CSEC Mathematics

June 2022 – Paper 2

Solutions

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SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, find the

(i) EXACT value of

(a) $\frac{7}{8} + \frac{1}{6} \div \frac{2}{9}$ [1]

Using a calculator,

$\frac{7}{8} + \frac{1}{6} \div \frac{2}{9} = \frac{13}{8}$ (exact value)

(b) $\frac{8}{0.4^3}$ [1]

Using a calculator,

$\frac{8}{0.4^3} = 125$ (exact value)

(ii) value of $\sqrt{26.8} - 2.5^{\frac{3}{2}}$, correct to 2 decimal places. [1]

Using a calculator,

$\sqrt{26.8} - 2.5^{\frac{3}{2}} = 1.22$ (to 2 decimal places)

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(b) Children go to a Rodeo camp during the Easter holiday. Ms Rekha buys bananas and oranges for the children at the camp.

(i) Bananas cost \$3.85 per kilogram. Ms Rekha buys 25 kg of bananas and received a discount of 12%. How much money does she spend on bananas? [2]

$1 \text{ kg of bananas} = \$3.85$

$25 \text{ kg of bananas} = \$3.85 \times 25$

$= \$96.25$

She receives a discount of 12%.

$\text{Percentage she pays} = (100 - 12)\%$

$= 88\%$

$\text{Amount she spends on bananas} = 88\% \text{ of } \$96.25$

$= \frac{88}{100} \times \$96.25$

$= \$84.70$

(ii) Ms Rekha spends \$165.31, inclusive of a sales tax of 15%, on oranges. Calculate the original price of the oranges. [2]

$\text{Oranges bought represent} = 100\% + 15\%$

$= 115\%$

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Now,

$115\% = \$165.31$

$1\% = \frac{\$165.31}{115}$

$100\% = \frac{\$165.31}{115} \times 100$

$= \$143.75$

$\therefore$ The original price of the oranges is $143.75.

(iii) The ratio of the number of bananas to the number of oranges is 2:3.

Furthermore, there are 24 more oranges than bananas.

Calculate the number of bananas Ms Rekha bought. [2]

Total number of parts $= 2 + 3$

$= 5$ parts

Difference $= 3 - 2$

$= 1$ part

There are 24 more oranges than bananas.

Therefore, 1 part $= 24$ fruits.

Total number of bananas $= 2 \times 24$

$= 48$ bananas

Total 9 marks

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2. (a) (i) Factorize completely the following quadratic expression. [2]

$$5x^2 - 9x + 4$$

$$5x^2 - 9x + 4$$

$$= 5x^2 - 5x - 4x + 4$$

$$= 5x(x - 1) - 4(x - 1)$$

$$= (5x - 4)(x - 1)$$

$$\therefore 5x^2 - 9x + 4 = (5x - 4)(x - 1)$$

(ii) Hence, solve the following equation. [1]

$$5x^2 - 9x + 4 = 0$$

$$5x^2 - 9x + 4 = 0$$

$$(5x - 4)(x - 1) = 0$$

Either $5x - 4 = 0$ or $x - 1 = 0$

$5x = 4$ $\qquad$ $x = 1$

$x = \frac{4}{5}$

$\therefore x = \frac{4}{5}$ or $x = 1$

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(b) Make v the subject of the formula. [3]

$$w = \frac{5+v}{v-3}$$

$$w = \frac{5+v}{v-3}$$

$$w(v-3) = 5+v$$

$$wv - 3w = 5 + v$$

$$wv - v = 5 + 3w$$

$$v(w-1) = 5 + 3w$$

$$v = \frac{5+3w}{w-1}$$

(c) The height, h, of an object is directly proportional to the square root of its perimeter, p.

(i) Write an equation showing the relationship between h and p. [1]

$h \propto \sqrt{p}$

$h = k\sqrt{p}$, where k is a constant

(ii) Given that $h = 5.4$ when $p = 1.44$, determine the value of h when $p = 2.89$. [2]

$h = k\sqrt{p}$

When $h = 5.4$ and $p = 1.44$,

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$5.4 = k\sqrt{1.44}$

$5.4 = k(1.2)$

$k = \frac{5.4}{1.2}$

$k = \frac{9}{2}$

Hence, $h = \frac{9}{2}\sqrt{p}$.

