Solutions to CSEC Maths P2 June 2010

Question 1(a)(i)

Required to calculate: $\frac{1\frac{1}{2}-\frac{2}{5}}{4\frac{2}{5}\times\frac{3}{4}}$

$$\text{Numerator} = 1\frac{1}{2} - \frac{2}{5}$$

$$= \frac{3}{2} - \frac{2}{5}$$

$$= \frac{15-4}{10}$$

$$= \frac{11}{10}$$

$$\text{Denominator} = 4\frac{2}{5} \times \frac{3}{4}$$

$$= \frac{22}{5} \times \frac{3}{4}$$

$$= \frac{66}{20}$$

$$= \frac{33}{10}$$

$$\therefore \text{Numerator} \div \text{Denominator} = \frac{11}{10} \div \frac{33}{10}$$

$$= \frac{11}{10} \times \frac{10}{33}$$

$$= \frac{11}{33}$$

$$= \frac{1}{3}$$

Question 1(a)(ii)

Required to calculate: $2.5^2 - \frac{2.89}{17}$

Using a calculator,

$$2.5^2 - \frac{2.89}{17} = 6.25 - 0.17$$

$$= 6.08$$

$$= 6.1 \qquad \text{(to 2 significant figures)}$$

Question 1(b)(i)

Required to calculate the cost of 1 T-shirt.

150 T-shirts = \$1920

1 T-shirt = $\frac{\$1920}{150}$

= \$12.80

Question 1(b)(ii)

Required to calculate the amount for 150 T-shirts at \$19.99 each.

1 T-shirt = \$19.99

150 T-shirts = \$19.99 × 150

= \$2998.50

Question 1(b)(iii)

Required to calculate the profit.

$$\text{Profit} = \text{Selling Price} - \text{Cost Price}$$

$$= \$2998.50 - \$1920$$

$$= \$1078.50$$

Question 1(b)(iv)

Required to calculate the percentage profit.

$$\text{Percentage profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$$

$$= \frac{1078.50}{1920} \times 100\%$$

$$= 56.2\%$$

$$= 56\% \qquad \text{(to the nearest whole number)}$$

Question 2(a)(i)

Required to find the value of $a + b + c$.

$a + b + c = (-1) + 2 + (-3)$

$= -1 + 2 - 3$

$= 2 - 4$

$= -2$

Question 2(a)(ii)

Required to find the value of $b^2 - c^2$.

$b^2 - c^2 = (2)^2 - (-3)^2$

$= 4 - 9$

$= -5$

Question 2(b)(i)

Required to express the statement given as an algebraic expression.

Phrase: "Seven times the sum of x and y."

Algebraic expression: $7(x + y)$

Question 2(b)(ii)

Required to express the statement given as an algebraic expression.

Phrase: "The product of TWO consecutive numbers when the smaller is y."

If the smaller number is y, then the next larger, consecutive number is $y + 1$.

Algebraic expression: $y(y + 1)$

Question 2(c)

Required to solve the given pair of simultaneous equations.

$2x + y = 7$ → Equation 1

$x - 2y = 1$ → Equation 2

Multiplying Equation 2 by 2 gives:

$2x - 4y = 2$ → Equation 3

Equation 1 – Equation 3 gives:

$5y = 5$

$y = \frac{5}{5}$

$y = 1$

Substituting $y = 1$ into Equation 2 gives:

$x - 2(1) = 1$

$x - 2 = 1$

$x = 1 + 2$

$x = 3$

$\therefore x = 3$ and $y = 1$

Question 2(d)(i)

Required to factorise completely $4y^2 - z^2$.

$4y^2 - z^2 = (2y + z)(2y - z)$ → difference of two squares

Question 2(d)(ii)

Required to factorise completely $2ax - 2ay - bx + by$.

$2ax - 2ay - bx + by$

$= 2a(x - y) - b(x - y)$

$= (2a - b)(x - y)$

Question 2(d)(iii)

Required to factorise completely $3x^2 + 10x - 8$.

$3x^2 + 10x - 8$

$= 3x^2 + 12x - 2x - 8$

$= 3x(x + 4) - 2(x + 4)$

$= (3x - 2)(x + 4)$

Question 3(a)(i)

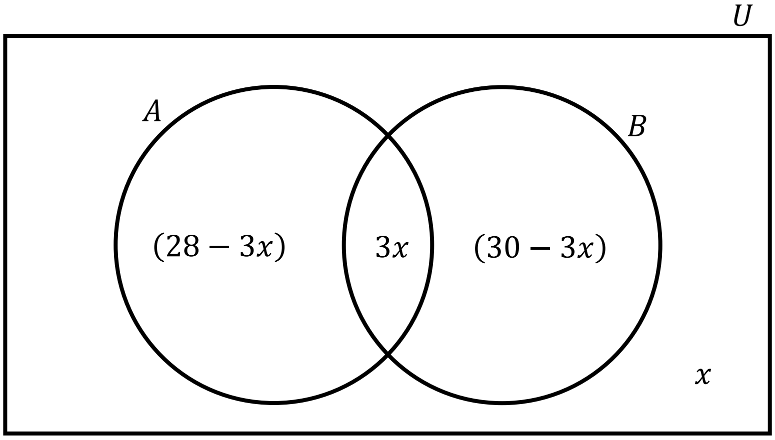
Required to copy and complete the Venn diagram to represent the given information.

