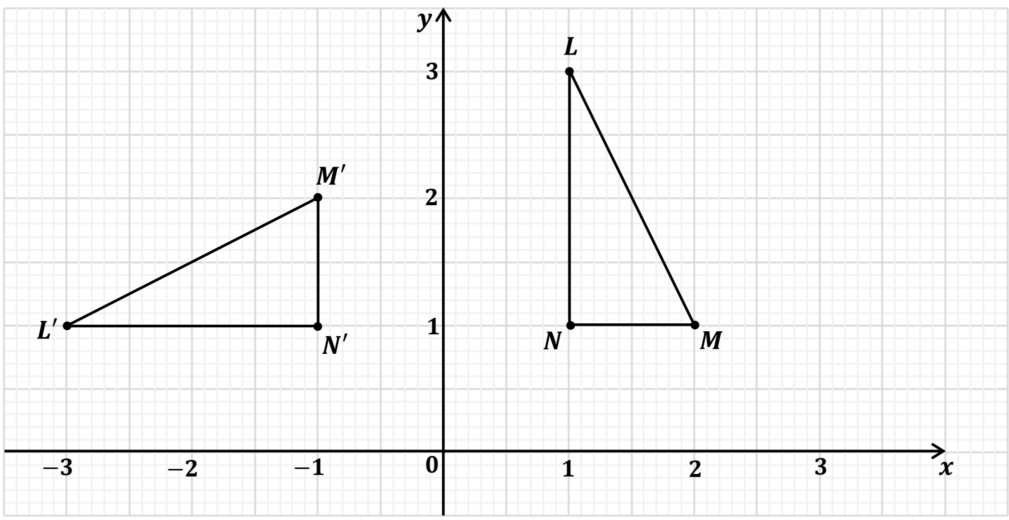
Co-interior angles are supplementary.

$y + 115° = 180°$

$y = 180° - 115°$

$y = 65°$

(b) The diagram below shows triangle LMN, and its image, triangle $L'M'N'$, after undergoing a rotation.



(i) Describe the rotation FULLY by stating

(a) the centre

(b) the angle

(c) the direction [3]

Triangle LMN maps onto its image, triangle $L'M'N'$, after undergoing an anticlockwise rotation of $90°$ about the origin.

(ii) State TWO geometric relationships between triangle LMN and its image, triangle $L'M'N'$. [2]

The two geometric relationships are:

1. ΔLMN maps onto $\Delta L'M'N'$ by a rotation which is a congruent transformation.

2. $\Delta LMN \equiv \Delta L'M'N'$, that is, all corresponding sides and all corresponding angles of the object are the same as that of the image.

(iii) Triangle LMN is translated by the vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

Determine the coordinates of the image of the point L under this transformation. [2]

Triangle LMN is translated by the vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

The coordinates of L is $(1, 3)$.

Now,

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 + 1 \\ 3 + (-2) \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\therefore$ The coordinates of the image of L is $(2, 1)$.

Total: 11 marks

7. A class of 24 students threw the cricket ball at sports. The distance thrown by each student was measured to the neares metre. The results are shown below.

22	50	35	52	47	30
48	34	45	23	43	40
55	29	46	56	43	59
36	63	54	32	49	60

(a) Copy and complete the frequency table for the data shown above. [3]

Distance (m)	**Frequency**
20 – 29	3
30 – 39	5
40 – 49	8
50 – 59	6
60 – 69	2

(b) State the lower boundary for the class interval 20 – 29. [1]

