THE STUDENT HUB

CSEC Mathematics

June 2023 – Paper 2

Solutions

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Find the EXACT value of

$$\frac{5}{6}+\frac{2}{3}-\frac{12}{35}\times\frac{7}{9}$$ [2]

$$\frac{5}{6}+\frac{2}{3}-\frac{12}{35}\times\frac{7}{9}=\left(\frac{5}{6}+\frac{4}{6}\right)-\frac{\cancel{12}^{4}}{{}_{5}\cancel{35}}\times\frac{\cancel{7}^{1}}{\cancel{9}_{3}}$$

$$=\frac{9}{6}-\frac{4}{15}$$

$$=\frac{3}{2}-\frac{4}{15}$$

$$=\frac{45-8}{30}$$

$$=\frac{37}{30}$$

(b) (i) Calculate the value of $\sqrt{1-(\cos 37°)^2}$ correct to 3 decimal places. [2]

With the use of a calculator,

$\sqrt{1-(\cos 37°)^2}=0.602$ (to 3 decimal places)

(ii) Write 0.00527 in standard form. [1]

$0.00527 = 5.27\times 10^{-3}$

(c) Haresh works at a call centre for 35 hours each week. He is paid an hourly rate of \$11.20.

(i) Calculate the amount of money Haresh earns in a **four-week month**. [2]

In one week, Haresh earns $= 35 \times \$11.20$

$= \$392$

In four weeks, Haresh earns $= \$392 \times 4$

$= \$1568$

(ii) In a certain week, Haresh works 8 hours overtime. Overtime hours are paid at $1\frac{1}{2}$ times the usual rate of \$11.20 per hour.

Find the TOTAL amount of money Haresh is paid for **that week**. [2]

Overtime rate $= 1.5 \times$ Basic rate

$= 1.5 \times \$11.20$

$= \$16.80$

Overtime wage $=$ Overtime rate $\times$ Overtime hours

$= \$16.80 \times 8$

$= \$134.40$

In one week, his basic wage $= 35 \times \$11.20$

$= \$392$

Therefore,

Total wage for that week = Basic wage + Overtime wage

$= \$392 + \$134.40$

$= \$526.40$

Total: 9 marks

2. (a) Simplify $\frac{4}{5x} \times \frac{15x}{16}$. [1]

$\frac{\cancel{4}}{\cancel{5x}} \times \frac{\overset{3}{\cancel{15x}}}{\underset{4}{\cancel{16}}} = \frac{3}{4}$

(b) Solve the inequality $12 - 4m \leq 5 - 8m$. [2]

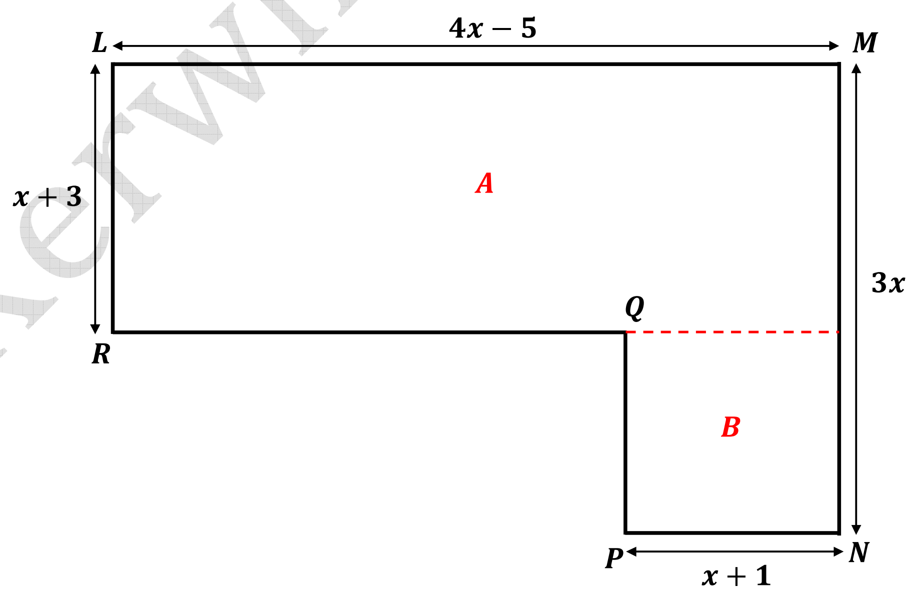
$12 - 4m \leq 5 - 8m$

$-4m + 8m \leq 5 - 12$

$4m \leq -7$

$m \leq -\frac{7}{4}$

(c) The diagram below shows a compound shape, $LMNPQR$, made from two rectangles. The lengths in the diagram, which are written in terms of x, are in centimetres.