28 visited Antigua

30 visited Barbados

$3x$ visited both Antigua and Barbados

x visited neither Antigua nor Barbados



Question 3(a)(ii)

Required to find an expression for the total number of tourists.

Total number of tourists $= (28 - 3x) + 3x + (30 - 3x) + x$

$= 28 - 3x + 3x + 30 - 3x + x$

$= 58 - 2x$

Question 3(a)(iii)

Required to calculate the value of x.

The survey was conducted among 40 tourists.

$58 - 2x = 40$

$2x = 58 - 40$

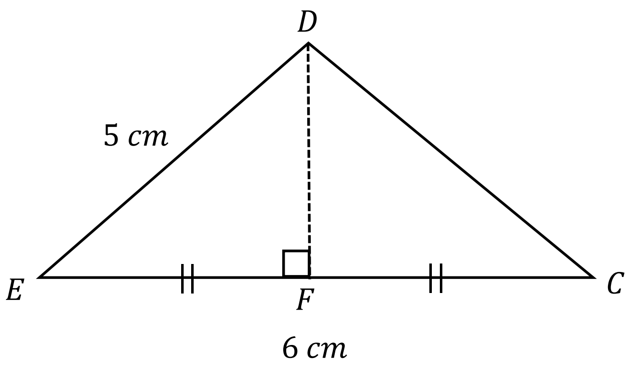
$2x = 18$

$x = \frac{18}{2}$

$x = 9$

Question 3(b)(i)

Required to calculate the length of EF.



F is the midpoint of EC.

$\therefore EF = \frac{6}{2}$

$= 3\ cm$

Question 3(b)(ii)

Required to calculate the length of DF.

By Pythagoras' Theorem,

$$(EF)^2 + (DF)^2 = (DE)^2$$

$$(3)^2 + (DF)^2 = (5)^2$$

$$9 + (DF)^2 = 25$$

$$(DF)^2 = 25 - 9$$

$$(DF)^2 = 16$$

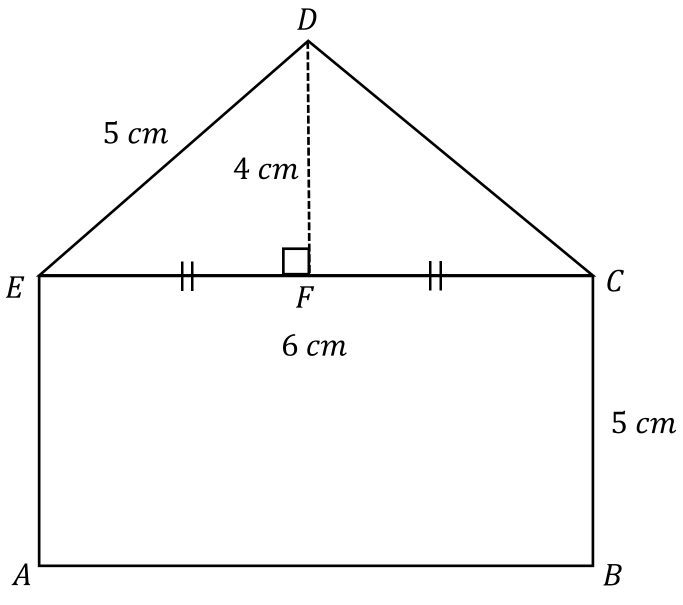
$$DF = \sqrt{16}$$

$$DF = 4\ cm$$

$\therefore$ The length of DF is $4\ cm$.

Question 3(b)(iii)

Required to calculate the area of the face $ABCDE$.



Area of $\Delta DEC = \frac{b \times h}{2}$

$= \frac{6 \times 4}{2}$

$= \frac{24}{2}$

$= 12\ cm^2$

Area of rectangle $ABCE = l \times b$

$= 6 \times 5$

$= 30\ cm^2$

$\therefore$ Area of the entire face $ABCDE = 12 + 30$

$= 42\ cm^2$

Question 4(a)(i)

Required to find the value of k.

Substituting $y = 50$ and $x = 10$ into $y = kx^2$ gives:

$50 = k(10)^2$

$50 = 100k$

$k = \frac{50}{100}$

$k = \frac{1}{2}$

Question 4(a)(ii)

Required to calculate the value of y when $x = 30$.