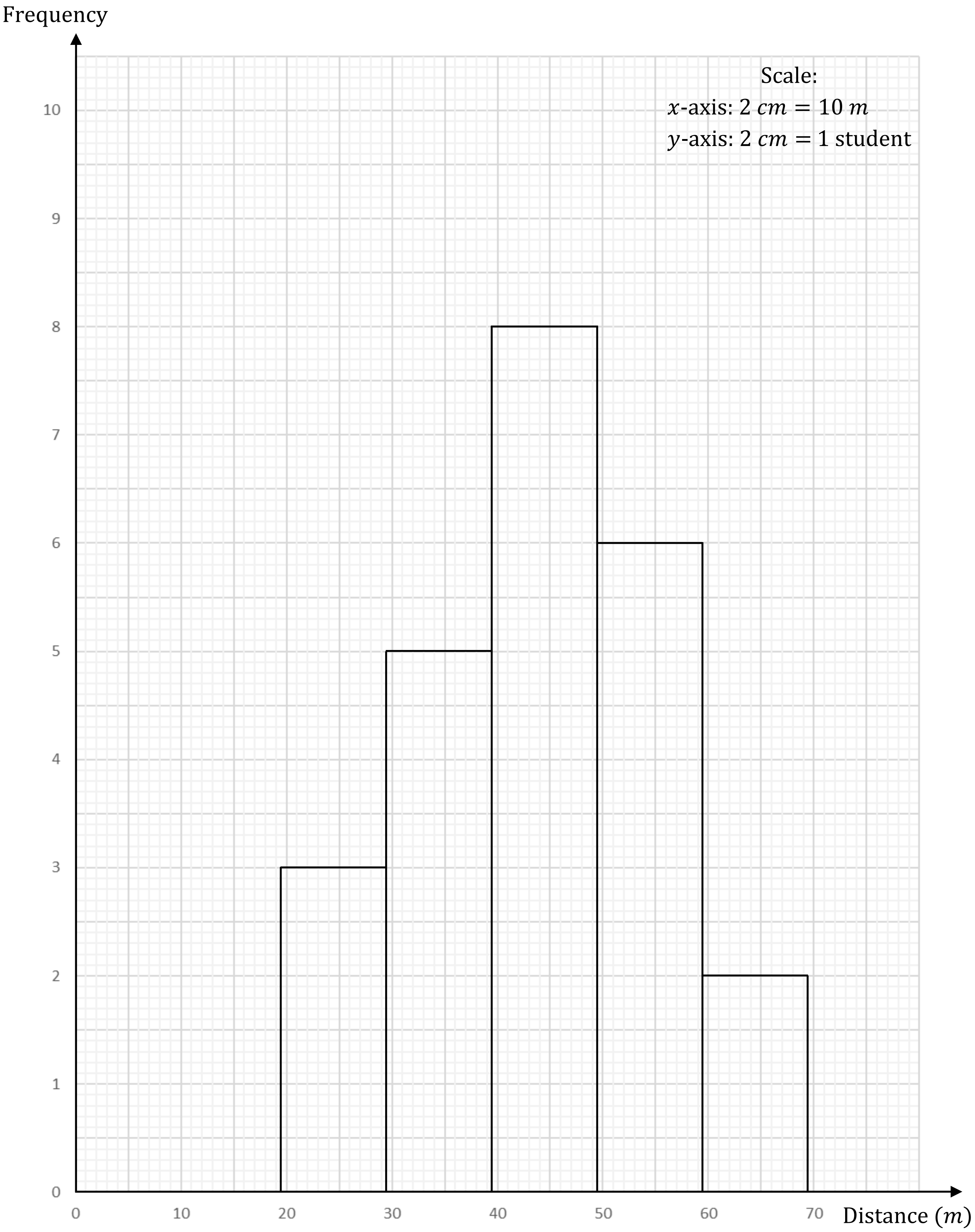
The lower class boundary is 19.5.

(c) Using a scale of **1 cm on the x-axis to represent 5 metres**, and a scale of **1 cm on the y-axis to represent 1 student**, draw a histogram to illustrate the data. [5]

Consider the table below:

Distance (m)	LCB	UCB	Frequency
$20-29$	19.5	29.5	3
$30-39$	29.5	39.5	5
$40-49$	39.5	49.5	8
$50-59$	49.5	59.5	6
$60-69$	59.5	69.5	2
			$\sum f=24$

Title: Histogram showing the information given.



(d) Determine

(i) the number of students who threw the ball a distance recorded as 50 metres or more [1]

The number of students who threw the ball 50 m or more $= 6 + 2$

$= 8$

(ii) the probability that a student, chosen at random, threw the ball a distance recorded as 50 metres or more. [1]

$$\text{Probability} = \frac{\text{number of students threw the ball} \geq 50\ m}{\text{total number of students}}$$