When $p = 2.89$,

$h = \frac{9}{2}\sqrt{2.89}$

$= \frac{153}{20}$ or 7.65

Total 9 marks

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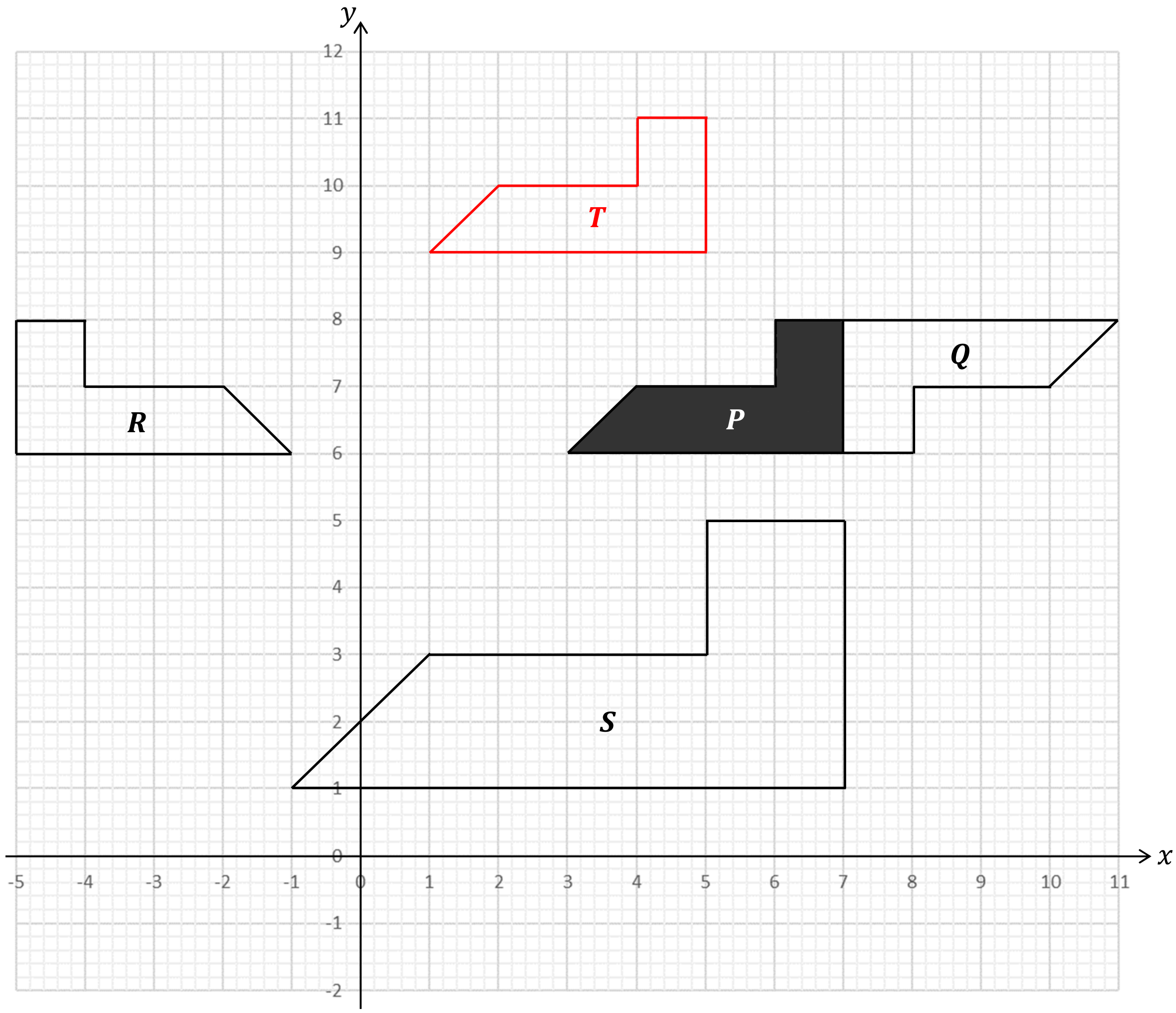
3. The diagram below shows four shapes, P, Q, R and S on a square grid.



(a) Describe fully the **single** transformation that maps shape P onto shape

(i) Q [3]

The single transformation that maps shape P onto shape Q is a clockwise rotation of $180°$ about the centre of rotation $(7, 7)$.

(ii) R [2]

The single transformation that maps shape P onto shape R is a reflection in the line $x = 1$.

(iii) S [3]

The single transformation that maps shape P onto shape S is an enlargement of scale factor 2 where the centre of enlargement is $(7, 11)$.

(b) On the grid provided **on page 10**, draw the image of shape P after a translation by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Label the image T. [1]

The vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ means to translate the object 2 units to the left and 3 units upwards.

See grid above.

Total 9 marks

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4. (a) The functions f and g are defined as follows:

$$f(x) = 5x + 7 \quad \text{and} \quad g(x) = 3x - 1.$$

For the functions given above, determine

(i) $g\left(\frac{1}{3}\right)$ [1]

$g(x) = 3x - 1$

$g\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) - 1$

$= 1 - 1$

$= 0$

(ii) $f^{-1}(-3)$ [2]

$f(x) = 5x + 7$

Let $y = f(x)$.

$y = 5x + 7$

Interchanging variables x and y.

$x = 5y + 7$

Make y the subject of the formula.

$x - 7 = 5y$

$\frac{x-7}{5} = y$

Hence, $f^{-1}(x) = \frac{x-7}{5}$.