$y = \frac{1}{2}x^2$

When $x = 30$,

$y = \frac{1}{2}(30)^2$

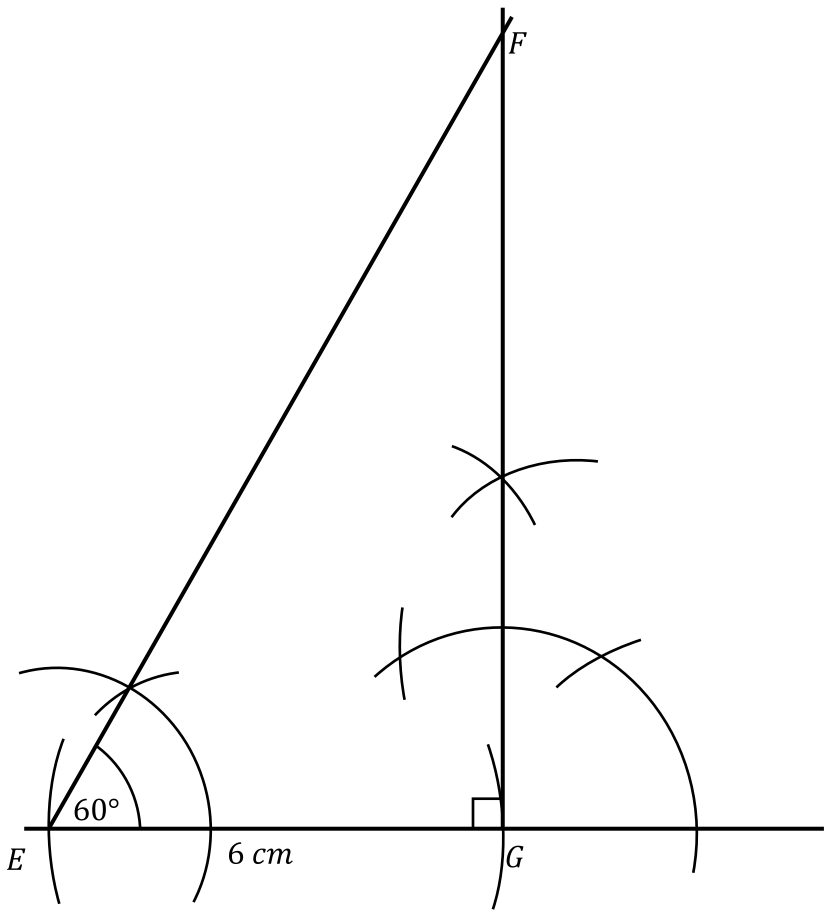
$= \frac{1}{2}(900)$

$= 450$

Question 4(b)(i)

Required to construct triangle EFG.

$EG = 6\ cm$, $\angle FEG = 60°$ and $\angle EGF = 90°$.



Question 4(b)(ii)(a)

Required to find the length of EF.

By measurement, $EF = 12.0\ cm$.

Question 4(b)(ii)(b)

Required to find the size of $\angle EFG$.

By measurement, $E\hat{F}G = 30°$.

Question 5(a)(i)(a)

Required to find the calculate the value of $f(4)$.

$f(x) = 2x - 5$

$f(4) = 2(4) - 5$

$\quad = 8 - 5$

$\quad = 3$

Question 5(a)(i)(b)

Required to find the calculate the value of $gf(4)$.

$gf(4) = g[f(4)]$

$\quad = g(3)$

$\quad = (3)^2 + 3$

$\quad = 9 + 3$

$\quad = 12$

$\therefore gf(4) = 12$

Question 5(a)(ii)

Required to find $f^{-1}(x)$.

$f(x) = 2x - 5$

Let $y = f(x)$.

$y = 2x - 5$

Interchanging variables x and y gives:

$x = 2y - 5$

Making y the subject of the formula gives:

$x + 5 = 2y$

$\frac{x+5}{2} = y$

$\therefore f^{-1}(x) = \frac{x+5}{2}$

Question 5(b)(i)

Required to use the graph to determine the scale used on the x-axis.

On the x-axis, the scale used is $2\ cm = 1\ unit$ or $1\ cm = 0.5\ unit$.

Question 5(b)(ii)

Required to find the value of y for which $x = -1.5$.

When $x = -1.5, y = -3.8$. (by read-off)

Question 5(b)(iii)

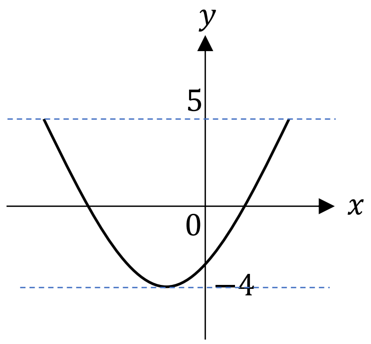
Required to find the values of x for which $y = 0$.

When $y = 0, x = -3$ and $x = 1$. (by read-off)

Question 5(b)(iv)

Required to determine the range of values of y, giving your answer in the form

$a \leq y \leq b$, where a and b are real numbers.



The range of values of y is: $-4 \leq y \leq 5$ which is of the form $a \leq y \leq b$.

Question 6(a)(i)

Required to determine the value of x.

Since alternate angles, $T\hat{U}V$ and $P\hat{V}W$, are equal, then

$x = 54°$

Question 6(a)(ii)

Required to determine the value of y.

Co-interior angles are supplementary.

$y + 115° = 180°$

$y = 180° - 115°$

$y = 65°$

Question 6(b)(i)

Required to describe the rotation fully by stating the centre, the angle and the direction.

Triangle LMN maps onto its image, triangle $L'M'N'$, after undergoing am anticlockwise rotation of 90° about the origin.

Question 6(b)(ii)

Required to state two geometric relationships between triangle LMN and its image, triangle $L'M'N'$.