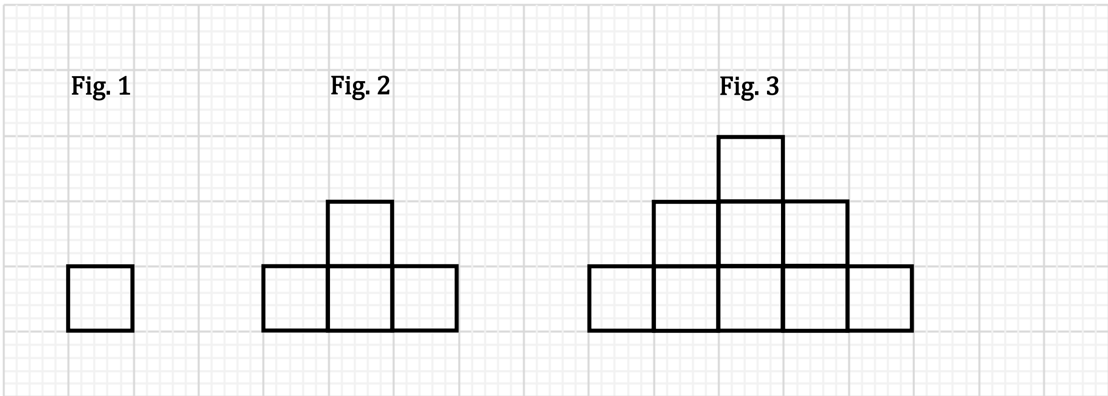
$$= \frac{8}{24}$$

$$= \frac{1}{3}$$

Total: 11 marks

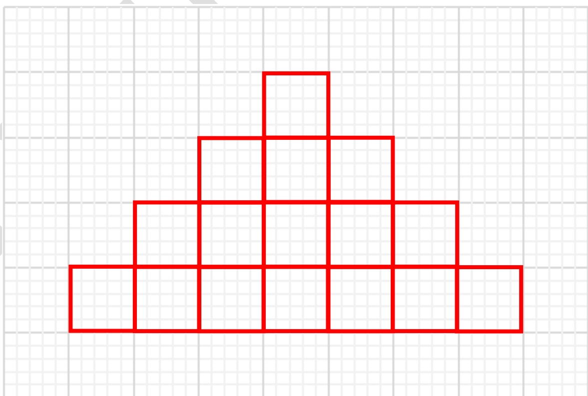
8. **An answer sheet is provided for this question.**

The diagram below shows the first three figures in a sequence of figures. Each figure is made up of squares of side 1 *cm*.



(a) **On your answer sheet**, draw the FOURTH figure (Fig. 4) in the sequence. [2]

The fourth diagram in the sequence is shown below:



(b) Study the patterns in the table shown below, and **on the answer sheet provided**, complete the rows numbered (i), (ii), (iii) and (iv).

	Figure	Area of Figure (cm^2)	Perimeter of Figure (cm)	
	1	1	$1 \times 6 - 2 = 4$	
	2	4	$2 \times 6 - 2 = 10$	
	3	9	$3 \times 6 - 2 = 16$	
(i)	4	16	$4 \times 6 - 2 = 22$	[2]
(ii)	5	25	$5 \times 6 - 2 = 28$	[2]
(iii)	15	225	$15 \times 6 - 2 = 88$	[2]
(iv)	n	n^2	$n \times 6 - 2 = 6n - 2$	[2]

Consider the nth figure.

Area of the figure $= n^2$

Perimeter of the figure $= n \times 6 - 2$

$= 6n - 2$

(i) When $n = 4$,

Area of the figure $= (4)^2$

$= 16$

Perimeter of the figure $= 4 \times 6 - 2$

$= 24 - 2$

$= 22$

(ii) When $n = 5$,

Area of the figure $= (5)^2$

$= 25$

Now,

Perimeter of the figure $= 5 \times 6 - 2$

$= 30 - 2$

$= 28$

(iii) When $n = 15$,

Area of the figure $= (15)^2$

$= 225$

Now,

Perimeter of the figure $= 15 \times 6 - 2$

$= 90 - 2$

$= 88$

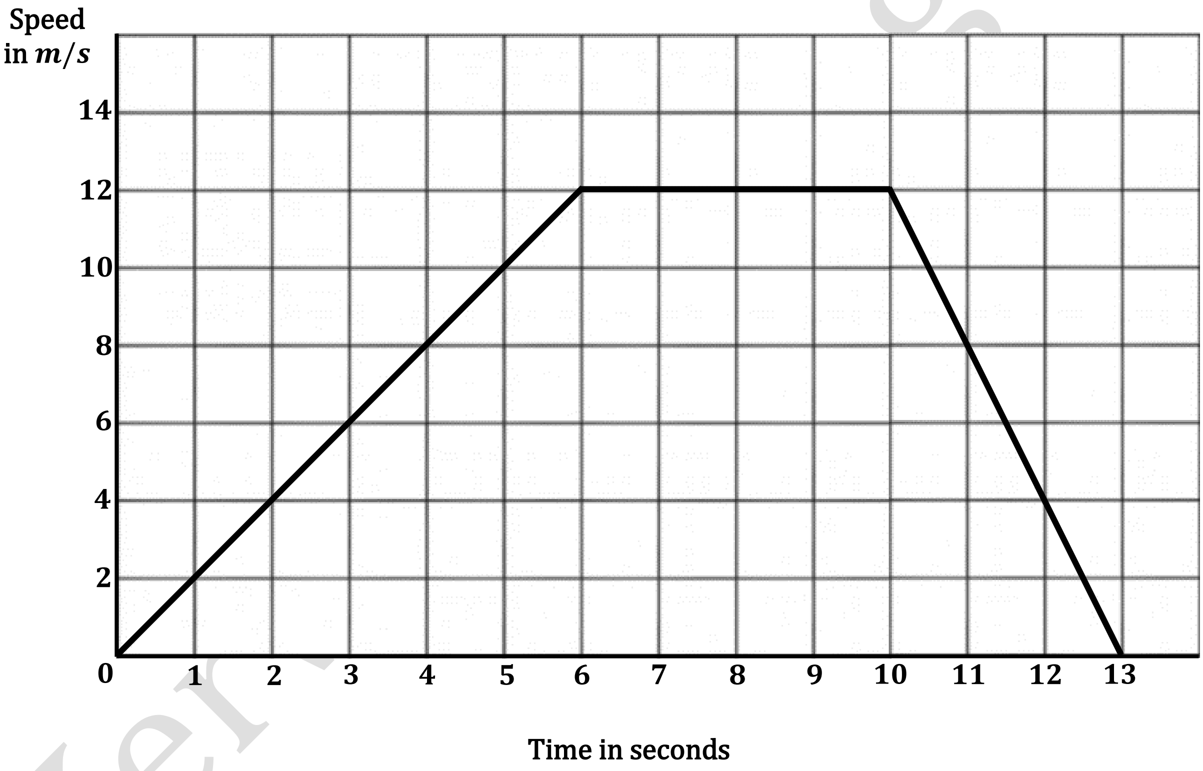
Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The diagram below shows the speed-time graph of the motion of an athlete during a race.