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Now,

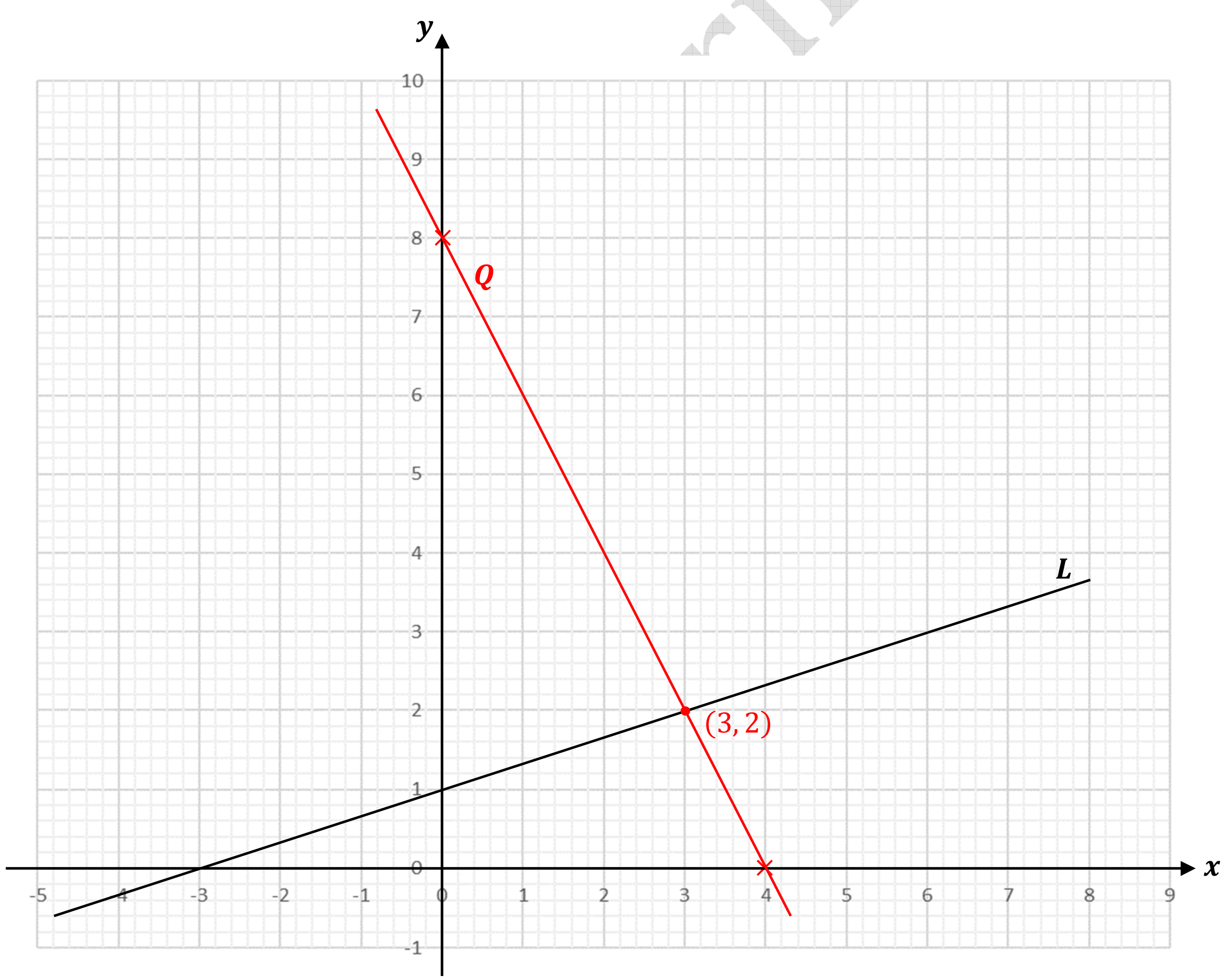
$f^{-1}(-3) = \frac{-3-7}{5}$

$= \frac{-10}{5}$

$= -2$

$\therefore f^{-1}(-3) = -2$

(b) The line L is shown on the grid below.



(i) Write the equation of the line L in the form $y = mx + c$. [2]

We need to find the gradient of the line.

Two points on the line are $(0, 1)$ and $(6, 3)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-1}{6-0}$

$= \frac{2}{6}$

$= \frac{1}{3}$

The value of the y-intercept is $c = 1$.

$\therefore$ The equation of the line is: $y = \frac{1}{3}x + 1$

which is of the form $y = mx + c$, where $m = \frac{1}{3}$ and $c = 1$.

(ii) The equation of a different line, Q, is $y = -2x + 8$.

(a) Write down the coordinates of the point where Q crosses the x-axis. [1]

When $y = 0$,

$0 = -2x + 8$

$2x = 8$

$x = \frac{8}{2}$

$x = 4$

$\therefore$ The coordinates of the point where Q crosses the x-axis is $(4, 0)$.

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(b) Write down the coordinates of the point where Q crosses the y-axis. [1]

When $x = 0$,

$y = -2(0) + 8$

$= 0 + 8$

$= 8$

$\therefore$ The coordinates of the point where Q crosses the y-axis is $(0, 8)$.

(c) On the grid **on page 14**, draw the graph of the line Q. [1]

See grid above.

(iii) Complete the statement below.

According to the graph, the solution of the system of equations consisting of L and Q is

………… $(3, 2)$ ………………………………………………………………………… [1]

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5. A school nurse records the height, h cm, of each of the 150 students in Class A who was vaccinated. The table below shows the information.

Height, $\boldsymbol{h}$ (cm)	Number of Students ($\boldsymbol{f}$)
$60 < h \leq 80$	4
$80 < h \leq 100$	20
$100 < h \leq 120$	35
$120 < h \leq 140$	67
$140 < h \leq 160$	20
$160 < h \leq 180$	4

(a) Complete the table below and use the information to calculate an estimate of the mean height of the students. Give your answer correct to 1 decimal place. [3]

Height, $\boldsymbol{h}$ (cm)	Number of Students ($\boldsymbol{f}$)	Midpoint ($\boldsymbol{x}$)	$\boldsymbol{f} \times \boldsymbol{x}$
$60 < h \leq 80$	4	70	280
$80 < h \leq 100$	20	90	1 800
$100 < h \leq 120$	35	110	3 850
$120 < h \leq 140$	67	130	8 710
$140 < h \leq 160$	20	150	3 000
$160 < h \leq 180$	4	170	680
	$\sum f = 150$		$\sum fx = 18\ 320$

Midpoint $= \frac{140+120}{2}$

$= \frac{260}{2}$

$= 130$

$f \times x = 67 \times 130$

$= 8710$

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$\text{Mean} = \frac{\sum fx}{\sum f}$

$= \frac{18\,320}{150}$

$= 122.1$ (to 1 decimal place)

$\therefore$ The mean height of the students is 122.1 cm.

(b) In Class B, the mean height of the students is 123.5 cm, and the standard deviation 29.87. For Class A, the standard deviation is 21.38.

Using the information provided, and your response in (a), comment on the distribution of the heights of the students in both Class A and Class B. [1]

In Class B, the heights of the students deviate (or are spread) further away from the mean height of 123.5 cm than the spread of heights in Class A from its mean of 122.1 cm.

(c) (i) Complete the cumulative frequency table below and use the information to construct the cumulative frequency curve on the grid provided **on page 19**. [1]

Height, h (cm)	Number of Students (f)	Cumulative Frequency
$60 < h \leq 80$	4	4
$80 < h \leq 100$	20	24
$100 < h \leq 120$	35	59
$120 < h \leq 140$	67	126
$140 < h \leq 160$	20	146
$160 < h \leq 180$	4	150

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(ii) Use your cumulative frequency curve to find

(a) an estimate of the median height of the group of students [1]

The median, Q_2, occurs at the $\frac{n}{2} = \frac{150}{2} = 75^{\text{th}}$ value.

From the graph, $Q_2 = 126$ cm

$\therefore$ The median height of the group of students is 126 cm.

(b) the probability that a student chosen at random would be taller than 130 cm. [1]

From the graph, the point is $(130, 90)$.