(i) Find an expression, in terms of x, for the length

(a) PQ [1]

$$PQ = MN - LR$$
$$= 3x - (x+3)$$
$$= 3x - x - 3$$
$$= 2x - 3$$

(b) RQ [1]

$$RQ = LM - PN$$
$$= 4x - 5 - (x+1)$$
$$= 4x - 5 - x - 1$$
$$= 3x - 6$$

(ii) Given that the TOTAL area of the shape is 414 cm^2, show that

$x^2 + x - 72 = 0.$ [4]

Area of section $A = LM \times LR$

$$= (4x-5)(x+3)$$
$$= 4x^2 + 12x - 5x - 15$$
$$= 4x^2 + 7x - 15$$

Area of section $B = PQ \times PN$

$$= (2x-3)(x+1)$$
$$= 2x^2 + 2x - 3x - 3$$
$$= 2x^2 - x - 3$$

So, we have,

Total area of the shape = Area of section A + Area of section B

$$= 4x^2 + 7x - 15 + 2x^2 - x - 3$$

$$= 6x^2 + 6x - 18$$

Since the total area of the shape is 414 cm^2, we have,

$$6x^2 + 6x - 18 = 414$$

$$6x^2 + 6x - 18 - 414 = 0$$

$$6x^2 + 6x - 432 = 0$$

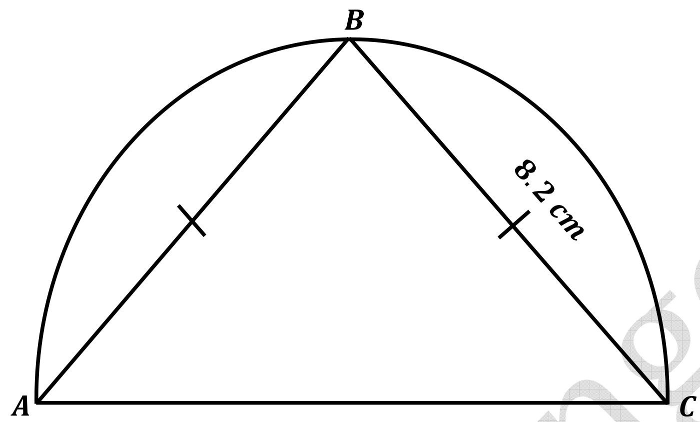
$(\div 6)$

$$x^2 + x - 72 = 0$$

Q.E.D.

Total: 9 marks

3. (a) The diagram below shows a semicircle with diameter AC. B is a point on the circumference and $AB = BC = 8.2\ cm$.



(i) State the geometrical name of the line AB. [1]

The geometrical name of the line AB is a chord.

(ii) Find the value of the radius of the circle. [3]

The angle in a semicircle is equal to 90°.

Therefore, $A\hat{B}C = 90°$.

By Pythagoras' Theorem,

$$c^2 = a^2 + b^2$$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (8.2)^2 + (8.2)^2$$

$$= 67.24 + 67.24$$

$$= 134.48$$

$AC = \sqrt{134.48}$

$= 11.597$ (to 3 decimal places)

Hence,

Value of the radius $= \frac{AC}{2}$

$= \frac{11.597}{2}$

$= 5.80\ cm$ (to 3 significant figures)

(b) Each interior angle of a regular polygon is 160°. Calculate the number of sides of the polygon. [2]

