Solutions to CSEC Maths P2 June 2018

Question 1(a)(i)

$73.18 - 5.23 \times 9.34$

$= 73.18 - (5.23 \times 9.34)$

$= 73.18 - 48.8482$

$= 24.3318$

$= 24.33$ (to 2 decimal places)

Question 1(a)(ii)

$\frac{3.1^2}{6.17} + 1.12$

$= \frac{9.61}{6.17} + 1.12$

$= 1.5575 + 1.12$

$= 2.6775$

$= 2.68$ (to 2 decimal places)

Question 1(b)(i)

$W_J = 600 + (0.9 \times 230)$

$= 600 + 207$

$= 807$

$\therefore$ Jenny's weekly wage is \$807 if she serves 230 customers.

Question 1(b)(ii)

$W_J = 600 + 0.9n$ and $W_J \geq 1000$

Hence,

$600 + 0.9n \geq 1000$

$0.9n \geq 1000 - 600$

$0.9n \geq 400$

$n \geq \frac{400}{0.9}$

$n \geq 444.4$

We need to round up in order for Jenny to earn at least $1000.

$\therefore$ The least number of customers Jenny must serve is 445.

Question 1(b)(iii)

W_s = Shawna's salary and m = number of customers

$\therefore W_s = 270 + 1.5m$

Question 1(b)(iv)

$W_J = 600 + 0.9n$ and $W_s = 270 + 1.5m$

If the number of customers served by both Jenny and Shawna is the same, then $n = m$.

Then,

$600 + 0.9n = 270 + 1.5n$

$600 - 270 = 1.5n - 0.9n$

$330 = 0.6n$

$n = \frac{330}{0.6}$

$n = 550$

$\therefore$ Jenny and Shawn each served 550 customers.

Question 2(a)(i)

$1 - 4h^2$

$= (1 - 2h)(1 + 2h)$ (difference of two squares)

Question 2(a)(ii)

$pq - q^2 - 3p + 3q$

$= q(p - q) - 3(p - q)$

$= (q - 3)(p - q)$

Question 2(b)(i)

$\frac{3}{2}y = 12$

$3y = 12 \times 2$

$3y = 24$

$y = \frac{24}{3}$

$y = 8$

Question 2(b)(ii)

$2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x(x + 3) - 1(x + 3) = 0$

$(2x - 1)(x + 3) = 0$

Either $2x - 1 = 0$ or $x + 3 = 0$

$2x = 1$ $x = -3$

$x = \frac{1}{2}$

Question 2(c)(i)

$F = \frac{m(v-u)}{t}$

When $m = 3, u = -1, v = 2$ and $t = 1$

$F = \frac{3\left(2-(-1)\right)}{1}$

$= 3(2 + 1)$

$= 3(3)$

$= 9$

Question 2(c)(ii)

$F = \frac{m(v-u)}{t}$

$\frac{F}{1} = \frac{m(v-u)}{t}$

Cross-multiplying gives:

$Ft = m(v - u)$

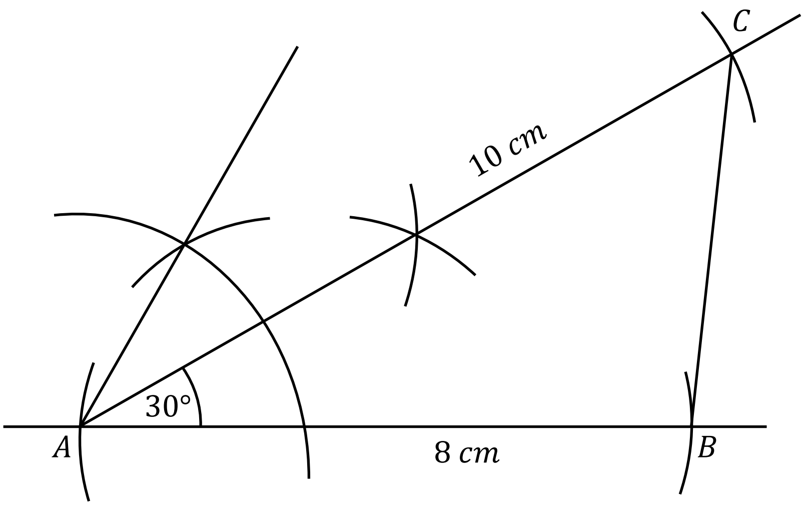
$Ft = mv - mu$

$Ft + mu = mv$

$mv = Ft + mu$

$v = \frac{Ft+mu}{m}$

Question 3(a)



Question 3(b)(i)

The coordinate of the point Q is $(4, 1)$.