The two geometric relationships are:

1. ΔLMN maps onto $\Delta L'M'N'$ by a rotation which is a congruent transformation.
2. $\Delta LMN \equiv \Delta L'M'N'$, that is, all corresponding sides and all corresponding angles of the object are the same as that of the image.

Question 6(b)(iii)

Required to determine the coordinates of the image of the point L under the transformation.

Triangle LMN is translated by the vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

The coordinates of L is $(1, 3)$.

Now,

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 + 1 \\ 3 + (-2) \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\therefore$ The coordinates of the image of L is $(2, 1)$.

Question 7(a)

Required to copy and complete the frequency table for the data given.

Distance (m)	**Frequency**
$20 - 29$	3
$30 - 39$	5
$40 - 49$	8
$50 - 59$	6
$60 - 69$	2

Question 7(b)

Required to state the lower boundary for the class interval $20 - 29$.

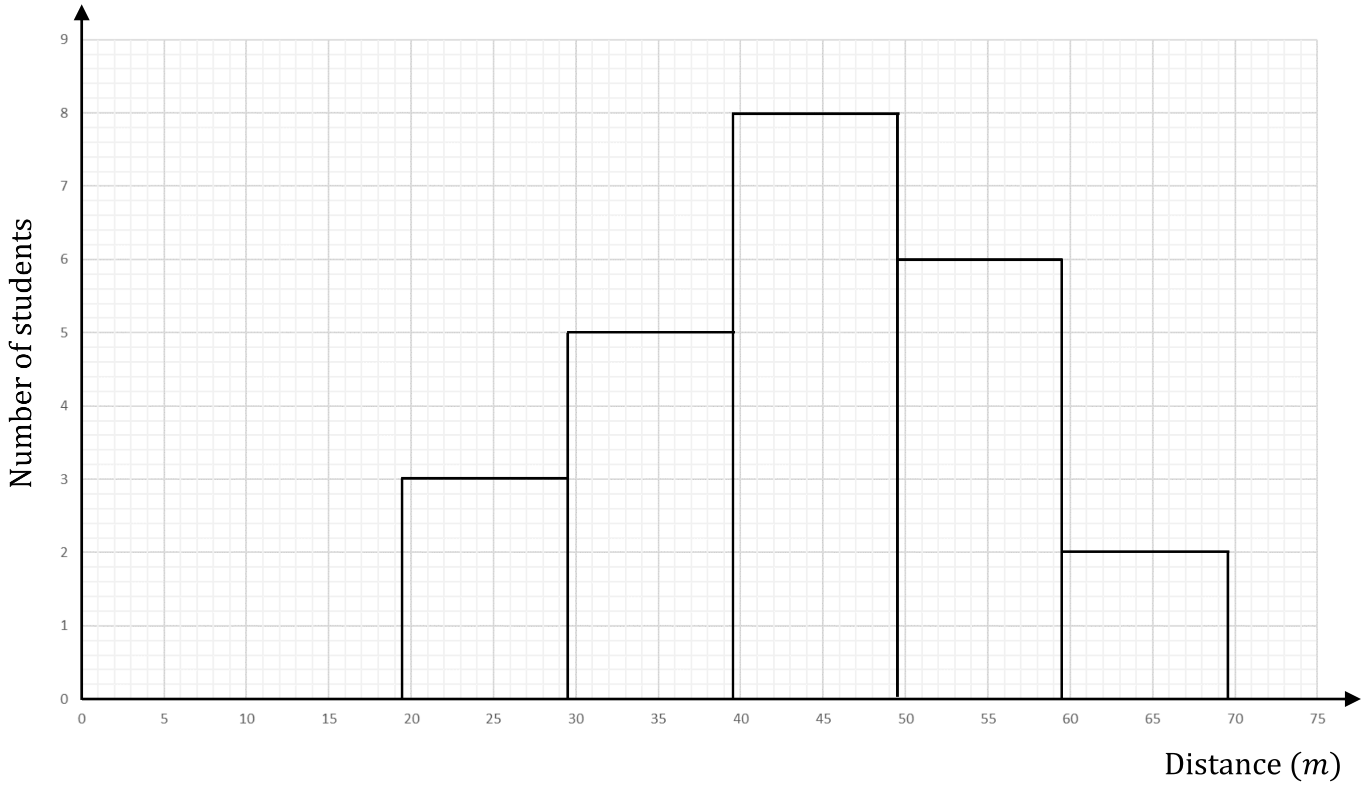
The lower class boundary is 19.5.

Question 7(c)

Required to draw a histogram to illustrate the data.

Distance (m)	**LCB**	**UCB**	**Frequency**
$20 - 29$	19.5	29.5	3
$30 - 39$	29.5	39.5	5
$40 - 49$	39.5	49.5	8
$50 - 59$	49.5	59.5	6
$60 - 69$	59.5	69.5	2
			$\sum f = 24$

Title: Histogram showing the information given.



Question 7(d)(i)

Required to determine the number of students who threw a ball a distance recorded as 50 metres or more.