Exterior angle $= 180° - 160°$

$= 20°$

Now,

Exterior angle $= \frac{360}{n}$

$20 = \frac{360}{n}$

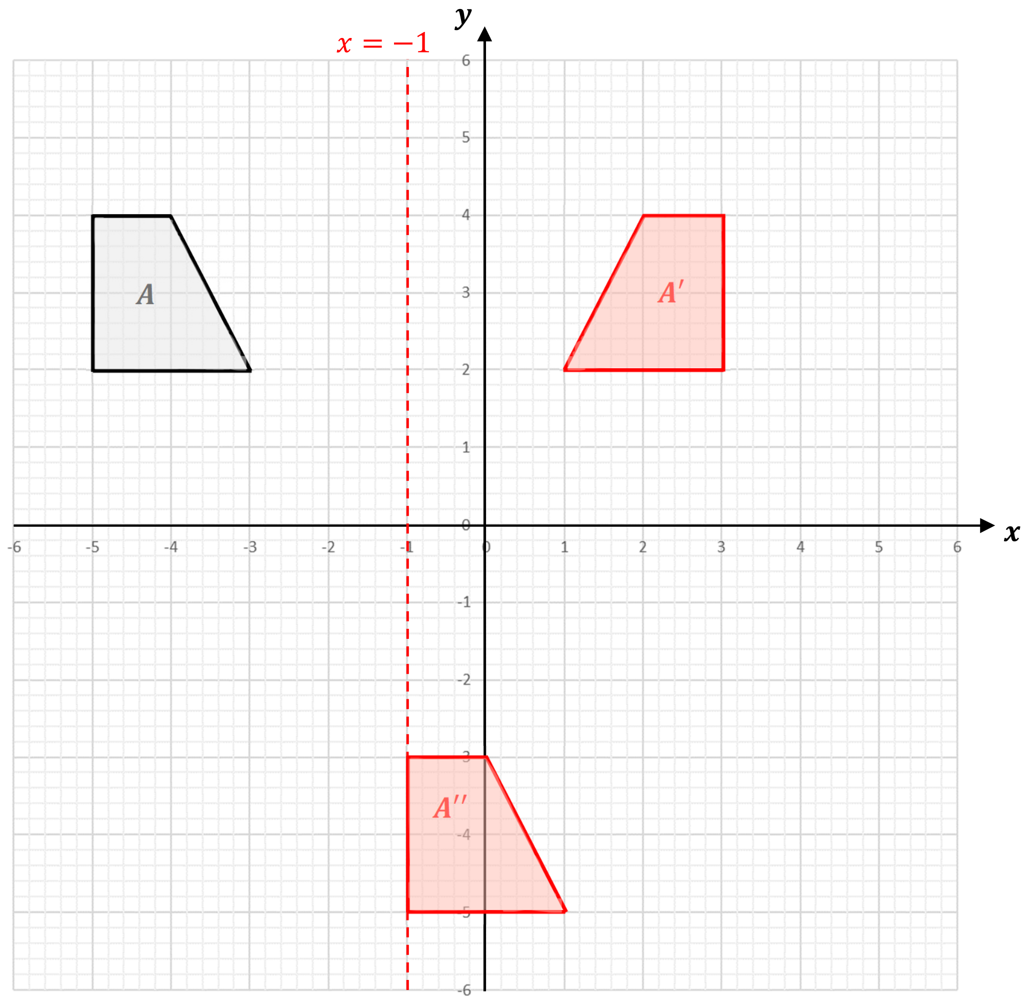
$20n = 360$

$n = \frac{360}{20}$

$n = 18$

$\therefore$ The number of sides of the polygon is 18 sides.

(c) The diagram below shows a trapezium, A, drawn on a square grid.



On the diagram above, draw the image of A after it undergoes a

(i) reflection in the line $x = -1$ and label this image A'. [2]

See graph above.

(ii) translation with vector $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ and label this image A''. [1]

Consider the coordinates of A.

A	$\rightarrow$	A''
$(-5, 2)$	$\rightarrow$	$(-1, -5)$
$(-5, 4)$	$\rightarrow$	$(-1, -3)$
$(-4, 4)$	$\rightarrow$	$(0, -3)$
$(-3, 2)$	$\rightarrow$	$(1, -5)$

Total: 9 marks

4. Consider the following functions.

$$f(x) = \frac{3}{x+2}, g(x) = 4x - 5 \text{ and } h(x) = x^2 + 1$$

(a) (i) For what value of x is $f(x)$ undefined? [1]

$f(x) = \frac{3}{x+2}$

$f(x)$ is undefined when the denominator is equal to zero.

$x + 2 = 0$

$x = -2$

$\therefore f(x)$ is undefined for $x = -2$

(ii) Find the value of

(a) $g\left(\frac{1}{4}\right)$ [1]

$g(x) = 4x - 5$

$g\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 5$

$= 1 - 5$

$= -4$

(b) $h(-3)$ [1]

$h(x) = x^2 + 1$

$h(-3) = (-3)^2 + 1$

$= 9 + 1$

$= 10$

(c) $ff(0)$ [2]

$f(x) = \frac{3}{x+2}$

$f(0) = \frac{3}{0+2}$

$= \frac{3}{2}$

Now,

$ff(0) = f[f(0)]$

$= f\left(\frac{3}{2}\right)$

$= \frac{3}{\left(\frac{3}{2}\right)+2}$

$= \frac{3}{\left(\frac{7}{2}\right)}$

$= 3 \div \frac{7}{2}$

$= 3 \times \frac{2}{7}$

$= \frac{6}{7}$

(b) Write an expression, in its **simplest** form, for $gh(x)$. [2]

$g(x) = 4x - 5$ and $h(x) = x^2 + 1$

$$gh(x) = g[h(x)]$$
$$= g(x^2 + 1)$$
$$= 4(x^2 + 1) - 5$$
$$= 4x^2 + 4 - 5$$
$$= 4x^2 - 1$$ which can be expressed as $(2x - 1)(2x + 1)$.

(c) Find $g^{-1}(-2)$. [2]