Question 3(b)(ii)

$P = (2, 2)$

Now,

$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$

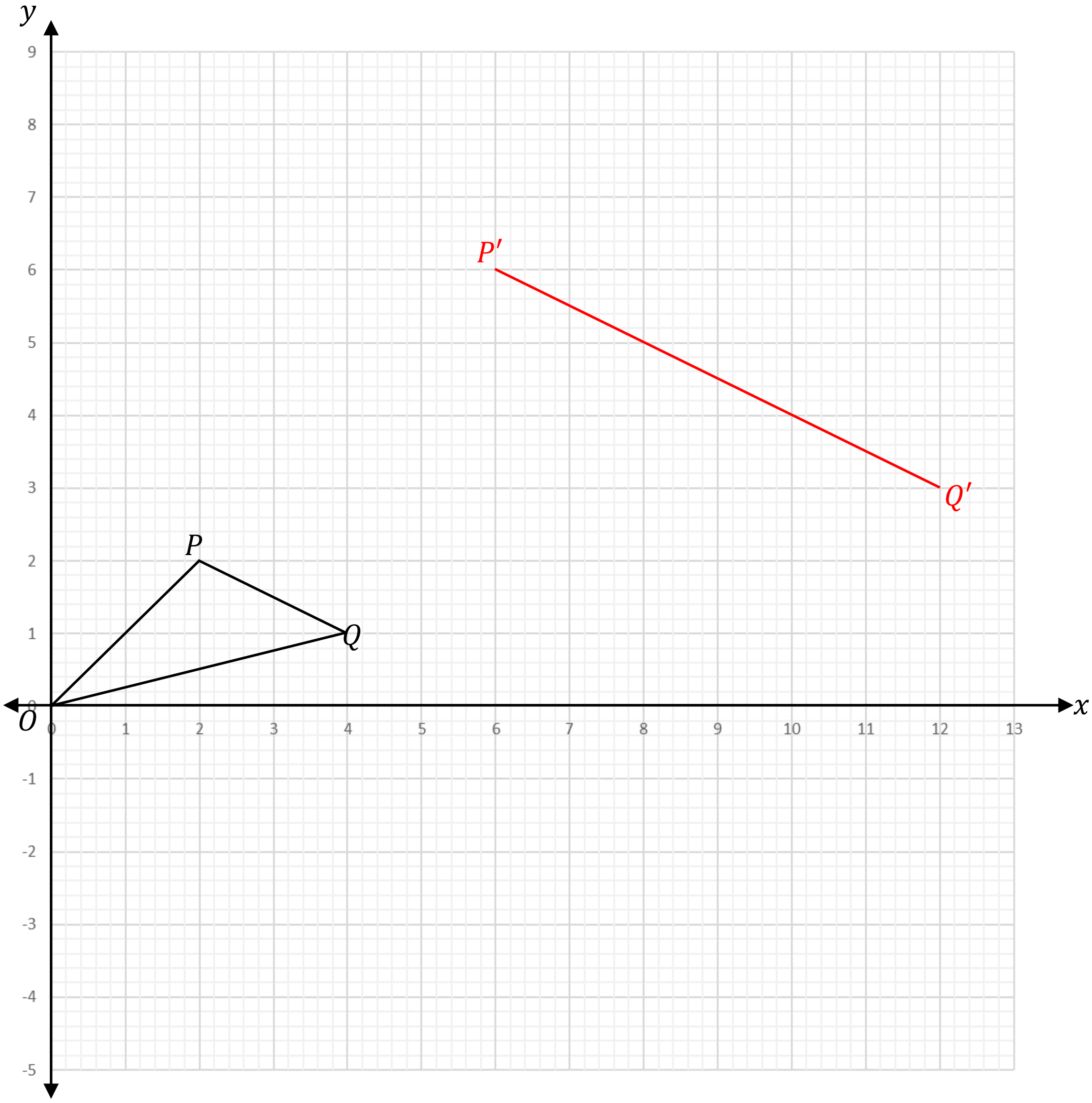
$\therefore P' = (6, 6)$

$Q = (4, 1)$

Now,

$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \end{pmatrix}$

$\therefore Q' = (12, 3)$

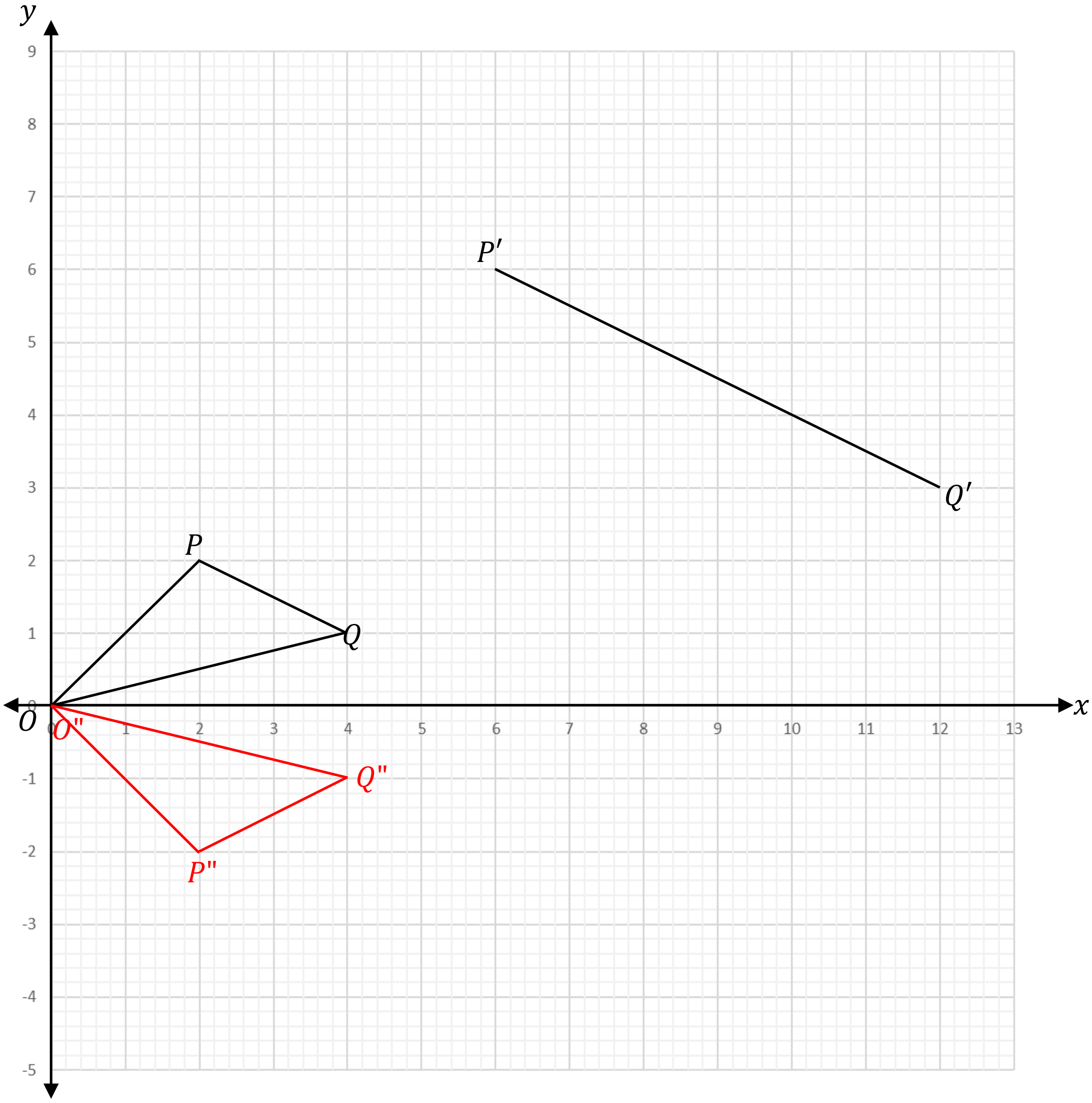


Question 3(b)(ii)

ΔOPQ is reflected in the line $y = 0$ to give $\Delta O"P"Q"$.

$y = 0$ is the equation of the x-axis.

The image $\Delta O"P"Q"$ is shown below:


y
x
O
P
Q
P′
Q′
O"
P"
Q"

Question 4(a)(i)

$f(1) = \frac{1}{2}(1) - 3$

$= \frac{1}{2} - 3$

$= -\frac{5}{2}$ or $-2\frac{1}{2}$

Question 4(a)(ii)

$f(x) = -2$

$\therefore \frac{1}{2}x - 3 = -2$

$\frac{1}{2}x = -2 + 3$

$\frac{1}{2}x = 1$

$x = 1 \times 2$

$x = 2$

Question 4(a)(iii)

From (a)(i), $f(1) = -\frac{5}{2}$

From (a)(ii), $f(2) = -2$

$f(3) = \frac{1}{2}(3) - 3$

$= \frac{3}{2} - 3$

$= -\frac{3}{2}$

So, the list of ordered pairs is $\left\{\left(1, -\frac{5}{2}\right), (2, -2), \left(3, -\frac{3}{2}\right)\right\}$.