The number of students who threw the ball 50 m or more $= 6 + 2$

$= 8$

Question 7(d)(ii)

Required to determine the probability that a student, chosen at random, threw the ball a distance recorded as 50 metres or more.

$$P(student\ threw\ the\ ball \geq 50\ m) = \frac{number\ of\ students\ threw\ the\ ball \geq 50\ m}{total\ number\ of\ students}$$

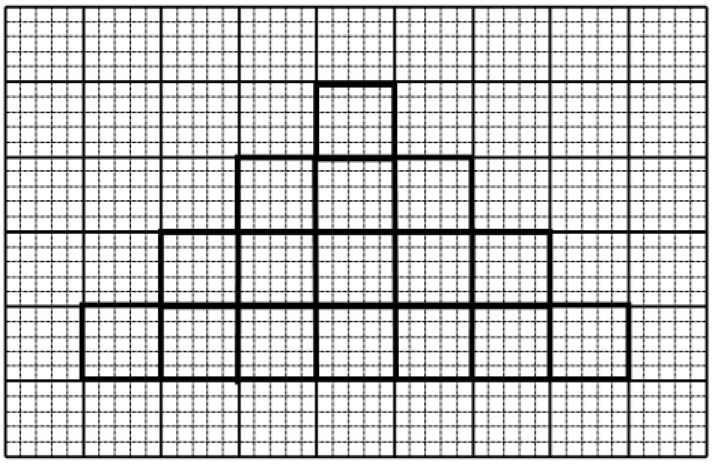
$$= \frac{8}{24}$$

$$= \frac{1}{3}$$

Question 8(a)

Required to draw the fourth diagram in the sequence.

The fourth diagram in the sequence is shown below:



Question 8(b)

Required to complete the table given.

Consider the nth figure.

Area of the figure $= n^2$

Perimeter of the figure $= n \times 6 - 2$

$= 6n - 2$

(i) When $n = 4$,

Area of the figure $= (4)^2$

$= 16$

Now,

Perimeter of the figure $= 4 \times 6 - 2$

$= 24 - 2$

$= 22$

(ii) When $n = 5$,

Area of the figure $= (5)^2$

$= 25$

Now,

Perimeter of the figure $= 5 \times 6 - 2$

$= 30 - 2$

$= 28$

(iii) When $n = 15$,

Area of the figure $= (15)^2$

$= 225$

Now,

Perimeter of the figure $= 15 \times 6 - 2$

$= 90 - 2$

$= 88$

The completed table is shown below.

	Figure	Area of Figure (cm^2)	Perimeter of Figure (cm)
	1	1	$1 \times 6 - 2 = 4$
	2	4	$2 \times 6 - 2 = 10$
	3	9	$3 \times 6 - 2 = 16$
(i)	4	16	$4 \times 6 - 2 = 22$
(ii)	5	25	$5 \times 6 - 2 = 28$
(iii)	15	225	$15 \times 6 - 2 = 88$
(iv)	n	n^2	$n \times 6 - 2 = 6n - 2$

Question 9(a)(i)(a)

Required to use the graph to determine the maximum speed.

The maximum speed is 12 ms^{-1}. (by read off)

Question 9(a)(i)(b)

Required to use the graph to determine the number of seconds for which the speed was constant.

We refer to the part of the graph that is a horizontal line.

The speed was constant for $(10 - 6) = 4$ seconds.

Question 9(a)(i)(c)

Required to use the graph to determine the total distance covered by the athlete during the race.

The total distance covered by the athlete is found by calculating the area under the graph.

Distance covered $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(4 + 13)12$

$= \frac{1}{2}(17)12$

$= 102\ m$

Question 9(a)(ii)(a)

Required to use the graph to determine which time-period of the race the speed of the athlete was increasing.

The athlete's speed was increasing from $t = 0$ to $t = 6$, that is, a period of 6 seconds.

This is illustrated by the part of the graph with the positive gradient.

Question 9(a)(ii)(b)

Required to use the graph to determine which time-period of the race the speed of the athlete was decreasing.

The athlete's speed was decreasing from $t = 10$ to $t = 13$, that is, a period of 3 seconds.

This is illustrated by the part of the graph with the negative gradient.

Question 9(a)(ii)(c)

Required to use the graph to determine which time-period of the race the acceleration of the athlete was zero.

If the acceleration is equal to zero, then this implies that velocity is horizontal.

The acceleration of the athlete was zero from $t = 6$ to $t = 10$, that is, a period of 4 seconds.

This is illustrated by the part of the graph that is horizontal.

Question 9(b)(i)

Required to write an inequality to represent the third condition.

Third condition: the total number of pumpkins and melons must not exceed 12.

Inequality: $x + y \leq 12$

Question 9(b)(ii)

Required to draw the graphs of the three lines associated with the three inequalities.