$g(x) = 4x - 5$

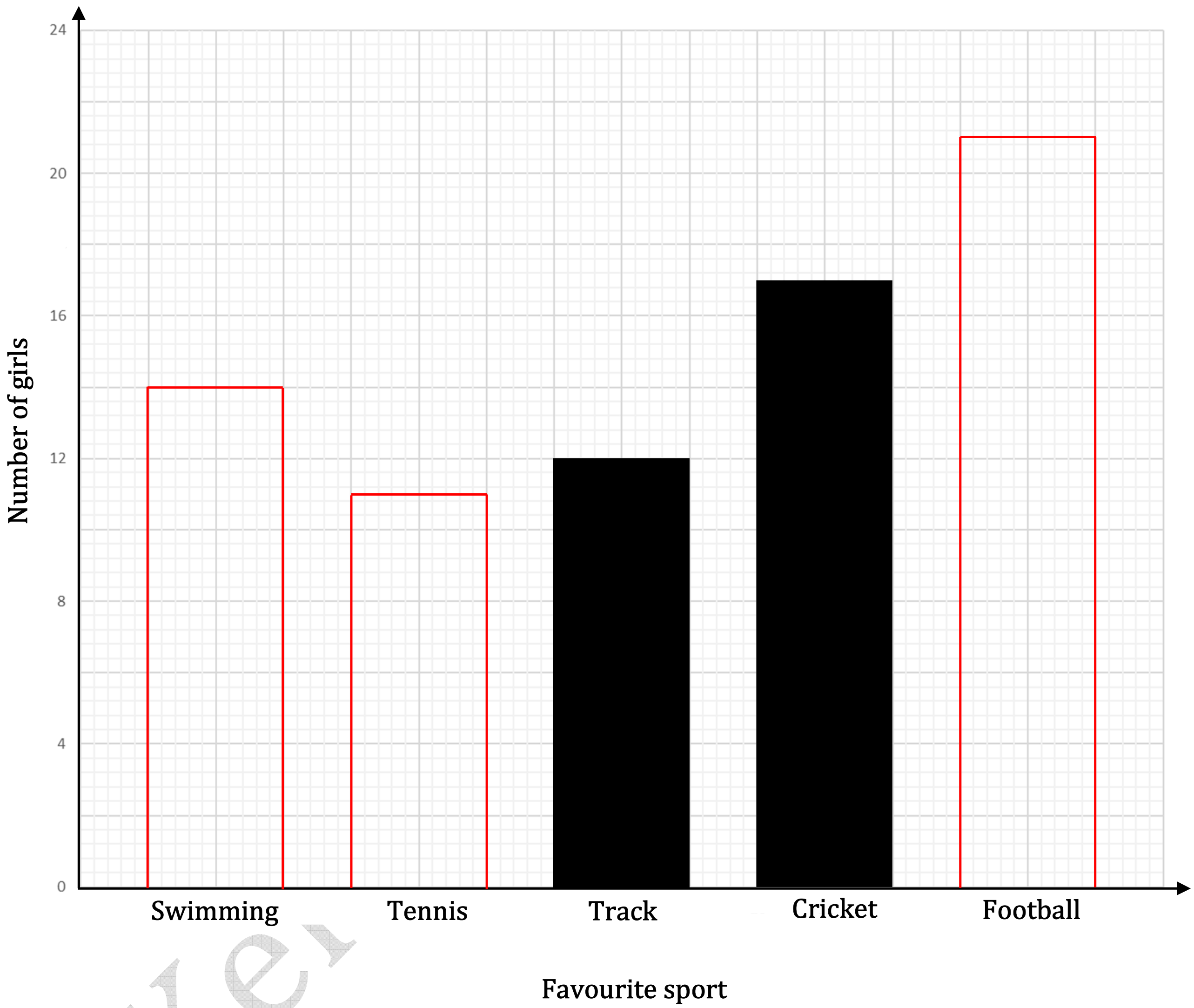
$g^{-1}(x) = \frac{x+5}{4}$

Now,

$$g^{-1}(-2) = \frac{(-2)+5}{4}$$
$$= \frac{3}{4}$$

Total: 9 marks

5. Each of 75 girls recorded the name of her favourite sport. The number of girls who chose track and cricket are shown on the bar chart below.



(a) How many **more** girls chose cricket than track as their favourite sport? [1]

Number of more girls who chose cricket than track $= 17 - 12$

$= 5$ girls

(b) Eleven girls recorded tennis as their favourite sports. For the remaining girls, the number who chose swimming compared to the number who chose football was in the ratio 2: 3.

Use this information to complete the bar chart above. [3]

Number of remaining girls $= 75 - (12 + 17 + 11)$

$= 75 - 40$

$= 35$

Swimming : Football

2 : 3

Number of girls who chose swimming $= \frac{2}{5} \times 35$

$= 14$ girls

Number of girls who chose football $= \frac{3}{5} \times 35$

$= 21$ girls

(c) Determine the modal sport. [1]

The modal sport is Football.

(d) One of the girls is selected at random. What is the probability that she chose NEITHER track NOR cricket as her favourite sport? [2]

Number of girls who chose track or cricket $= 12 + 17$

$= 29$ girls

Number of girls who chose neither track nor cricket $= 75 - 29$

$= 46$ girls

Probability that she chose neither track nor cricket $= \frac{Number\ of\ desired\ outcomes}{Total\ number\ of\ outcomes}$

$= \frac{46}{75}$

(e) The information on the favourite sport of the 75 girls is to be shown on a pie chart. Calculate the sector angle for football. [2]

Sector angle for football $= \frac{21}{75} \times 360°$

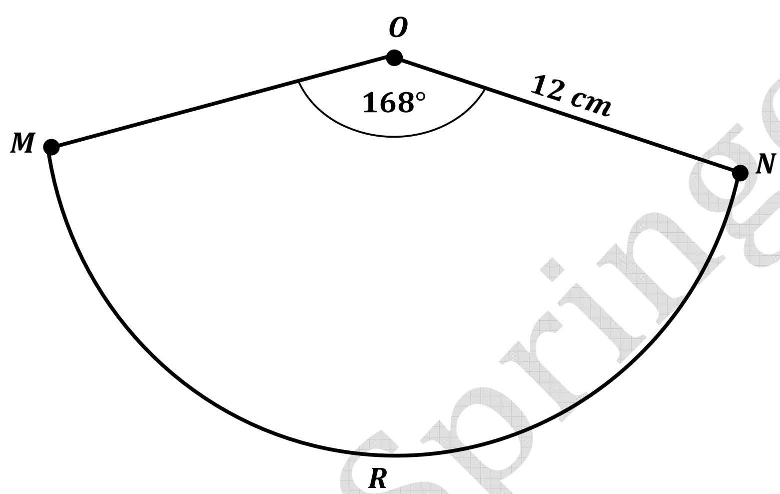
$= \frac{504°}{5}$

$= 100.8°$

Total: 9 marks

6. [In this question, take $\pi = \frac{22}{7}$ and the volume, V, of a cone with radius r and height h as $V = \frac{1}{3}\pi r^2 h$.]

The diagram below shows a sector $OMRN$, of a circle with centre O, radius 12 cm and sector angle 168°, which was formed using a thin sheet of metal.