(i) **Using the graph**, determine

(a) the MAXIMUM speed

The maximum speed is 12 ms^{-1}. (from graph)

(b) the number of seconds for which the speed was constant

In the graph, the part for which the speed was constant is represented by the horizontal line.

Number of seconds $= 10 - 6$

$= 4$ seconds

$\therefore$ The number of seconds for which the speed was constant was 4 seconds.

(c) the TOTAL distance covered by the athlete during the race. [4]

The total distance covered by the athlete is found by calculating the area under the graph.

Distance covered $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(4 + 13)12$

$= \frac{1}{2}(17)12$

$= 102\ m$

$\therefore$ The total distance covered by the athlete during the race is $102\ m$.

(ii) During which time-period of the race was

(a) the speed of the athlete increasing

The athlete's speed was increasing from $t = 0$ to $t = 6$, that is, a period of 6 seconds. This is illustrated by the part of the graph with the positive gradient.

$\therefore$ The time-period for which the speed of the athlete increasing is $t = 0$ to $t = 6$.

(b) the speed of the athlete decreasing

The athlete's speed was decreasing from $t = 10$ to $t = 13$, that is, a period of 3 seconds. This is illustrated by the part of the graph with the negative gradient.

$\therefore$ The time-period for which the speed of the athlete decreasing is $t = 10$ to $t = 13$.

(c) the acceleration of the athlete zero? [3]

The acceleration of the athlete was zero from $t = 6$ to $t = 10$, that is, a period of 4 seconds. This is illustrated by a horizontal line in a speed-time graph.

$\therefore$ The time-period for which the acceleration of the athlete is zero is $t = 6$ to $t = 10$.

(b) A farmer supplies his neighbours with x pumpkins and y melons daily, using the following conditions:

First condition : $y \geq 3$

Second condition : $y \leq x$

Third condition : the total number of pumpkins and melons must not exceed 12

(i) Write an inequality to represent the THIRD condition. [1]

Third condition: the total number of pumpkins and melons must not exceed 12.

Inequality: $x + y \leq 12$

(ii) Using a scale of **1 cm to represent one pumpkin on the x-axis and 1 cm to represent one melon** on the y-axis, draw the graphs of the THREE lines associated with the THREE inequalities. [4]

The three inequalities are:

$y \geq 3$

$y \leq x$

$x + y \leq 12$

The equations of the lines are:

$y = 3$

$y = x$

$x + y = 12$

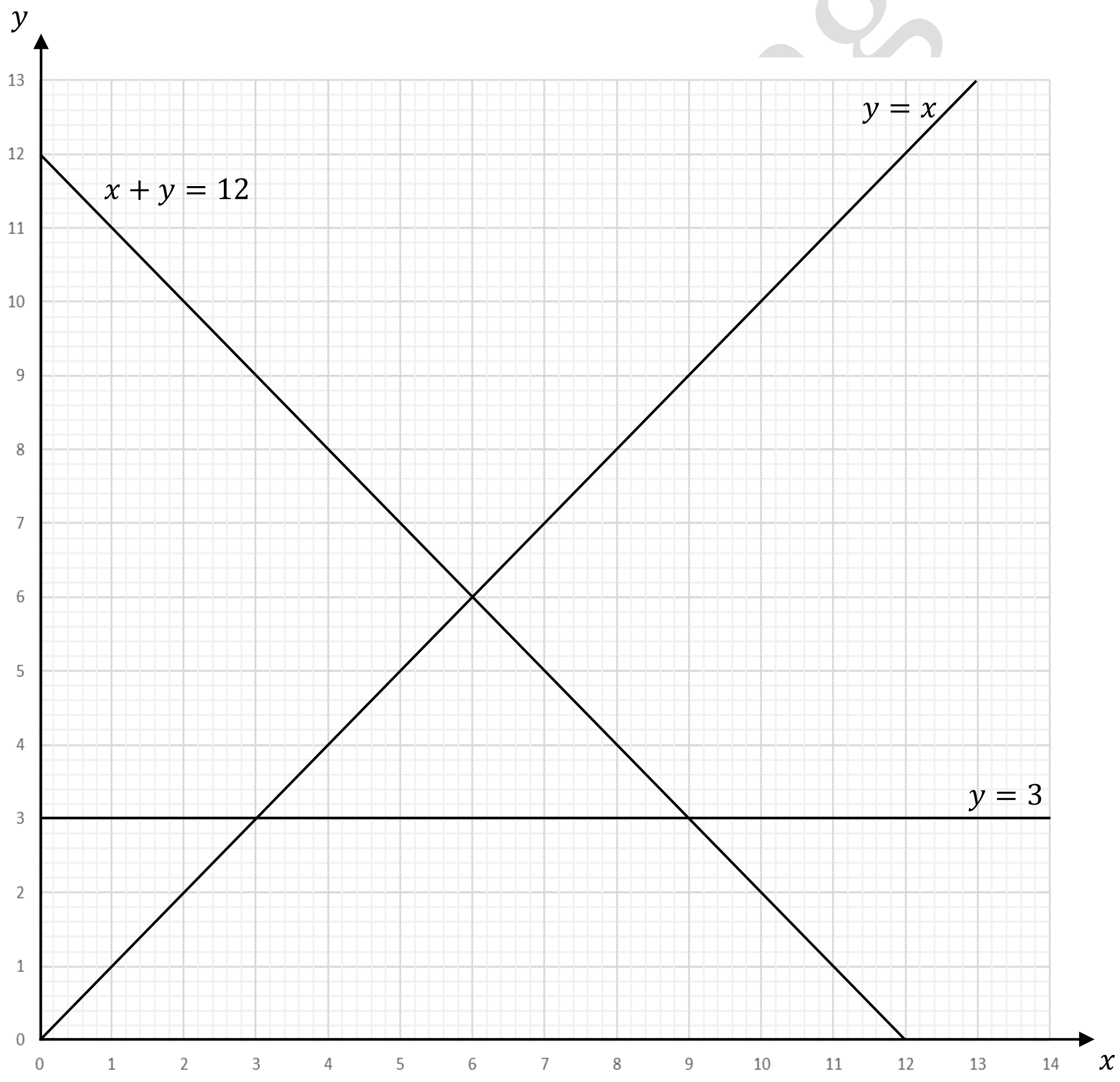
Consider $x + y = 12$.

When $x = 0, y = 12$.

When $y = 0, x = 12$.

Points to be plotted are $(0, 12)$ and $(12, 0)$.

The graph is shown below:



(iii) Identify, by shading, the region which satisfies the THREE inequalities. [1]