The three inequalities are:

$y \geq 3$

$y \leq x$

$x + y \leq 12$

The equations of the lines are:

$y = 3$

$y = x$

$x + y = 12$

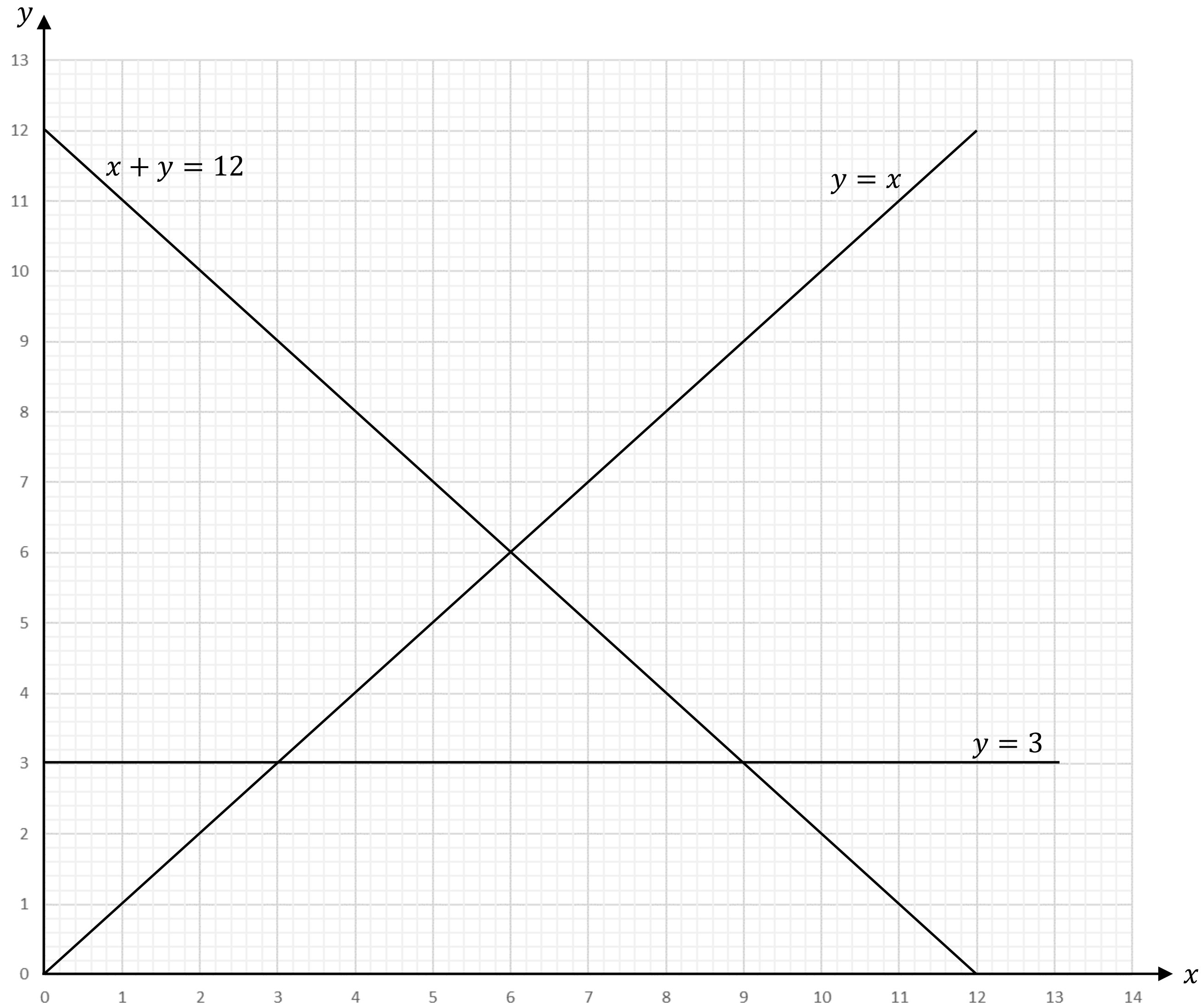
Consider $x + y = 12$.

When $x = 0, y = 12$.

When $y = 0, x = 12$.

Points to be plotted are $(0, 12)$ and $(12, 0)$.

The graph is shown below:



Question 9(b)(iii)