Number of students taller than 130 cm $= 150 - 90$

$= 60$ students

Hence,

$$P(\text{student taller than 130 cm}) = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{60}{150}$$

$$= \frac{2}{5} \quad \text{or} \quad 0.4$$

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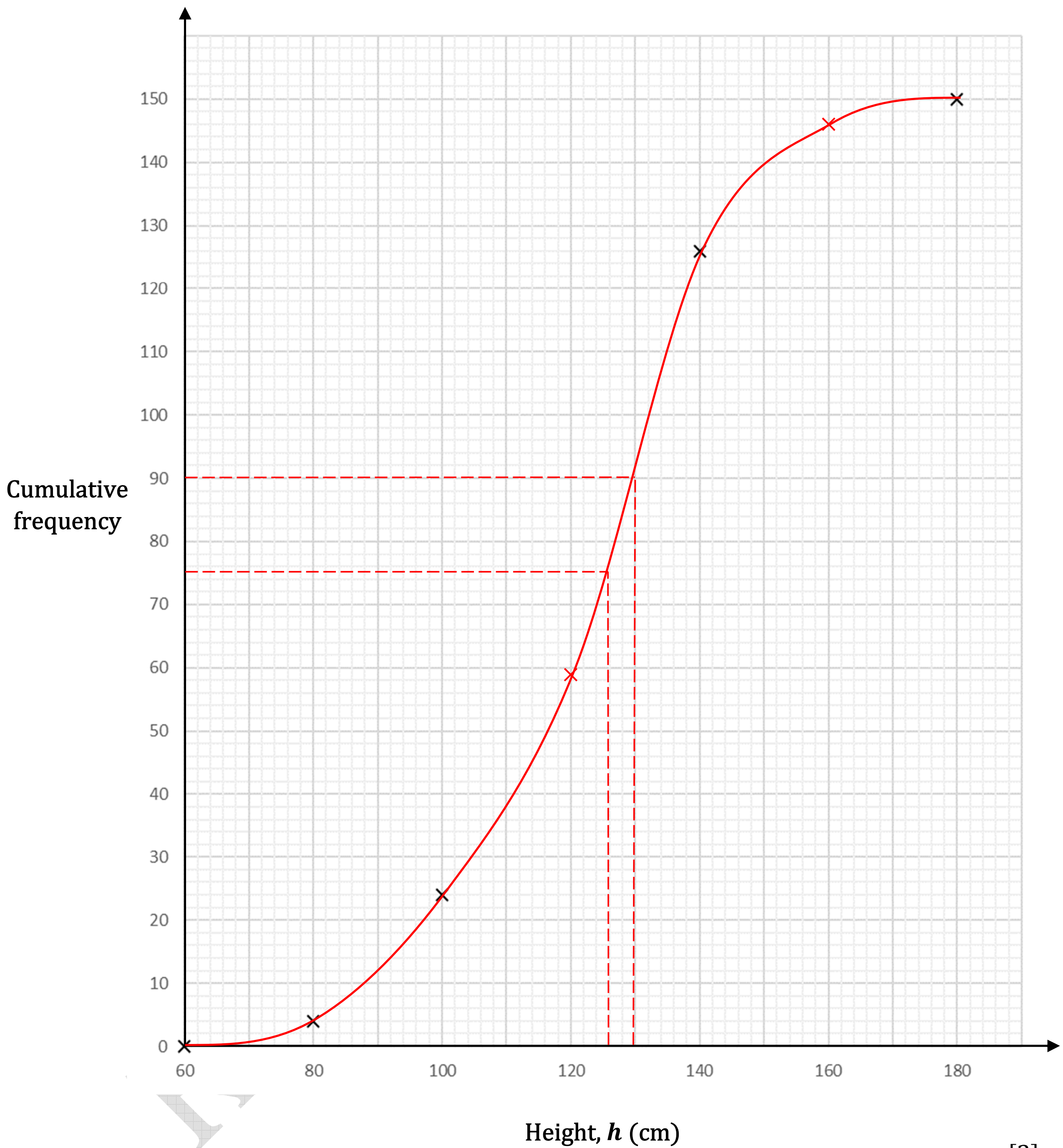
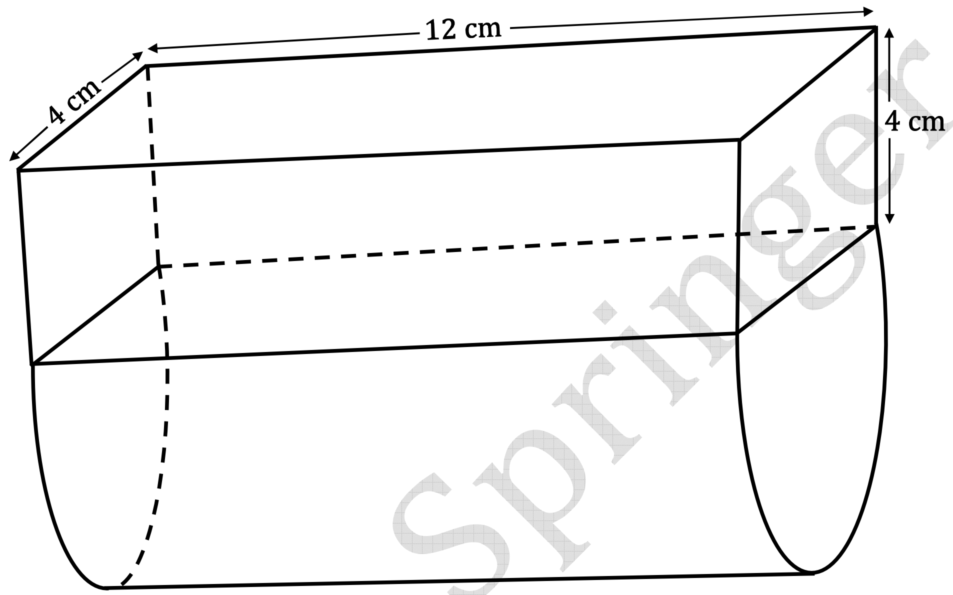


[2]

Total 9 marks

6. The diagram below shows a solid made from a semi-circular cylindrical base, with a rectangular prism above it. The diameter of the cylindrical base and the width of the rectangular prism are 4 cm each.



(a) Calculate the TOTAL surface area of the solid. [4]

[The surface area, A, of a cylinder with radius r is $A = 2\pi r^2 + 2\pi rh$].

Surface area of a cylinder, $A = 2\pi r^2 + 2\pi rh$

Surface area of half of a cylinder, $A = \frac{1}{2}(2\pi r^2 + 2\pi rh)$

$= \pi r^2 + \pi rh$

Radius $= \frac{Diameter}{2}$

$= \frac{4}{2}$

$= 2$ cm

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Now,

Surface area of half of a cylinder, $A = \pi r^2 + \pi rh$

$= \pi(2)^2 + \pi(2)(12)$

$= 4\pi + 24\pi$

$= 28\pi\ cm^2$

Surface area of rectangle = (Front + Back) + (Top) + (Side + Side)

$= 2(4 \times 12) + (12 \times 4) + 2(4 \times 4)$

$= 2(48) + 48 + 32$

$= 96 + 48 + 32$

$= 176\ cm^2$

Hence,

Total Surface Area = Surface area of half cylinder + Surface area of rectangle

$= 176 + 28\pi$

$= 263.96\ cm^2$

$\approx 264\ cm^2$ (to the nearest cm)

$\therefore$ The total surface area of the solid is $264\ cm^2$.

