CSEC Mathematics

June 2024 – Paper 2

Solutions

THE STUDENT HUB

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) (i) Calculate the value of $\sqrt{(7.1)^2 + (2.9)^2}$, giving your answer correct to

(a) 2 significant figures [1]

$\sqrt{(7.1)^2 + (2.9)^2} = 7.7$ (to 2 significant figures)

(b) 2 decimal places [1]

$\sqrt{(7.1)^2 + (2.9)^2} = 7.67$ (to 2 decimal places)

(ii) Write the following quantities in ascending order. [1]

$\frac{12}{25}$, 0.46 , 47%

$\frac{12}{25} = 0.48$, 0.46 , $47\% = 0.47$

In ascending order, the numbers should be arranged as 0.46, 47% , $\frac{12}{25}$.

………… 0.46 ………….. < ………… 47% ………….. < ………… $\frac{12}{25}$ …………..

(b) Mahendra and Jaya shared \$7 224 in the ratio 7:5. How much MORE money does Mahendra receive than Jaya? [2]

$\text{Difference} = 7 - 5$

$= 2 \text{ parts}$

$\text{Total parts} = 7 + 5$

$= 12 \text{ parts}$

Now,

$12 \text{ parts} = \$7\,224$

$1 \text{ part} = \frac{\$7\,224}{12}$

$= \$602$

$2 \text{ parts} = 2 \times \$602$

$= \$1204$

$\therefore$ Mahendra receives \$1204 more than Jaya.

(c) The present population of Portmouth is 550 000. It is expected that this population will increase 42% by the year 2030.

(i) Write the number 550 000 in standard form. [1]

$550\ 000 = 5.5 \times 10^5$

(ii) Calculate the expected population in Portmouth in 2030. [1]

Final percentage $= (100 + 42)\%$

$= 142\%$

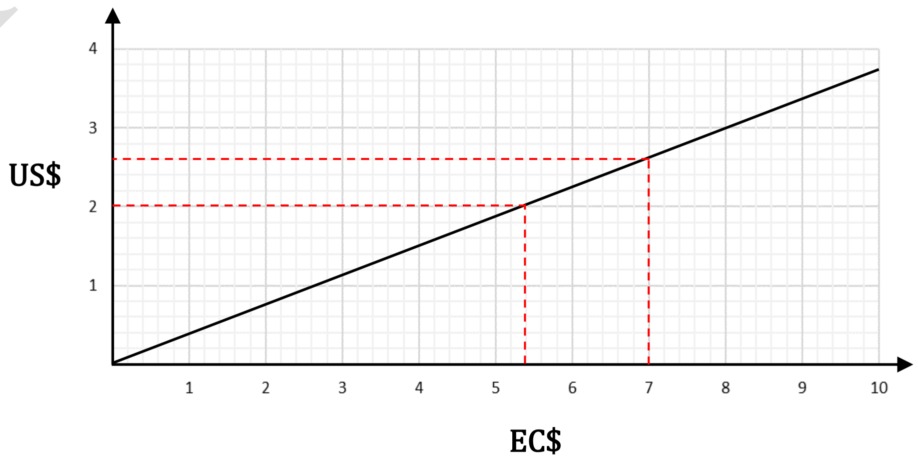
Expected population $= 142\%$ of $550\ 000$

$= \frac{142}{100} \times \frac{550\ 000}{1}$

$= 781\ 000$

$\therefore$ The expected population in Portmouth in 2030 is 781 000.

(d) The graph below can be used to convert between United States dollars (US$) and Eastern Caribbean dollars (EC$).



Using the graph,

(i) convert US$2 to EC$ [1]

From graph, US$2 = EC$5.40.

(ii) convert EC$70 to US$ [1]

From graph, EC$7 = US$2.60.