(a) Calculate the perimeter of the sector above, made from the thin sheet of metal. [3]

$$\text{Length of arc } MRN = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{168°}{360°} \times 2 \times \frac{22}{7} \times 12$$

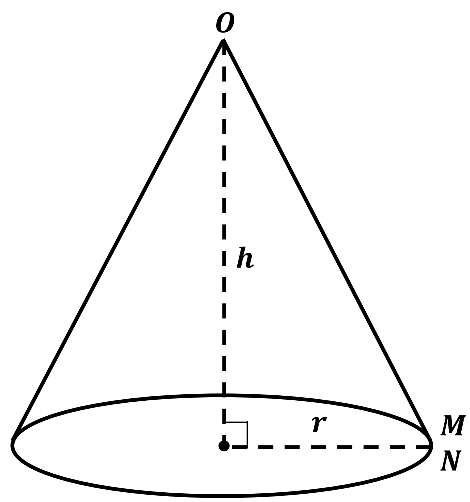
$$= 35.2\ cm$$

$$\text{Perimeter of sector} = OM + ON + MRN$$

$$= 12 + 12 + 35.2$$

$$= 59.2\ cm$$

(b) A cone is made from the sector in (a) by joining OM to ON, as shown below.



(i) Calculate the

(a) radius, r, of the cone [2]

From the diagram, the circumference of the base of the cone = Arc length MN

$\therefore 2\pi r = 35.2$

$r = \frac{35.2}{2\pi}$

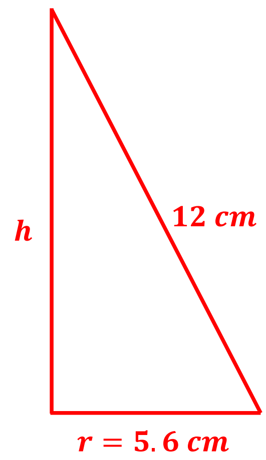
$= \frac{35.2}{2\left(\frac{22}{7}\right)}$

$= 5.6\ cm$

$\therefore$ Radius of the cone, $r = 5.6\ cm$

(b) height, h, of the cone [2]

Consider the triangle below:



By Pythagoras' Theorem,

$$a^2 + b^2 = c^2$$

$$h^2 + (5.6)^2 = (12)^2$$

$$h^2 = (12)^2 - (5.6)^2$$

$$= 144 - 31.36$$

$$= 112.64$$

$$h = \sqrt{112.64}$$

$$= 10.6\ cm \qquad \text{(to 3 significant figures)}$$

$\therefore$ The height, h, of the cone is $10.6\ cm$.

(ii) Calculate the capacity of the cone, **in litres**. [2]

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (5.6)^2 \times 10.6$$

$$= 348.2\ cm^3 \qquad \text{(to 1 decimal place)}$$

Now,

$$1000\ cm^3 = 1 \text{ litre}$$

$$1\ cm^3 = \frac{1}{1000} \text{ litre}$$

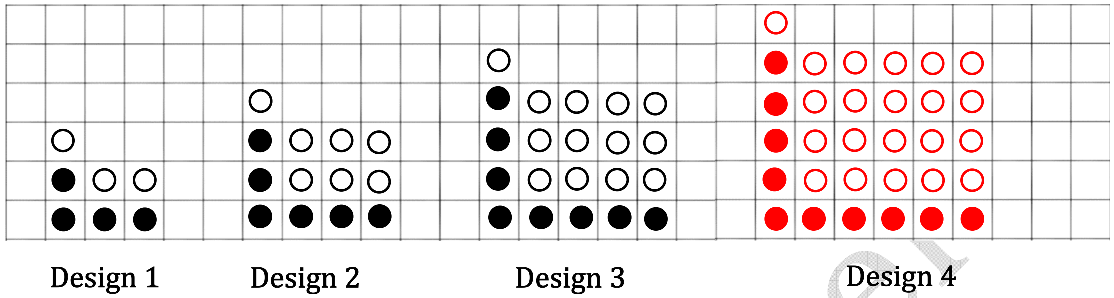
$$348.2\ cm^3 = \frac{1}{1000} \times 348.2$$

$$= 0.348\ l \qquad \text{(to 3 significant figures)}$$

$\therefore$ The capacity of the cone is $0.348\ l$.

Total: 9 marks

7. A sequence of designs is made using black discs and white discs. The first 3 designs in the sequence are shown below.



(a) In the space provided on the grid above, draw Design 4. [2]

See grid above.

(b) The number of white discs, W, the number of black discs, B, and the total number of discs, T, that form each design follow a pattern. The values for W, B and T for the first 3 designs are shown in the table below. Study the pattern of numbers in the table.

Complete Rows (i), (ii) and (iii) in the table below.

	Design Number (**P**)	Number of White Discs (**W**)	Number of Black Discs (**B**)	Total Number of Discs (**T**)	
	1	$(1 \times 1) + 1 + 1 = 3$	4	7	
	2	$(2 \times 2) + 2 + 1 = 7$	6	13	
	3	$(3 \times 3) + 3 + 1 = 13$	8	21	
	⋮	⋮	⋮	⋮	
(i)	9	$(9 \times 9) + 9 + 1 = 91$	20	111	[2]
	⋮	⋮	⋮	⋮	
(ii)	20	$(20 \times 20) + 20 + 1 = 421$	42	463	[2]
	⋮	⋮	⋮	⋮	
(iii)	n	$(n \times n) + n + 1 = n^2 + n + 1$	$2n + 2$	$n^3 + 3n + 3$	[3]

Consider the nth term,

Number of black discs, $B = 2n + 2$

Total number of discs = Number of white discs + Number of black discs

$$T = W + B$$

(i) When $n = 9$,

$B = 2(9) + 2$

$= 18 + 2$

$= 20$

(ii) When $n = 20$,

$B = 2(20) + 2$

$= 40 + 2$

$= 42$

$T = W + B$

$= 421 + 42$

$= 463$

(iii) For the nth term,

$T = W + B$

$= (n^2 + n + 1) + (2n + 2)$

$= n^2 + n + 1 + 2n + 2$

$= n^2 + 3n + 3$

(c) Stephen has 28 black discs and 154 white discs, and wants to make Design 12.