Question 4(a)(iv)

If $f(x) = 5$, then

$\frac{1}{2}x - 3 = 5$

$\frac{1}{2}x = 5 + 3$

$\frac{1}{2}x = 8$

$x = 8 \times 2$

$x = 16$

But 16 is out of the domain, A, of the function which includes $\{1, 2, 3\}$.

$\therefore$ $f(x)$ cannot be equal to 5.

Question 4(b)(i)(a)

$3x - 1 < 11$

$3x < 11 + 1$

$3x < 12$

$x < \frac{12}{3}$

$x < 4$

Question 4(b)(i)(b)

$2 \leq 3x - 1$

$2 + 1 \leq 3x$

$3 \leq 3x$

$\frac{3}{3} \leq x$

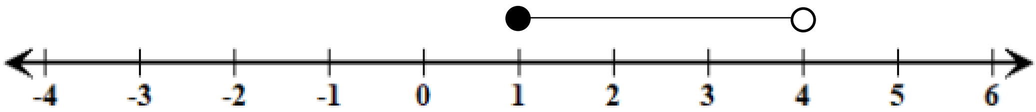
$1 \leq x$

$x \geq 1$

Question 4(b)(ii)

The two inequalities are $x < 4$ and $x \geq 1$.

The solution to $2 \leq 3x - 1 < 11$ is $1 \leq x < 4$.



Question 5(a)(i)

The sum of the angles in a circle add up to 360°.

$x = 360° - (94° + 45°)$

$= 221°$

Question 5(a)(ii)

The percentage of students who chose cricket

$= \frac{Angle\ corresponding\ to\ cricket}{360°} \times 100\%$

$= \frac{94°}{360°} \times 100\%$

$= 26.11\%$ (to 2 decimal places)

Question 5(a)(iii)

Percentage of students who chose Tennis $= \frac{45°}{360°} \times 100\%$

$= 12.5\%$

If 12.5% of students equals 40 students, then

100% of students $= \frac{40}{12.5} \times 100$

$= 320$

∴ The total number of students is 320.

Question 5(b)(i)

The completed table is shown below:

Number of matches(f)	5	7	3	3	4	2	1
Number of goals scored(x)	0	1	2	3	4	5	6

Question 5(b)(ii)

The team scored 1 goal in 7 matches, this score occurs the most, having the highest frequency. Therefore, the modal number of goals scored by the team is 1.

Question 5(b)(iii)

25 matches were played in total. As median is the middle value, we determine the number of goals scored during the 13th match (since half of 25 is 12.5, rounding to 13). At the 13th match, the median score is 2.

Question 5(b)(iii)

Number of matches(f)	5	7	3	3	4	2	1
Number of goals scored(x)	0	1	2	3	4	5	6
fx	0	7	6	9	16	10	6

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{0+7+6+9+16+10+6}{5+7+3+3+4+2+1}$$

$$= \frac{54}{25}$$

$$= 2.16 \text{ goals}$$

Question 6(a)(i)

Circumference $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 21$$

$$= 132\ mm$$

Question 6(a)(ii)

Area of whole disc $= \pi r^2$

$$= \frac{22}{7} \times (21)^2$$

$$= 1386\ mm^2$$

Area of square centre $= s \times s$

$$= 6 \times 6$$

$$= 36\ mm^2$$

Area of cross-section of the disc = Area of whole disc – Area of the square centre

$$= 1386 - 36$$

$$= 1350\ mm^2$$

Question 6(a)(iii)

Volume of 1 disc = Cross-sectional area × thickness

$$= 1350 \times 2$$

$$= 2700\ mm^3$$

Converting to cm^3:

$1000\ mm^3 = 1\ cm^3$

$1\ mm^3 = \frac{1}{1000}\ cm^3$

$2700\ mm^3 = \frac{2700}{1000}\ cm^3$

$= 2.7\ cm^3$

Number of discs that can be made $= \frac{Volume\ of\ metal}{Volume\ of\ 1\ disc}$

$= \frac{1000}{2.7}$

$= 370.4$ discs (to 1 decimal place)

$\therefore$ 370 completed discs can be made.