(b) Calculate the volume of the solid. [3]

Volume of cylinder $= \pi r^2 h$

Volume of semi-cylinder $= \frac{1}{2}\pi r^2 h$

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The radius is 2 cm and the height is 12 cm. Hence,

$$\text{Volume of semi-cylinder} = \frac{1}{2}\pi(2)^2(12)$$

$$= \frac{1}{2}\pi(48)$$

$$= 24\pi\ cm^3$$

$$\text{Volume of cuboid} = l \times b \times h$$

$$= 4 \times 4 \times 12$$

$$= 192\ cm^3$$

Therefore,

$$\text{Total volume of the solid} = \text{Volume of semi-cylinder} + \text{Volume of cuboid}$$

$$= 192 + 24\pi$$

$$= 267.4\ cm^3 \quad \text{(to 1 decimal place)}$$

(c) The solid is made from gold. One **cubic centimetre** of gold has a mass of 19.3 grams. The cost of 1 gram of gold is $42.62.

Calculate the cost of the gold used to make the solid. [2]

$$\text{Mass of solid} = 267.4 \times 19.3$$

$$= 5160.82 \text{ grams}$$

$$\text{Cost of the gold used to make the solid} = 5160.82 \times \$42.62$$

$$= \$240\ 597.43$$

Total 9 marks

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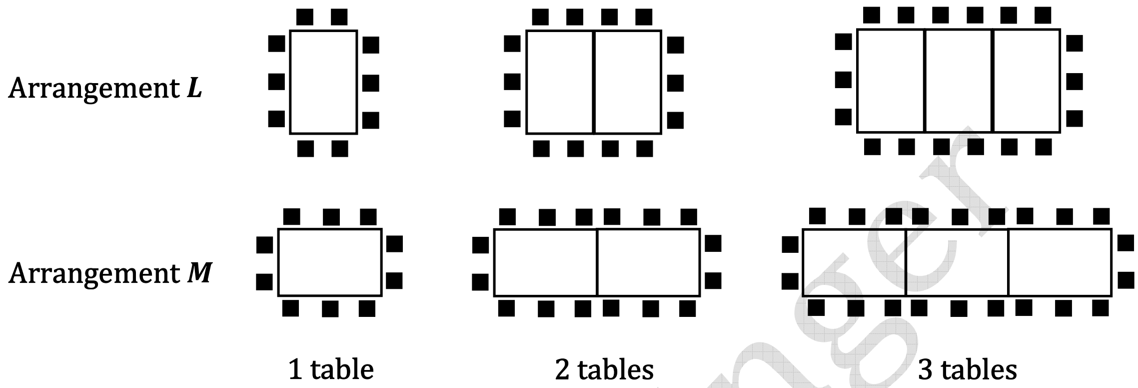
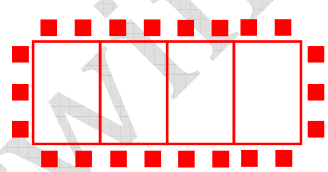
7. At an entertainment hall, tables and chairs can be arranged in two different ways as shown in the diagram below.



(a) Draw the diagram for **4 tables** using **Arrangement *L***. [2]

The diagram for 4 tables using Arrangement L is shown below:



(b) The number of chairs, C, that can be placed around a given number of tables, T, for either arrangement, L or M, forms a pattern. The values for C for the first 3 diagrams for both arrangements are shown in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

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	Number of Tables ($\boldsymbol{T}$)	Arrangement $\boldsymbol{L}$	Arrangement $\boldsymbol{M}$	
		Number of Chairs ($\boldsymbol{C}$)	Number of Chairs ($\boldsymbol{C}$)	
	1	10	10	
	2	14	16	
	3	18	22	
(i)	4	22	28	[2]
	⋮	⋮	⋮	
(ii)	21	90	130	[2]
	⋮	⋮	⋮	
(iii)	n	$4n + 6$	$6n + 4$	[2]

For the nth number of tables,

Arrangement L: Number of chairs, $C = 4n + 6$

Arrangement M: Number of chairs, $C = 6n + 4$

When $n = 4$,

Arrangement L: Number of chairs, $C = 4n + 6$

$= 4(4) + 6$

$= 16 + 6$

$= 22$

Arrangement M: Number of chairs, $C = 6n + 4$

$= 6(4) + 4$

$= 24 + 4$

$= 28$

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When $C = 130$ for Arrangement M, then

$6n + 4 = 130$

$6n = 130 - 4$

$6n = 126$

$n = \frac{126}{6}$

$n = 21$

When $n = 21$,

Arrangement L: Number of chairs, $C = 4n + 6$

$= 4(21) + 6$

$= 84 + 6$

$= 90$

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(c) Leon needs to arrange tables to seat 70 people for a birthday party. Which of the arrangements, L or M, will allow him to rent the LEAST number of tables?

Use calculations to justify your answer. [2]

Using Arrangement L,

$4n + 6 = 70$

$4n = 70 - 6$

$4n = 64$

$n = \frac{64}{4}$

$n = 16$ tables

Using Arrangement M,

$6n + 4 = 70$

$6n = 70 - 4$

$6n = 66$

$n = \frac{66}{6}$

$n = 11$ tables

$\therefore$ Arrangement M will allow him to rent the least number of tables.

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Total 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. A rental company has x cars and y minivans. The company has at least 8 vehicles altogether. The number of minivans is less than or equal to the number of cars. The number of cars is no more than 9.

(a) Write down THREE inequalities, in terms of x and/or y, other than $x \geq 0$ and $y \geq 0$, to represent this information. [3]

The company has at least 8 vehicles altogether.

Inequality: $x + y \geq 8$

The number of minivans is less than or equal to the number of cars.