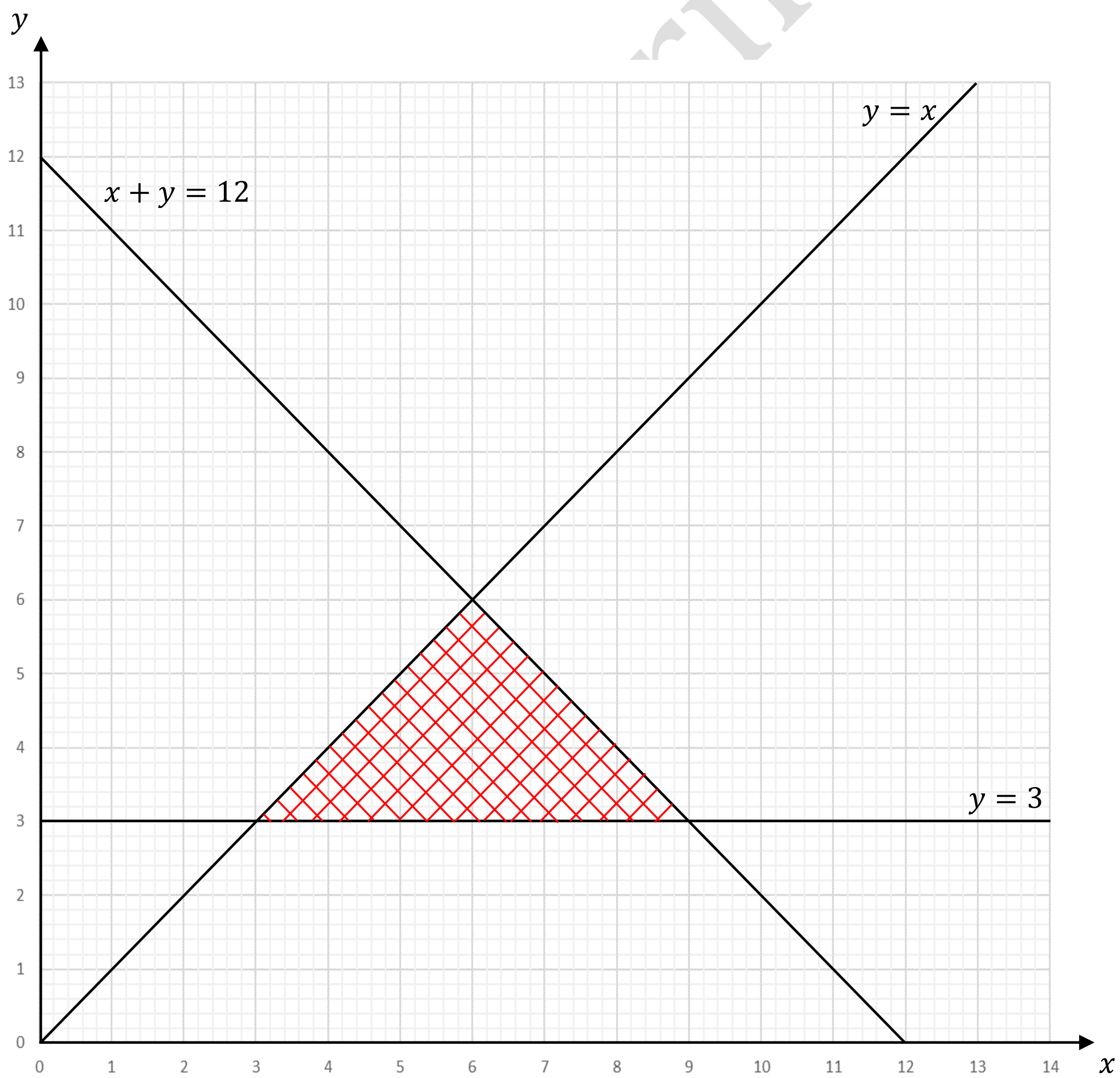
The three inequalities are:

$y \geq 3$

$y \leq x$

$x + y \leq 12$

The region which satisfies all three inequalities is shown below:



(iv) Determine, from your graph, the **minimum** values of x and y which satisfy the conditions. [2]

The vertices of the shaded region are $(3, 3)$, $(6, 6)$ and $(9, 3)$.

Consider $x + y \leq 12$.

Now,

For the point $(3, 3)$, we have

$3 + 3 = 6 \leq 12$

For the point $(6, 6)$, we have

$6 + 6 = 12 \leq 12$

For the point $(9, 3)$, we have

$9 + 3 = 12 \leq 12$

$\therefore$ The minimum x and y that satisfies the inequalities are $x = 3$ and $y = 3$.

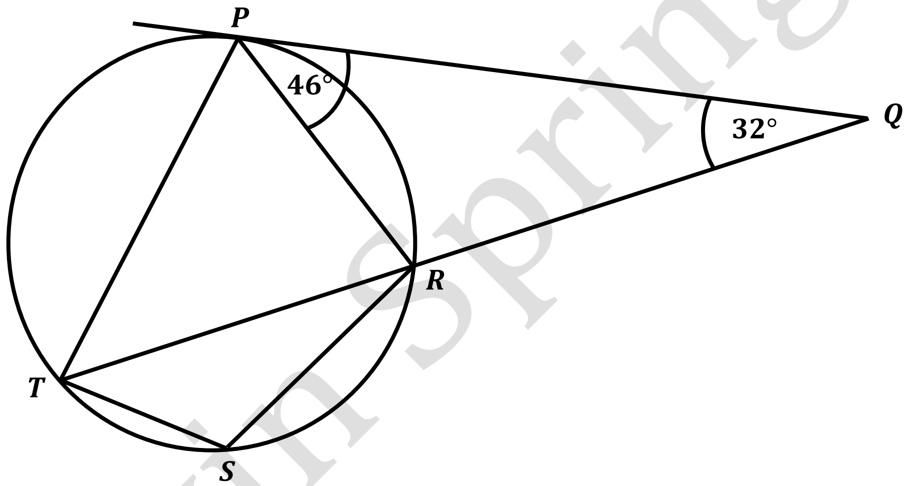
Total: 15 marks

10. (a) In the diagram below, **not drawn to scale**, PQ is a tangent to the circle $PTSR$, so that

$\angle RPQ = 46°$

$\angle RQP = 32°$

and TRQ is a straight line.



Calculate, giving a reason for EACH step of your answer,

(i) $\angle PTR$ [2]

The angle made by the tangent PQ to a circle and a chord, PR, at the point of contact, R, is equal to the angle in the alternate segment.