Question 6(b)(i)

$40\ 000\ km$ is represented by $160\ cm$

$\therefore$ $1\ km$ will be represented by $\frac{160}{40\ 000}\ cm$

$500\ km$ will be represented by $\frac{160}{40\ 000} \times 500 = 2\ cm$

Question 6(b)(ii)

$160\ cm$ is represented by $40\ 000\ km$

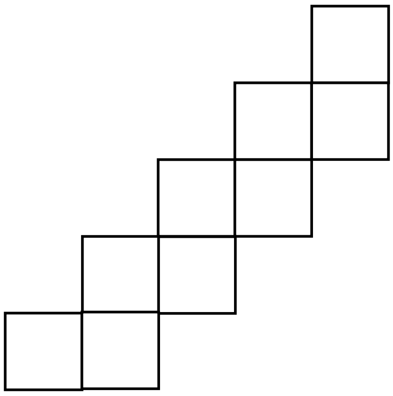
$\therefore$ $1\ cm$ will be represented by $\frac{40\ 000}{160}\ km$

$25\ cm$ will be represented by $\frac{40\ 000}{160} \times 25 = 6250\ km$

$\therefore$ The distance $PQ = 6250\ km$

Question 7(a)

Figure 4 of the sequence is shown below



Question 7(b)

	Figure, n	Number of Squares in Figure (S)	Perimeter of Figure (P)
	1	3	8
	2	5	12
	3	7	16
(i)	4	9	20
	⋮	⋮	⋮
(ii)	21	43	88
(iii)	n	$S = 2n + 1$	$P = 4n + 4$

Consider the nth term.

The number of squares, $S = 2n + 1$

The perimeter of the figure, $P = 4n + 4$

(i) When $n = 4$,

$S = 2(4) + 1$

$= 8 + 1$

$= 9$

$P = 4(4) + 4$

$= 16 + 4$

$= 20$

(ii) When $S = 43$,

$43 = 2n + 1$

$2n = 43 - 1$

$2n = 42$

$n = \frac{42}{2}$

$n = 21$

When $n = 21$,

$P = 4(21) + 4$

$= 84 + 4$

$= 88$

Question 7(c)

$S = 2n + 1$

$P = 4n + 4$

Rearranging equation S to make n the subject of the formula gives,

$n = \frac{S-1}{2}$

Substituting this equation into the equation for P gives,

$P = 4\left(\frac{S-1}{2}\right) + 4$

$= 2(S - 1) + 4$

$= 2S - 2 + 4$

$= 2S + 2$

Therefore, the relationship between the perimeter of the figure, P, and the number of squares in the figure, S, is: $P = 2S + 2$

Question 8(a)(i)

The completed table is shown below:

x	0.1	0.2	0.5	1	1.5	2	2.2	2.5
y	10.3	5.6	$\underline{\mathbf{3.5}}$	4	5.2	$\underline{\mathbf{6.5}}$	7.1	7.9