Inequality: $y \leq x$

The number of cars is no more than 9.

Inequality: $x \leq 9$

(b) A car can carry 4 people and a minivan can carry 6 people. There are at most 60 persons to be taken on a tour.

Show that $2x + 3y \leq 30$. [1]

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From the information given in the question,

$4x + 6y \leq 60$

$(\div 2)$

$2x + 3y \leq 30$

Q.E.D.

(c) On the grid below, plot the four lines associated with the inequalities in (a) and (b). Shade and label the region that satisfies ALL four inequalities R. [5]

Consider $x + y \geq 8$.

Equation is $x + y = 8$.

When $x = 0, y = 8$.

When $y = 0, x = 8$.

Coordinates to be plotted are $(0, 8)$ and $(8, 0)$.

Consider $2x + 3y \leq 30$.

Equation is $2x + 3y = 30$.

When $x = 0$,

$3y = 30$

$y = \frac{30}{3}$

$y = 10$

When $y = 0$,

$2x = 30$

$x = \frac{30}{2}$

$x = 15$

Coordinates to be plotted are $(0, 10)$ and $(15, 0)$.

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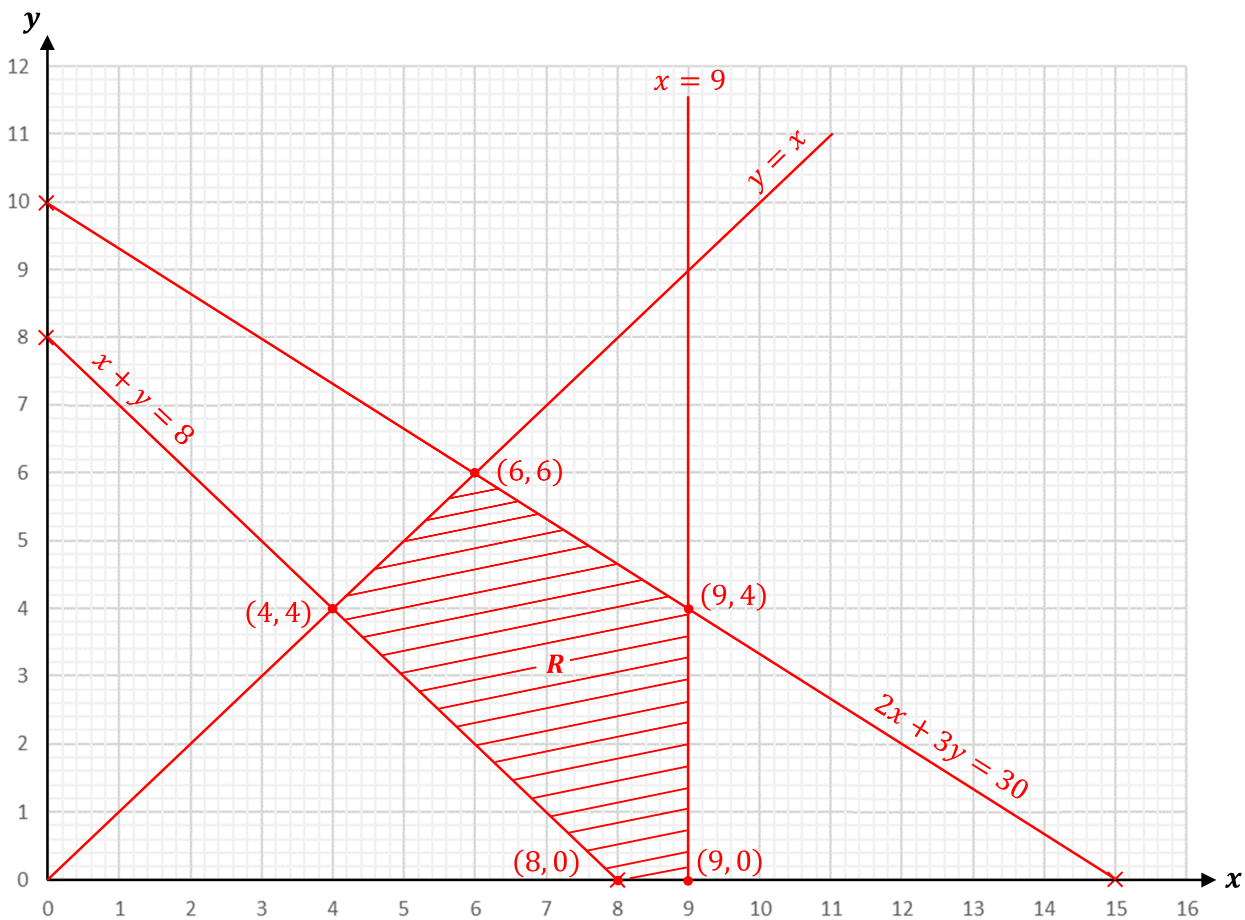

(d) (i) Determine the **two combinations** for the MINIMUM number of cars and minivans that can be used to carry EXACTLY 60 people on the tour. [2]

From the graph, points are $(6, 6)$ and $(9, 4)$.

$\therefore$ To carry exactly 60 people, we can use 6 cars and 6 minivans

OR 9 cars and 4 minivans.

(ii) The company charges \$75 to rent a car and \$90 to rent a minivan. Show that the MINIMUM rental cost for this tour is \$990. [1]

Consider 6 cars and 6 minivans.

Cost for 6 cars and 6 minivans $= 6(\$75) + 6(\$90)$

$= \$450 + \$540$

$= \$990$

Consider 9 cars and 4 minivans.

Cost for 9 cars and 4 minivans $= 9(\$75) + 4(\$90)$

$= \$675 + \$360$

$= \$1035$

$\therefore$ Minimum possible cost is \$990.

Q.E.D.