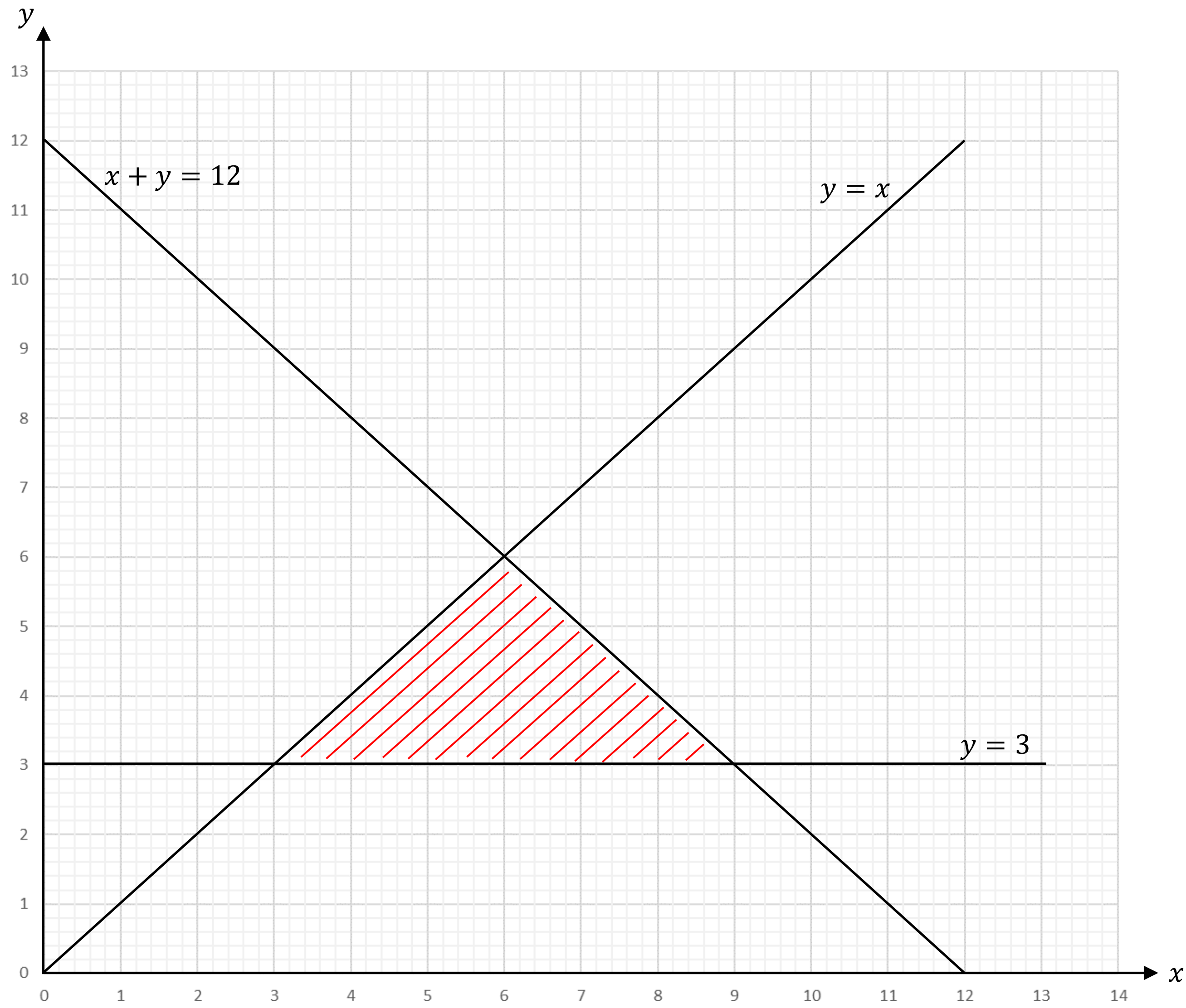
Required to shade the region that satisfies all three inequalities.

The three inequalities are:

$y \geq 3$

$y \leq x$

$x + y \leq 12$



Question 9(b)(iv)

Required to determine the minimum values of x and y which satisfy the conditions.

The vertices of the shaded region are $(3, 3)$, $(6, 6)$ and $(9, 3)$.

The minimum x and y that satisfies the inequalities are $x = 3$ and $y = 3$.

Question 10(a)(i)

Required to calculate $\angle PTR$.

The angle made by the tangent PQ to a circle and a chord, PR, at the point of contact, R, is equal to the angle in the alternate segment.

$\therefore P\hat{T}R = 46°$

Question 10(a)(ii)

Required to calculate $\angle TPR$.

The sum of the angles in a triangle add up to 180°.

$T\hat{P}R = 180° - (46° + 46° + 32°)$

$= 180° - 144°$

$= 56°$

Question 10(a)(iii)

Required to calculate $\angle TSR$.

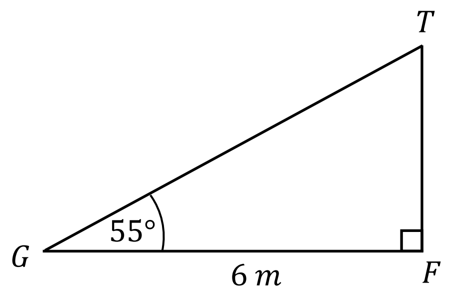
The opposite angles in a cyclic quadrilateral $PRST$ are supplementary.

$T\hat{S}R = 180° - 56°$

$= 124°$

Question 10(b)(i)

Required to calculate the height, FT, of the flagpole.



The triangle TFG is a right-angled triangle.

Now,

$\tan T\hat{G}F = \frac{opp}{adj}$

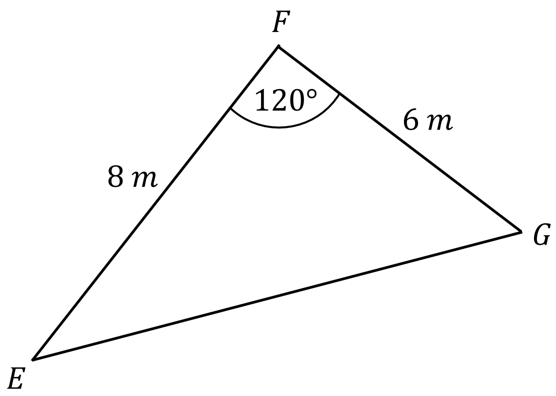
$\tan 55° = \frac{FT}{6}$

$FT = 6 \tan 55°$

$FT = 8.57\ m$ (to 3 significant figures)

Question 10(b)(ii)

Required to calculate the length of EG.