When $x = 0.5$,

$$y = 3(0.5) + \frac{1}{0.5}$$

$$= 1.5 + 2$$

$$= 3.5$$

When $x = 2$,

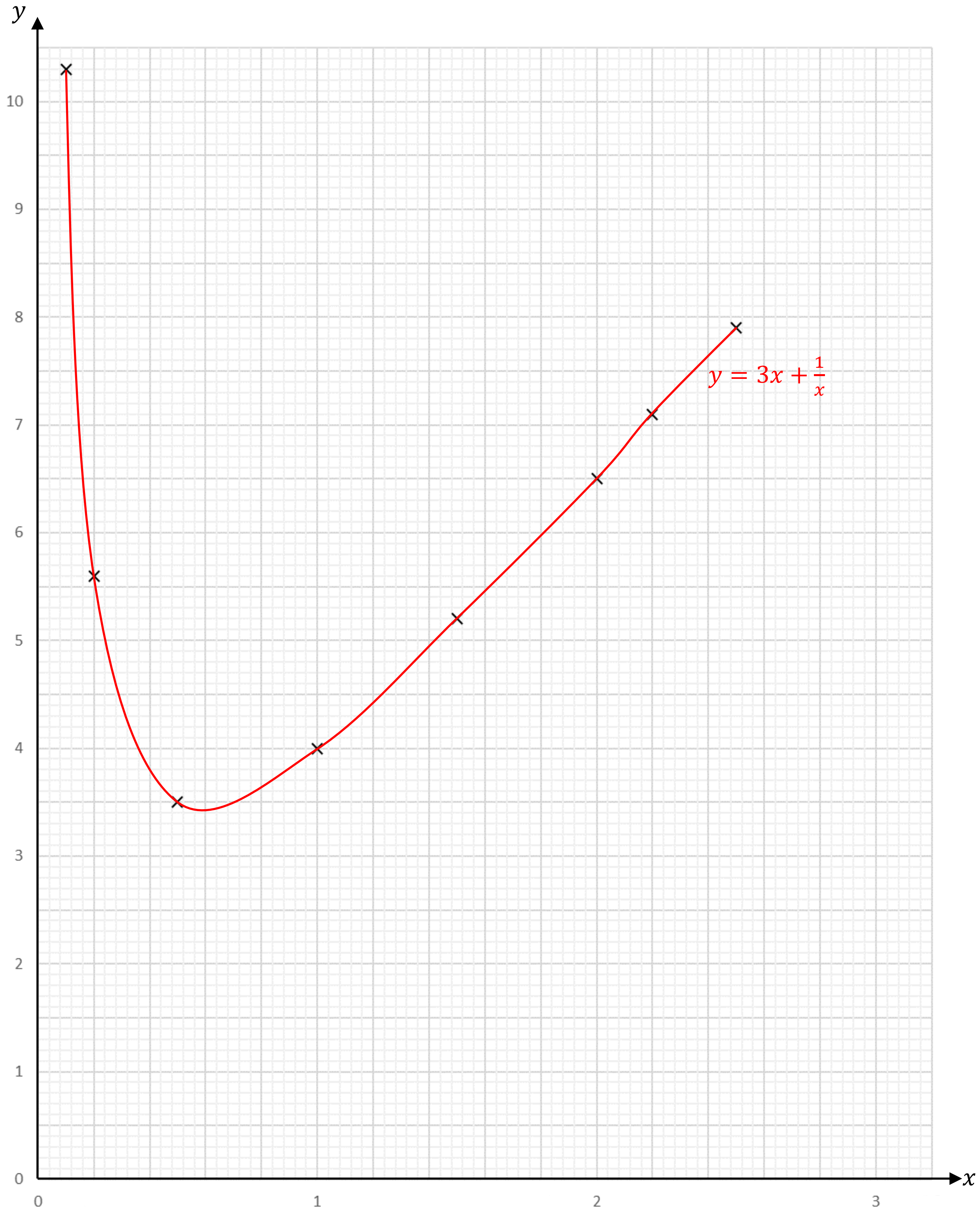
$$y = 3(2) + \frac{1}{2}$$

$$= 6 + 0.5$$

$$= 6.5$$

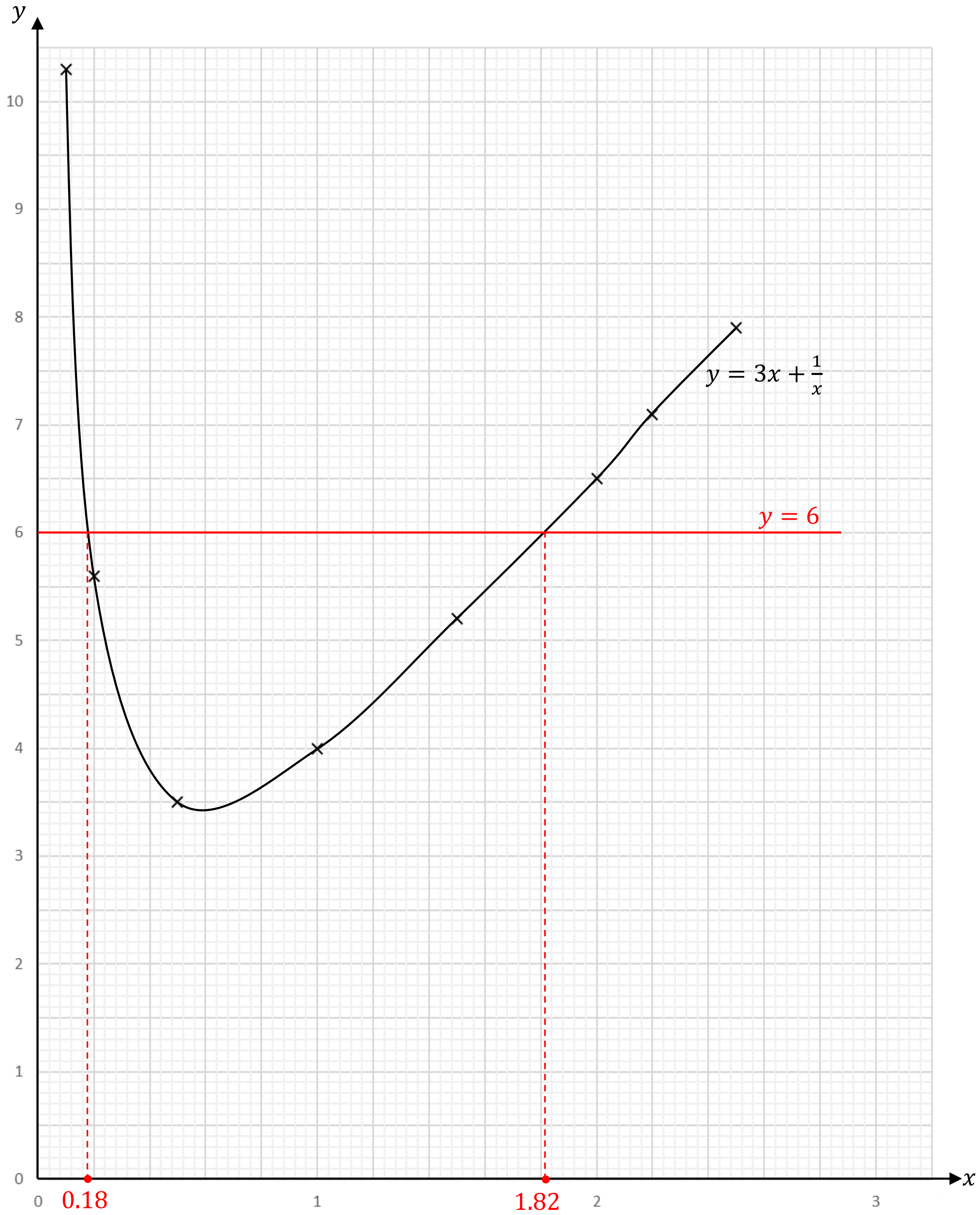
Question 8(a)(ii)