Now,

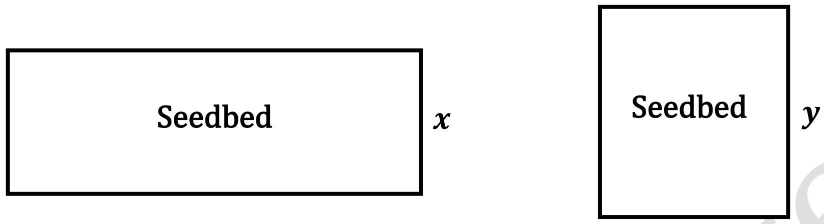
$$\text{EC\$70} = \text{US\$2.60} \times 10$$

$$= \text{US\$26.00}$$

Total: 9 marks

2. Laura needs to put mesh around **two** seedbeds to protect her seedlings.

Altogether, she uses 60 m of mesh. One of the seedbeds is a rectangle and the other is a square, as shown in the diagram below.



The width of the rectangular seedbed is x metres. The length of the rectangular seedbed is 3 times its width. The length of a side of the square seedbed is y metres.

(a) Using the information given above, derive a simplified expression for y in terms of x. [2]

Width of rectangular seedbed $= x$

Length of rectangular seedbed $= 3x$

Now,

Perimeter of rectangular seedbed $= 2(x + 3x)$

$= 2(4x)$

$= 8x$

Length of a side of the square seedbed $= x$

Now,

Perimeter of square seedbed $= 4y$

We are told that altogether, she uses 60 m of mesh.

So, we have,

$8x + 4y = 60$

$(\div 4)$

$2x + y = 15$

$y = 15 - 2x$

$\therefore$ A simplified expression for y in terms of x is $y = 15 - 2x$.

(b) The area of the rectangular seedbed is **equal** to the area of the square seedbed.

(i) Use this information and your answer in (a) to write down a quadratic equation, in terms of x, and show it simplifies to

$$x^2 - 60x + 225 = 0$$ [2]

Area of rectangular seedbed $=$ length $\times$ width

$= 3x \times x$

$= 3x^2$

Area of square seedbed $=$ side $\times$ side

$= y \times y$

$= y^2$

We are given that the area of the rectangular seedbed is equal to the area of the square seedbed.

So, we have,

$3x^2 = y^2$

$3x^2 = (15 - 2x)^2$ [from part (i)]

$3x^2 = 225 - 60x + 4x^2$

$0 = 225 - 60x + 4x^2 - 3x^2$

$0 = 225 - 60x + x^2$

$\therefore x^2 - 60x + 225 = 0$

Q.E.D.

(ii) Solve the equation $x^2 - 60x + 225 = 0$ using the quadratic formula. [3]

$x^2 - 60x + 225 = 0$ which is in the form $ax^2 + bx + c = 0$, where $a = 1, b = -60$ and $c = 225$.

Using the quadratic formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-60) \pm \sqrt{(-60)^2 - 4(1)(225)}}{2(1)}$

$= \frac{60 \pm \sqrt{3600 - 900}}{2}$

$= \frac{60 \pm \sqrt{2700}}{2}$

Either $x = \frac{60 - \sqrt{2700}}{2}$ or $x = \frac{60 + \sqrt{2700}}{2}$

$= 4.019$ (to 4 s.f.) $= 55.98$ (to 4 s.f.)

(iii) Calculate the TOTAL area of the **two** seedbeds. [2]

Recall that: $y = 15 - 2x$

Consider when $x = 55.98$.

Then,

$$y = 15 - 2(55.98)$$
$$= 15 - 111.96$$
$$= -96.96\ m \quad \text{(which is negative)}$$

So, $x = 55.98$ is invalid for a dimension.