Consider the ΔEFG.

Using the cosine rule,

$(EG)^2 = (EF)^2 + (FG)^2 - 2(EF)(FG)\cos E\hat{F}G$

$(EG)^2 = (8)^2 + (6)^2 - 2(8)(6)\cos 120°$

$(EG)^2 = 64 + 36 - 96\left(-\frac{1}{2}\right)$

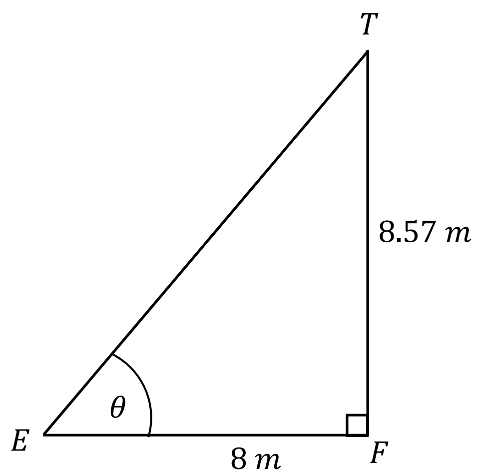
$(EG)^2 = 148$

$EG = \sqrt{148}$

$EG = 12.2\ m$ (to 3 significant figures)

Question 10(b)(iii)

Required to calculate the angle of elevation of T from E.



Triangle EFT is a right-angled triangle.

Let θ be the angle of elevation of T from E.

Now,

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{8.57}{8}$

$\theta = \tan^{-1}\left(\frac{8.57}{8}\right)$

$\theta = 47.0°$ (to 1 decimal place)

Question 11(a)(i)

Required to find AB.

We are given that $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$.

$$AB = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \times 5) + (2 \times -2) & (1 \times -2) + (2 \times 1) \\ (2 \times 5) + (5 \times -2) & (2 \times -2) + (5 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 + (-4) & -2 + 2 \\ 10 + (-10) & -4 + 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Question 11(a)(ii)

Required to determine B^{-1}, the inverse of B.

$$\det(B) = ad - bc$$

$$= (5)(1) - (-2)(-2)$$

$$= 5 - 4$$

$$= 1$$

$$adj(B) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -(-2) \\ -(-2) & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$B^{-1} = \frac{1}{\det(B)} \times adj(B)$

$= \frac{1}{1}\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$

Question 11(a)(iii)

Required to write $\begin{pmatrix} x \\ y \end{pmatrix}$ as the product of two matrices.

$\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Question 11(a)(iv)

Required to calculate the values of x and y.

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} (1 \times 2) + (2 \times 3) \\ (2 \times 2) + (5 \times 3) \end{pmatrix}$

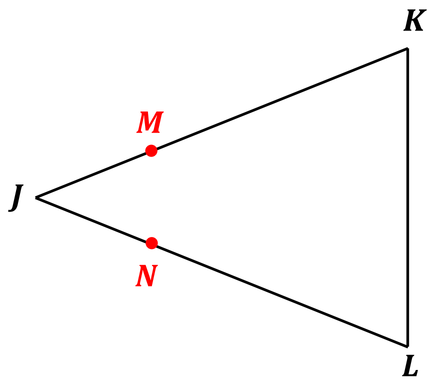
$= \begin{pmatrix} 2 + 6 \\ 4 + 15 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 19 \end{pmatrix}$

$\therefore x = 8$ and $y = 19$.

Question 11(b)(i)

Required to copy and complete the diagram, showing the points M and N.



Question 11(b)(ii)(a)

Required to write an expression for $\overrightarrow{JK}$.

$$\overrightarrow{JK} = 3\overrightarrow{JM}$$

$$= 3\boldsymbol{u}$$

Question 11(b)(ii)(b)

Required to write an expression for $\overrightarrow{MN}$.

$$\overrightarrow{MN} = \overrightarrow{MJ} + \overrightarrow{JN}$$

$$= -\overrightarrow{JM} + \overrightarrow{JN}$$

$$= -\boldsymbol{u} + \boldsymbol{v}$$

$$= \boldsymbol{v} - \boldsymbol{u}$$

Question 11(b)(ii)(b)

Required to write an expression for $\overrightarrow{KL}$.

$\overrightarrow{KL} = \overrightarrow{KJ} + \overrightarrow{JL}$

$= -3\boldsymbol{u} + 3\boldsymbol{v}$

$= 3\boldsymbol{v} - 3\boldsymbol{u}$

$= 3(\boldsymbol{v} - \boldsymbol{u})$

Question 11(b)(iii)

Required to deduce two relationships between KL and MN.

$\overrightarrow{KL} = 3 \times \overrightarrow{MN}$, that is a scalar multiple.

Hence, $\overrightarrow{KL}$ is parallel to $\overrightarrow{MN}$.

So, $|\overrightarrow{KL}| = 3|\overrightarrow{MN}|$, that is, the length of $\overrightarrow{KL}$ is 3 times the length of $\overrightarrow{MN}$.