Required to plot the missing points and draw the graph of $y = 3x + \frac{1}{x}$.



Question 8(a)(iii)

Required to find the approximate solutions to the equation $3x + \frac{1}{x} = 6$.



From this diagram, we may identify that the two possible values of x for which

$3x + \frac{1}{x} = 6$ are $x = 0.18$ and $x = 1.82$.

Question 8(b)(i)

To find the acceleration during stage B, we find the gradient of the graph from the beginning of stage B to the end of stage B.

First coordinate would be (40,15).

Second coordinate would be (60,40).

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{40-15}{60-40}$$

$$= \frac{25}{20}$$

$$= 1.25\ ms^{-2}$$

Question 8(b)(ii)

Required to find the average speed of the car in stage B.

By finding area of the trapezium, we may find the total distance from the beginning and end of stage B.

$$\text{Area of trapezium} = \frac{1}{2}(a+b)d$$

$$= \frac{1}{2}(40+15)(60-40)$$

$$= \frac{1}{2}(55)(20)$$

$$= 550\ m$$

Total time taken $= 60 - 40$

$= 20\ s$

Average speed $= \frac{Total\ Distance\ covered}{Total\ time\ taken}$

$= \frac{550}{20}$

$= 27.5\ ms^{-1}$

Question 8(b)(iii)

Required to determine how long it will take the car to come to rest.

We are given that at time $t = 60$ seconds, the car starts to slow down with a uniform deceleration of $2.5\ ms^{-2}$.

Gradient of the deceleration $= \frac{40-0}{60-(60+t)}$

$= \frac{40}{-t}$

Since the rate of deceleration is equal to the gradient, then

$\frac{40}{-t} = -2.5$

$2.5t = 40$

$t = \frac{40}{2.5}$

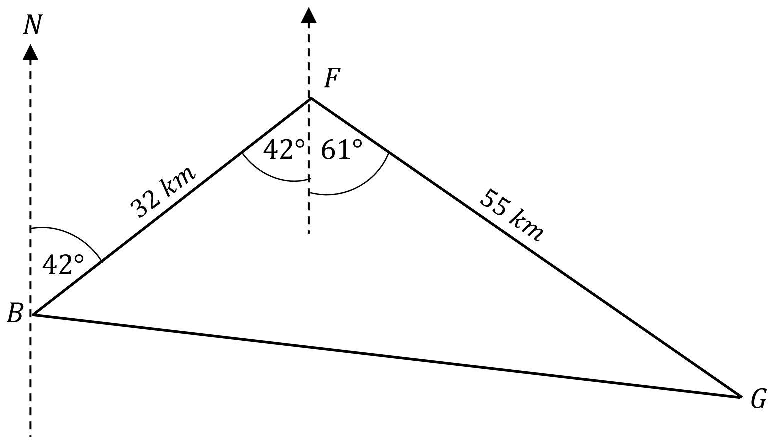
$t = 16\ s$

Hence, the total time to come to rest is

$= 60 + 16$

$= 76$ seconds from the start of travel.

Question 9(a)(i)



Alternate angles are equal to each other.

So, the angle made at F indicated on the diagram is $42°$.

Therefore, the remaining angle is $103° - 42° = 61°$.

The bearing of B from $F = 180° + 42°$

$= 222°$

Question 9(a)(ii)

Consider the triangle BFG.