Explain why it is NOT for him to make Design 12. [1]

Number of black discs in Design 12 $= 2(12) + 2$

$= 24 + 2$

$= 26$

Number of white discs in Design 12 $= (12)^2 + 12 + 1$

$= 144 + 12 + 1$

$= 157$

Since Stephen does not have enough white discs, he is unable to make Design 12.

Total: 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) Complete the table for the function $y = -x^2 + x + 7$. [2]

x	-3	-2	-1	0	1	2	3	4
y	-5	1	5	7	7	5	1	-5

When $x = -3$,

$y = -(-3)^2 + (-3) + 7$

$= -9 - 3 + 7$

$= -5$

When $x = 1$,

$y = -(1)^2 + (1) + 7$

$= -1 + 1 + 7$

$= 7$

When $x = -1$,

$y = -(-1)^2 + (-1) + 7$

$= -1 - 1 + 7$

$= 5$

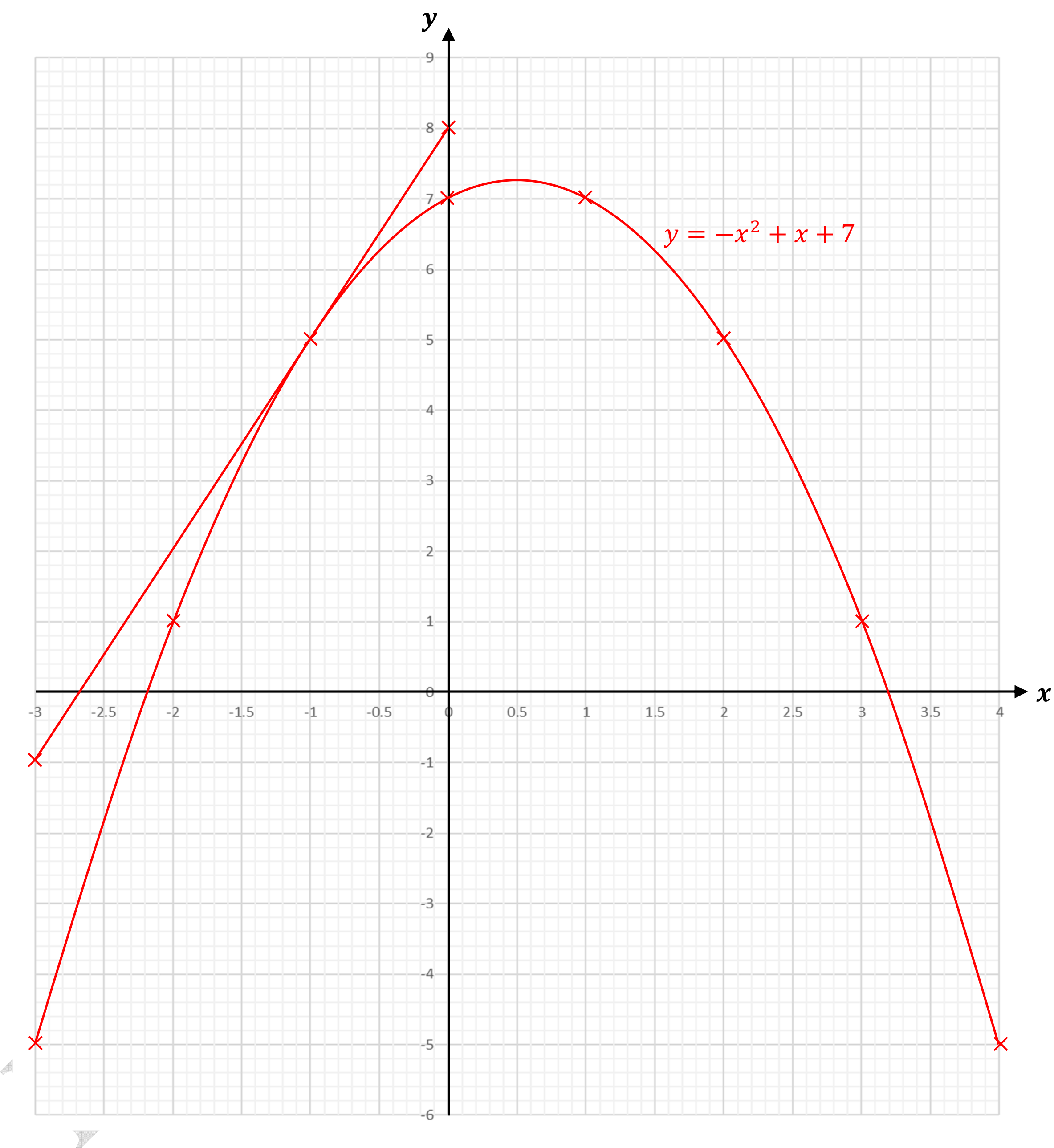
When $x = 3$,

$y = -(3)^2 + (3) + 7$

$= -9 + 3 + 7$

$= 1$

(b) On the grid below, draw the graph of $y = -x^2 + x + 7$ for $-3 \leq x \leq 4$. [3]



(c) Write down the coordinates of the maximum/minimum point of the graph. [1]

(……0.5……, ……7.2……)

(d) Write down the equation of the axis of symmetry of the graph. [1]

The equation of the axis of symmetry is $x = 0.5$.