Total 12 marks

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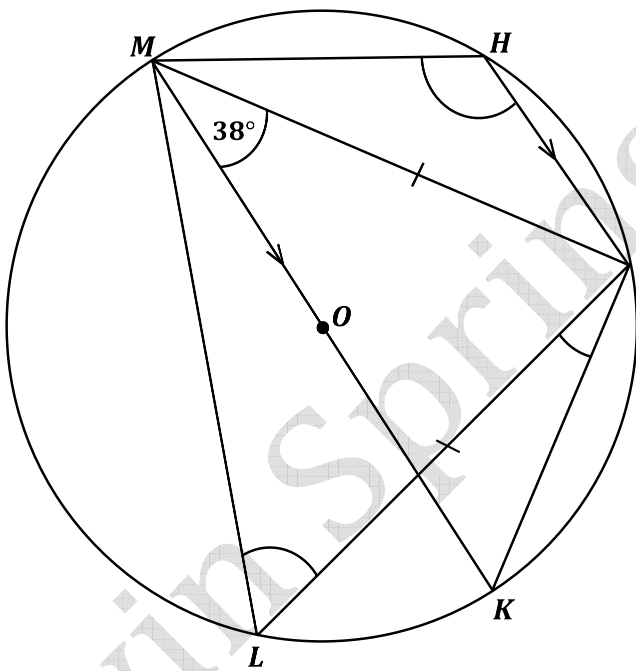
9. (a) H, J, K, L and M are points on the circumference of a circle with centre O. MK is a diameter of the circle and it is parallel to HJ. $MJ = JL$ and angle $JMK = 38°$.



(i) Explain, giving a reason, why angle

(a) $HJM = 38°$ [1]

HJ and MK are parallel lines.

Since alternate angles are equal, then

$\angle HJM = \angle KMJ$

$= 38°$

(b) $MJK = 90°$ [1]

Since MK is a diameter of the circle and the angle in a semicircle equals $90°$, then $MJK = 90°$.

(ii) Determine the value of EACH of the following angles. Show detailed working where appropriate.

(a) Angle MLJ [2]

Consider ΔMJK.

Angles in a triangle sum to $180°$.

$\angle MKJ = 180° - (\angle KMJ + \angle MJK)$

$= 180° - (38° + 90°)$

$= 180° - 128°$

$= 52°$

Angles from a common chord in the same segment are equal.

$\angle MLJ = \angle MKJ$

$= 52°$

(b) Angle LJK [1]

Consider ΔMJL.

Since ΔMJL is an isosceles triangle and $\angle MLJ = 52°$, then

$\angle LMJ = 52°$

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Now,

$\angle LMK = 52° - 38°$

$= 14°$

Angles from a common chord, LK, in the same segment are equal.

$\angle LJK = \angle LMK$

$= 14°$

(c) Angle JHM [1]

Opposite angles in a cyclic quadrilateral, $HJLM$, add up to 180°.

$\angle MLJ + \angle MHJ = 180°$

$52° + \angle MHJ = 180°$

$\angle MHJ = 180° - 52°$

$\angle MHJ = 128°$

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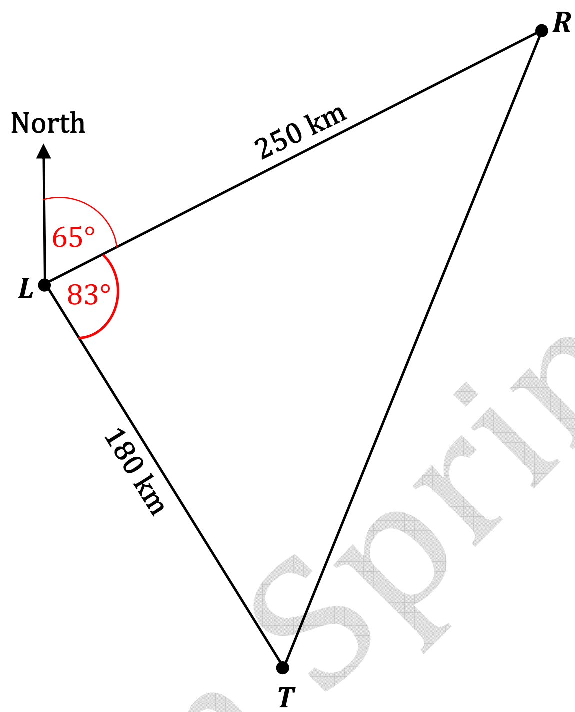
(b) From a port, L, ship R is 250 kilometres on a bearing of 065°. Ship T is 180 kilometres from L on a bearing of 148°. This information is illustrated in the diagram below.



(i) Complete the diagram above by inserting the value of angle RLT. [1]

$\angle RLT = 148^\circ - 65^\circ$

$\angle RLT = 83^\circ$

See diagram above.

(ii) Calculate RT, the distance between the two ships. [2]

Using the cosine rule,

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$$(RT)^2 = (LR)^2 + (LT)^2 - 2(LR)(LT)\cos R\hat{L}T$$

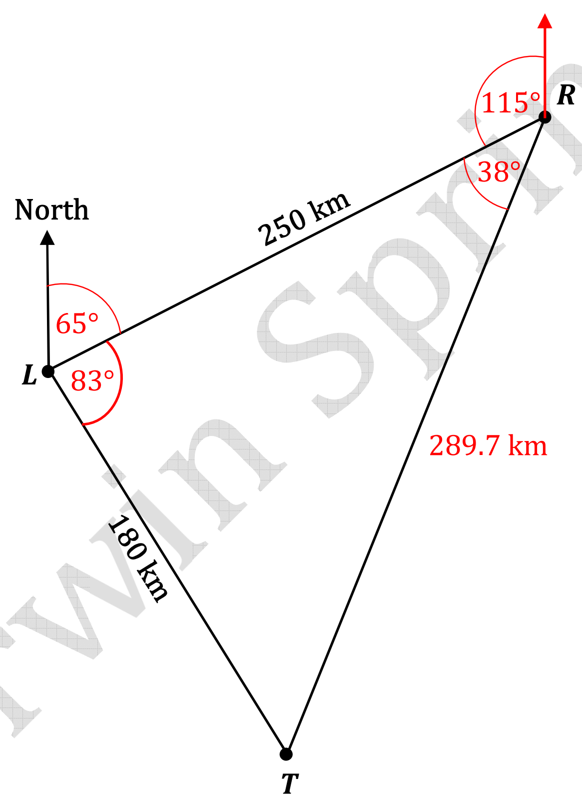
$$= (250)^2 + (180)^2 - 2(250)(180)\cos 83°$$

$$= 83931.75909$$

$$RT = \sqrt{83931.75909}$$

$$= 289.7 \text{ km} \qquad \text{(to 1 decimal place)}$$

(iii) Determine the bearing of T from R. [3]



Using the sine rule,

$$\frac{\sin L\hat{R}T}{LT} = \frac{\sin R\hat{L}T}{RT}$$

$$\frac{\sin L\hat{R}T}{180} = \frac{\sin 83°}{289.7}$$

$$\sin L\hat{R}T = \frac{180 \times \sin 83°}{289.7}$$

$$\sin L\hat{R}T = 0.6167$$

$L\hat{R}T = \sin^{-1}(0.6167)$

$= 38°$ (to the nearest degree)

Therefore,

Bearing of T from $R = 360° - (38° + 115°)$

$= 360° - 153°$

$= 207°$

Total 12 marks

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VECTORS AND MATRICES

10. (a) The transformation matrix $\boldsymbol{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation of $90°$ anticlockwise about the origin O.