$\therefore P\hat{T}R = 46°$

(ii) $\angle TPR$ [3]

The sum of the angles in a triangle add up to 180°.

$$T\hat{P}R = 180° - (46° + 46° + 32°)$$

$$= 180° - 144°$$

$$= 56°$$

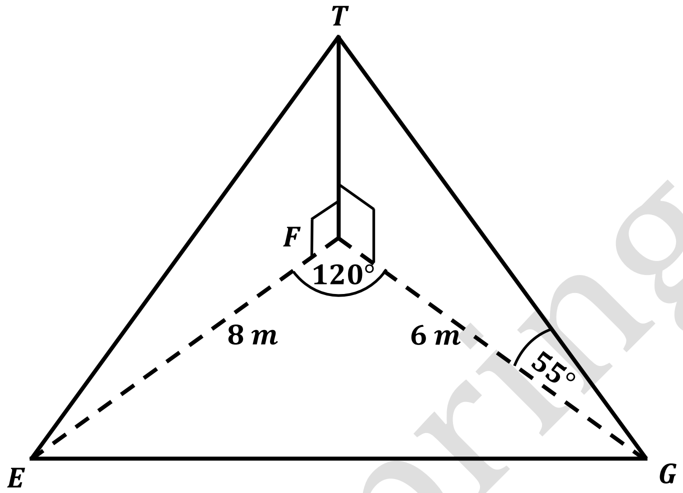
(iii) $\angle TSR$ [2]

The opposite angles in a cyclic quadrilateral $PRST$ are supplementary.

$$T\hat{S}R = 180° - 56°$$

$$= 124°$$

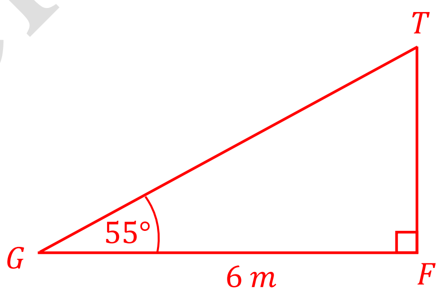
(b) The diagram below, **not drawn to scale**, shows a vertical flagpole, FT, with its foot, F, on the horizontal plane EFG. ET and GT are wires which support the flagpole in its position. The angle of elevation of T from G is 55°, $EF = 8\ m$, $FG = 6\ m$ and $\angle EFG = 120°$.



Calculate, giving your answer correct to 3 significant figures

(i) the height, FT, of the flagpole [2]

Consider triangle FGT below.



The triangle FGT is a right-angled triangle.

Now,

$\tan T\hat{G}F = \frac{opp}{adj}$

$\tan 55° = \frac{FT}{6}$

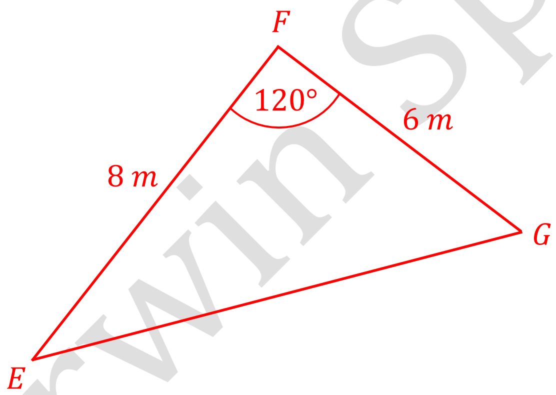
$FT = 6 \tan 55°$

$FT = 8.57\ m$ (to 3 significant figures)

$\therefore$ The height of the flagpole is $8.57\ m$.

(ii) the length of EG [3]

Consider the triangle EFG.



Using the cosine rule,

$(EG)^2 = (EF)^2 + (FG)^2 - 2(EF)(FG) \cos E\hat{F}G$

$(EG)^2 = (8)^2 + (6)^2 - 2(8)(6) \cos 120°$

$(EG)^2 = 64 + 36 - 96\left(-\frac{1}{2}\right)$

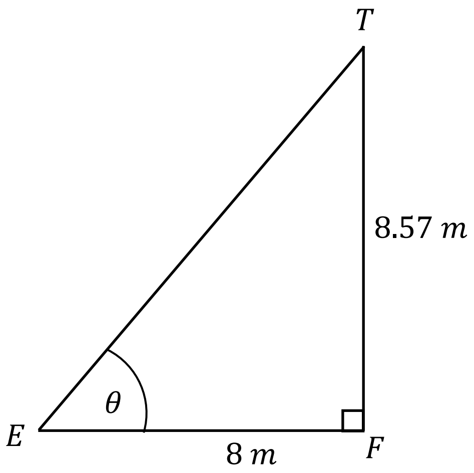
$(EG)^2 = 148$

$EG = \sqrt{148}$

$EG = 12.2\ m$ (to 3 significant figures)

(iii) the angle of elevation of T from E [3]

Consider the right-angled triangle EFT.