(e) Use your graph to find the solutions of the equation $-x^2 + x + 7 = 0$. [2]

$x =$ ......-2.2...... or $x =$ ......3.2......

(f) (i) On the grid **on page 24**, draw a line through the points $(-3, -1)$ and $(0, 8)$. [1]

See graph above.

(ii) Determine the equation of this line in the form $y = mx + c$. [2]

Points are $(-3, -1)$ and $(0, 8)$.

$$\text{Gradient, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8-(-1)}{0-(-3)}$$

$$= \frac{8+1}{0+3}$$

$$= \frac{9}{3}$$

$$= 3$$

Substituting $m = 3$ and point $(0, 8)$ into $y - y_1 = m(x - x_1)$ gives:

$y - 8 = 3(x - 0)$

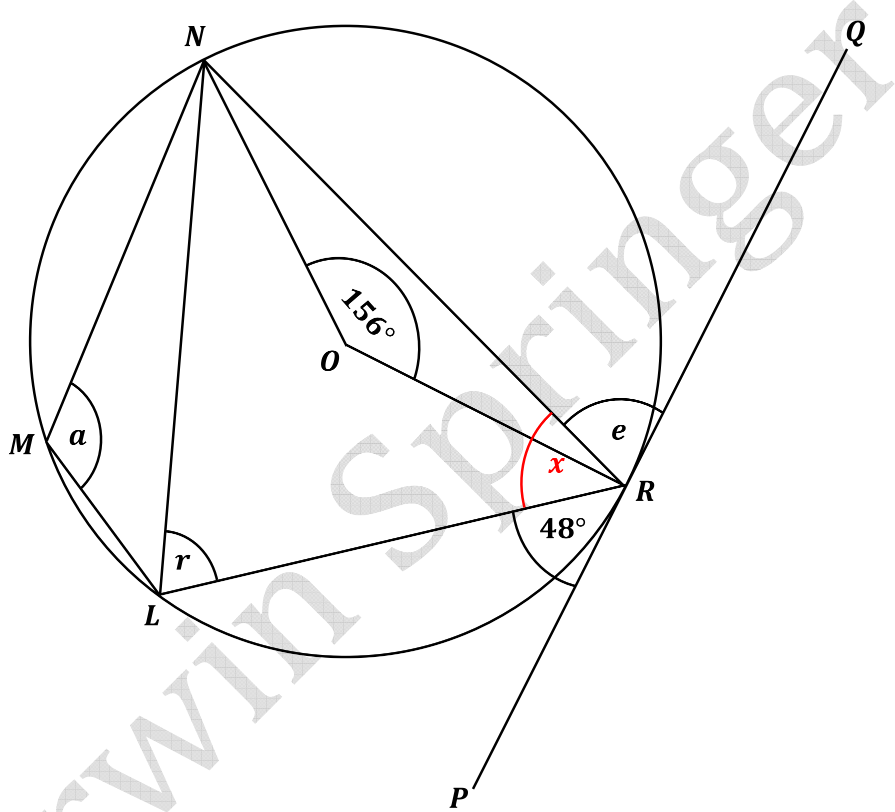
$y - 8 = 3x$

$y = 3x + 8$

$\therefore$ The equation of this line is $y = 3x + 8$.

Total: 12 marks

9. (a) L, M, N and R are points on the circumference of a circle, with centre O. PQ is a tangent to the circle at R. Angle $PRL = 48°$ and Angle $RON = 156°$.



Find the value of EACH of the following angles, giving reasons for EACH of your answers. Show ALL working where appropriate.

(i) Angle r [2]

The angle at the centre of the circle is twice the angle at the circumference of the circle standing on the same chord.

Angle $r = 156° \div 2$

$= 78°$

(ii) Angle e [2]

Angle e is the angle that the chord NR makes with the tangent PQ.

So, angle e is equal to the angle in the alternate segment, Angle r.

$\therefore$ Angle $e = 78°$

(iii) Angle a [2]

The angles at R are supplementary and add up to 180°.

Angle x + Angle $e + 48° = 180°$

Angle $x + 78° + 48° = 180°$

Angle $x = 180° - 78° - 48°$

$= 54°$

Opposite angles in a cyclic quadrilateral add up to 180°.