Using the cosine rule,

$BG^2 = (32)^2 + (55)^2 - 2(32)(55)\cos 103°$

$= 1024 + 3025 - 3520(-0.22495)$

$= 4049 + 791.828$

$= 4840.828$

$BG = 69.58$

$= 69.6\ km$ (to 1 decimal place)

Question 9(a)(iii)

Using the sine rule,

$$\frac{69.58}{\sin 103°} = \frac{55}{\sin\theta}$$

$$\therefore \sin\theta = \frac{55 \times \sin 103°}{69.58}$$

$$= 0.7702$$

$$\theta = \sin^{-1}(0.7702)$$

$$= 50.4°$$

The angle $NBG = 42° + 50.4°$

The angle $NBG = 92.4°$

$\therefore$ The bearing of G from B is 092° (to the nearest degree).

Question 9(b)(i)

Angle $ACB = 90°$ since the angle in a semi-circle is a right angle.

The sum of the angles in a triangle add up to 180°.

Therefore,

Angle $ABC = 180° - (90° + 58°)$

$$= 32°$$

Question 9(b)(ii)

Since opposite angles in a cyclic quadrilateral are supplementary.

Angle $CMB = 180° - 58°$

$$= 122°$$

Question 9(b)(iii)

Since angles in a straight line add up to 180°, then

$$\text{Angle } CMN = 180° - 122°$$

$$= 58°$$

The sum of the angles in a triangle add up to 180°.

Therefore,

$$\text{Angle } NCM = 180° - (90° + 58°)$$

$$= 32°$$

Question 10(a)(i)

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} (2 \times -2) + (-1 \times 3) \\ (2 \times -2) + (0 \times 3) \end{pmatrix}$$

$$= \begin{pmatrix} -4 - 3 \\ -4 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} -7 \\ -4 \end{pmatrix}$$

$\therefore\ a = -7$ and $b = -4$

Question 10(a)(ii)

$$T = \begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$$

$$\det(T) = ad - bc$$

$$= (2)(0) - (-1)(2)$$

$$= 0 + 2$$

$$= 2$$

$$adj\ (T) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -2 & 2 \end{pmatrix}$$

$$\therefore\ T^{-1} = \frac{1}{det} \times adj\ (T)$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{-2}{2} & \frac{2}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{1}{2} \\ -1 & 1 \end{pmatrix}$$

Question 10(a)(iii)(a)

$P = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$

The combined transformation T followed by P is,

$$\begin{aligned} PT &= \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} (0 \times 2) + (1 \times 2) & (0 \times -1) + (1 \times 0) \\ (1 \times 2) + (-2 \times 2) & (1 \times -1) + (-2 \times 0) \end{pmatrix} \\ &= \begin{pmatrix} 0 + 2 & 0 + 0 \\ 2 + (-4) & -1 + 0 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix} \end{aligned}$$

Question 10(a)(iii)(b)

The image of the point $(1, 4)$ under this transformation is

$$\begin{aligned} \begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} &= \begin{pmatrix} (2 \times 1) + (0 \times 4) \\ (-2 \times 1) + (-1 \times 4) \end{pmatrix} \\ &= \begin{pmatrix} 2 + 0 \\ -2 - 4 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ -6 \end{pmatrix} \end{aligned}$$

$\therefore$ The image is $(2, -6)$.

Question 10(b)(i)

Since $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel, they are scalar multiples of each other.

In fact, $\overrightarrow{DC}$ is larger than $\overrightarrow{AB}$ by a scalar factor of 3.

Since $\overrightarrow{DC} = 3\overrightarrow{AB}$, then $|\overrightarrow{AB}| = \frac{1}{3}|\overrightarrow{DC}|$.

Therefore, the completed statement would be:

$\overrightarrow{AB}$ and $\overrightarrow{DC}$ are ………parallel………… and $|\overrightarrow{AB}|$ is ….. $\frac{1}{3}$ ….. times $|\overrightarrow{CD}|$.

Question 10(b)(ii)

According to the Triangle Law,

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -m + \overrightarrow{AC}$

Now, $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

$= n + 3m$

So, $\overrightarrow{BC} = -m + (n + 3m)$

$= n + 2m$

Question 10(b)(iii)

$\overrightarrow{AL} = \overrightarrow{AD} + \overrightarrow{DL}$

$= \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC}$

$= n + \frac{1}{2}(3m)$

So,

$\overrightarrow{BL} = \overrightarrow{BA} + \overrightarrow{AL}$

$= -m + n + \frac{1}{2}(3m)$

$= \frac{1}{2}m + n$