The transformation matrix $\boldsymbol{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line with equation $y = -x$.

(i) Write the coordinates of P', the image of the point $P(7, 11)$ after it undergoes a rotation by $90°$ anticlockwise about the origin, O. [1]

$P' = A \times P$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 7 \\ 11 \end{pmatrix}$

$= \begin{pmatrix} (0 \times 7) + (-1 \times 11) \\ (1 \times 7) + (0 \times 11) \end{pmatrix}$

$= \begin{pmatrix} 0 - 11 \\ 7 + 0 \end{pmatrix}$

$= \begin{pmatrix} -11 \\ 7 \end{pmatrix}$

$\therefore$ The coordinates of $P' = (-11, 7)$.

(ii) T is the combined transformation of A followed by B. Determine the elements of the matrix representing the transformation T. [2]

T is the combined transformation of A followed by B.

In other words, $T = BA$.

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$T = BA$

$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} (0 \times 0) + (-1 \times 1) & (0 \times -1) + (-1 \times 0) \\ (-1 \times 0) + (0 \times 1) & (-1 \times -1) + (0 \times 0) \end{pmatrix}$$

$$= \begin{pmatrix} 0 - 1 & 0 + 0 \\ 0 + 0 & 1 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(iii) Describe, geometrically, the transformation represented by T. [2]

$$T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The transformation represented by T is a reflection in the y-axis.

(b) The 2×2 matrix C is defined, in terms of a scalar constant k, by

$$C = \begin{pmatrix} 3 & k \\ 6 & 4 \end{pmatrix}.$$

Determine the value of k, given that the matrix C is singular. [2]

Since the matrix C is singular, then $\det(C) = 0$.

$\det(C) = 0$

$ad - bc = 0$

$(3)(4) - (k)(6) = 0$

$12 - 6k = 0$

$6k = 12$

$k = \frac{12}{6}$

$k = 2$

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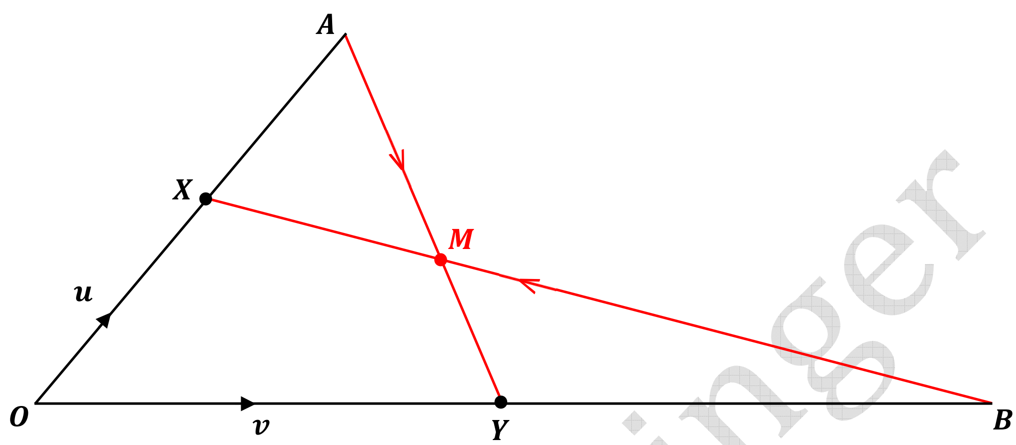
(c) In the diagram below, O is the origin, $\overrightarrow{OX} = u$ and $\overrightarrow{OY} = v$. OX and OY are extended so that X and Y are the midpoints of OA and OB respectively.



(i) Express $\overrightarrow{BX}$ in terms of u and v. [1]

Y is the midpoint of OB.

Hence,

$\overrightarrow{OY} = \frac{1}{2}\overrightarrow{OB}$

$v = \frac{1}{2}\overrightarrow{OB}$

$\overrightarrow{OB} = 2v$

Now,

$\overrightarrow{BX} = \overrightarrow{BO} + \overrightarrow{OX}$

$= -\overrightarrow{OB} + \overrightarrow{OX}$

$= -2v + u$

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(ii) Given that YA and BX intersect at M and $BM = 2MX$,

(a) express $\overrightarrow{BM}$ in terms of u and v. [1]

$\overrightarrow{BM} = 2\overrightarrow{MX}$

So,

$$\overrightarrow{BM} = \frac{2}{3}\overrightarrow{BX}$$

$$= \frac{2}{3}(-2v + u)$$

$$= -\frac{4}{3}v + \frac{2}{3}u$$

(b) using a vector method, show that the ratio YM: YA is 1:3.

Show ALL working. [3]

$$\overrightarrow{YA} = \overrightarrow{YO} + \overrightarrow{OA}$$

$$= -v + u + u$$

$$= -v + 2u$$

Now,

$$\overrightarrow{YM} = \overrightarrow{YB} + \overrightarrow{BM}$$

$$= v + -\frac{4}{3}v + \frac{2}{3}u$$

$$= -\frac{1}{3}v + \frac{2}{3}u$$

$$= \frac{1}{3}(-v + 2u)$$

So, $\overrightarrow{YA}$ is related to $\overrightarrow{YM}$ by a scalar factor of 3.

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$\therefore \overrightarrow{YM} = \frac{1}{3}\overrightarrow{YA}$

Hence,

$YM : YA$

$1 : 3$

Total 12 marks

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END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.