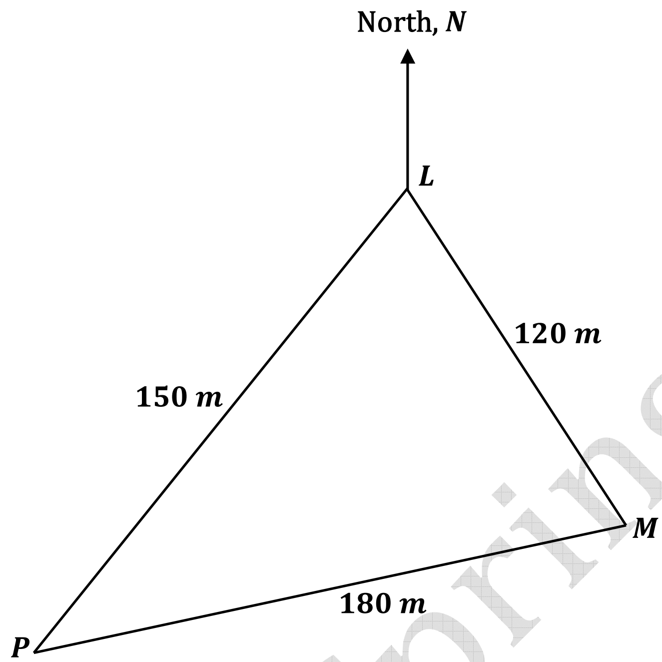
Angle a + Angle $x = 180°$

Angle $a + 54° = 180°$

Angle $a = 180° - 54°$

$= 126°$

(b) The diagram below shows a triangular field, LMP, on horizontal ground.



(i) Calculate the value of Angle MLP. [3]

Using the cosine rule,

$$(PM)^2 = (PL)^2 + (LM)^2 - 2(PL)(LM)\cos M\hat{L}P$$

$$(180)^2 = (150)^2 + (120)^2 - 2(150)(120)\cos M\hat{L}P$$

$$32400 = 22500 + 14400 - 36000\cos M\hat{L}P$$

$$32400 - 22500 - 14400 = -36000\cos M\hat{L}P$$

$$-4500 = -36000\cos M\hat{L}P$$

$$\cos M\hat{L}P = \frac{-4500}{-36000}$$

$$\cos M\hat{L}P = \frac{1}{8}$$

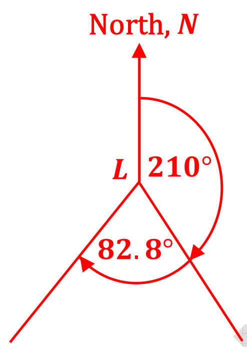
$$M\hat{L}P = \cos^{-1}\left(\frac{1}{8}\right)$$

$$M\hat{L}P = 82.8^\circ \qquad \text{(to 1 decimal place)}$$

(ii) The bearing of P from L is $210°$.

(a) Find the bearing of M from L. [1]

Consider:



Bearing of M from $L = 210° - 82.8°$

$= 127.2°$

(b) Calculate the value of Angle NLP and hence, find the bearing of L from P. [2]

Angle $NLP = 360° - 210°$

$= 150°$

Since Angle NLP and the bearing of L from P are co-interior angles,

Bearing of L from $P = 180° - 150°$

$= 30°$

Total: 12 marks

10. (a) The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ represent the transformations given below.

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents an anticlockwise rotation of 90° about the origin, O.

$B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the straight line with equation $y = -x$.

(i) Determine the elements of the matrix $\boldsymbol{C}$ which represents an anticlockwise rotation of 90° about the origin, O, followed by a reflection in the straight line $y = -x$. [2]

$C = BA$

$$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} (0 \times 0) + (-1 \times 1) & (0 \times -1) + (-1 \times 0) \\ (-1 \times 0) + (0 \times 1) & (-1 \times -1) + (0 \times 0) \end{pmatrix}$$

$$= \begin{pmatrix} 0 + (-1) & 0 + 0 \\ 0 + 0 & 1 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(ii) Describe, geometrically, the single transformation represented by $\boldsymbol{C}$. [2]

The matrix C represents a reflection in the y-axis.

(b) A transformation, T, is defined by the following 2×2 matrix.

$$T = \begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

T maps the point $(2, 3)$ onto the point $(8, 15)$.

Determine the value of k. [2]

$TP = P'$

$$\begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$$

Using the last row,

$2k + (-3) = 15$

$2k - 3 = 15$

$2k = 15 + 3$

$2k = 18$

$k = \frac{18}{2}$

$k = 9$

(c) The following vectors are defined as shown below.

$$\overrightarrow{WX} = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \qquad \overrightarrow{XY} = \begin{pmatrix} -3 \\ 7 \end{pmatrix} \qquad \overrightarrow{ZY} = \begin{pmatrix} 8 \\ -7 \end{pmatrix}$$

Determine EACH of the following.

(i) A vector, other than $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$, that is parallel to $\overrightarrow{WX}$ [1]

A vector parallel to $\overrightarrow{WX}$ is $\begin{pmatrix} 10 \\ -2 \end{pmatrix}$ since $\begin{pmatrix} 10 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

(ii) $\overrightarrow{WY}$ [1]

$$\overrightarrow{WY} = \overrightarrow{WX} + \overrightarrow{XY}$$

$$= \begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 5 + (-3) \\ -1 + 7 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

(iii) $\overrightarrow{XZ}$ [2]

$$\overrightarrow{XZ} = \overrightarrow{XY} + \overrightarrow{YZ}$$

$$= \overrightarrow{XY} - \overrightarrow{ZY}$$

$$= \begin{pmatrix} -3 \\ 7 \end{pmatrix} - \begin{pmatrix} 8 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} -3 - 8 \\ 7 - (-7) \end{pmatrix}$$

$$= \begin{pmatrix} -11 \\ 14 \end{pmatrix}$$

(iv) $|\overrightarrow{XY}|$ [2]

$$|\overrightarrow{XY}| = \sqrt{(-3)^2 + (7)^2}$$

$$= \sqrt{9 + 49}$$

$$= \sqrt{58}$$

$$= 7.62 \quad \text{(to 3 significant figures)}$$

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.