Let θ be the angle of elevation of T from E.

Now,

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{8.57}{8}$

$\theta = \tan^{-1}\left(\frac{8.57}{8}\right)$

$\theta = 47.0°$ (to 1 decimal place)

$\therefore$ The angle of elevation of T from E is $47°$.

Total: 15 marks

VECTORS AND MATRICES

11. (a) A and B are two 2×2 matrices such that

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

(i) Find AB. [2]

We are given that $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$.

$$AB = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \times 5) + (2 \times -2) & (1 \times -2) + (2 \times 1) \\ (2 \times 5) + (5 \times -2) & (2 \times -2) + (5 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 + (-4) & -2 + 2 \\ 10 + (-10) & -4 + 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(ii) Determine B^{-1}, the inverse of B. [1]

$$B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

$$\det(B) = ad - bc$$

$$= (5)(1) - (-2)(-2)$$

$$= 5 - 4$$

$$= 1$$

$$adj(B) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -(-2) \\ -(-2) & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$$B^{-1} = \frac{1}{\det(B)} \times adj(B)$$

$$= \frac{1}{1}\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$\therefore$ The inverse of B is $B^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$.

(iii) Given that

$$\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

write $\begin{pmatrix} x \\ y \end{pmatrix}$ as the product of TWO matrices. [2]

$$\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

(iv) Hence, calculate the values of x and y. [2]

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

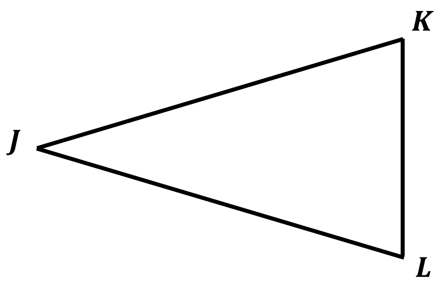
$$= \begin{pmatrix} (1 \times 2) + (2 \times 3) \\ (2 \times 2) + (5 \times 3) \end{pmatrix}$$

$$= \begin{pmatrix} 2 + 6 \\ 4 + 15 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 19 \end{pmatrix}$$

$\therefore x = 8$ and $y = 19$.

(b) The diagram below, not drawn to scale, shows triangle JKL.

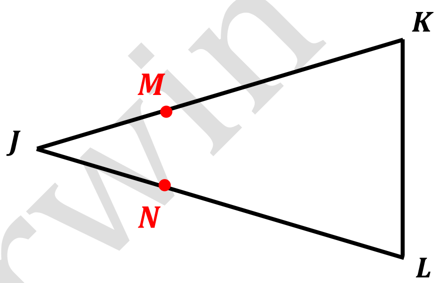


M and N are points on JK and JL respectively, such that

$JM = \frac{1}{3}JK$ and $JN = \frac{1}{3}JL$.

(i) Copy the diagram in your answer booklet and show the points M and N. [2]

The diagram is shown below:



(ii) Given that $\overrightarrow{JM} = \boldsymbol{u}$ and $\overrightarrow{JN} = \boldsymbol{v}$,

write, in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$, an expression for

(a) $\overrightarrow{JK}$

$$\overrightarrow{JK} = 3\overrightarrow{JM}$$

$$= 3\boldsymbol{u}$$

(b) $\overrightarrow{MN}$

Using the triangle law,

$\overrightarrow{MN} = \overrightarrow{MJ} + \overrightarrow{JN}$

$= -\overrightarrow{JM} + \overrightarrow{JN}$

$= -\boldsymbol{u} + \boldsymbol{v}$

$= \boldsymbol{v} - \boldsymbol{u}$

(c) $\overrightarrow{KL}$ [4]

Using the triangle law,

$\overrightarrow{KL} = \overrightarrow{KJ} + \overrightarrow{JL}$

$= -3\boldsymbol{u} + 3\boldsymbol{v}$

$= 3\boldsymbol{v} - 3\boldsymbol{u}$

$= 3(\boldsymbol{v} - \boldsymbol{u})$

(iii) Using your findings in (b)(ii), deduce TWO geometrical relationships between KL and MN. [2]

Two geometrical relationships between KL and MN are:

1. $\overrightarrow{KL} = 3\overrightarrow{MN}$, that is, a scalar multiple. Hence, $\overrightarrow{KL}$ is parallel to $\overrightarrow{MN}$.
2. $|\overrightarrow{KL}| = 3|\overrightarrow{MN}|$, that is, the length of $\overrightarrow{KL}$ is 3 times the length of $\overrightarrow{MN}$.

Total: 15 marks

END OF TEST