Now,

When $x = 4.019$,

$$y = 15 - 2(4.019)$$
$$= 15 - 8.038$$
$$= 6.962\ m$$

Total area of seedbeds $= 3x^2 + y^2$

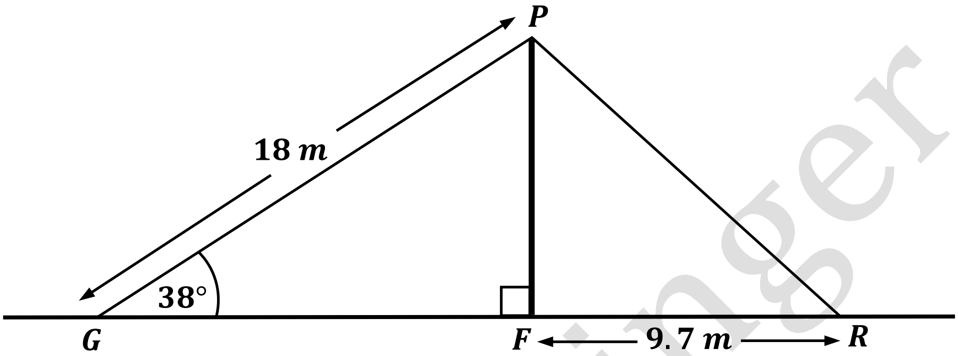
$$= 3(4.019)^2 + (6.962)^2$$
$$= 96.9\ m^2 \quad \text{(to 3 significant figures)}$$

$\therefore$ The total area of the two seedbeds is $96.9\ m^2$.

Total: 9 marks

3. (a) A vertical flagpole, FP, stands on horizontal ground and is held by two ropes, PG and PR, as shown in the diagram below.

$PG = 18\ m$, $FR = 9.7\ m$ and angle $FGP = 38°$.



(i) Calculate the height of the flagpole, FP. [2]

Consider the right-angled triangle FGP.

$\sin\theta = \frac{opp}{hyp}$

$\sin 38° = \frac{FP}{18}$

$FP = 18\sin 38°$

$FP = 11.1\ m$ (to 3 significant figures)

$\therefore$ The height of the flagpole, FP, is $11.1\ m$.

(ii) Find PR, the length of one of the pieces of rope used to hold the flagpole. [2]

Consider the right-angled triangle FPR.

Using Pythagoras' Theorem,

$$c^2 = a^2 + b^2$$

$$(PR)^2 = (FR)^2 + (FP)^2$$

$$(PR)^2 = (9.7)^2 + (18 \sin 38°)^2$$

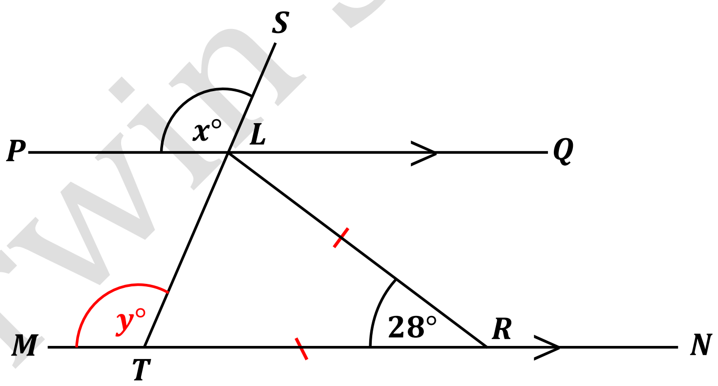
$$(PR)^2 = 216.8986529$$

$$PR = \sqrt{216.8986529}$$

$PR = 14.7\ m$ (to 3 significant figures)

$\therefore$ The length of PR is $14.7\ m$.

(b) In the diagram below, PQ is parallel to MN, LRT is an isosceles triangle and SLT is a straight line.



Find the value of x. [2]

Consider the triangle LRT. The base angles in an isosceles triangle are equal.

$$\angle LRT = \frac{180° - 28°}{2}$$

$$= \frac{152°}{2}$$

$$= 76°$$

Now, $\angle LRT$ and Angle y as shown in the diagram lie on a straight line and therefore, add up to $180°$.

$y = 180° - 76°$

$\quad = 104°$

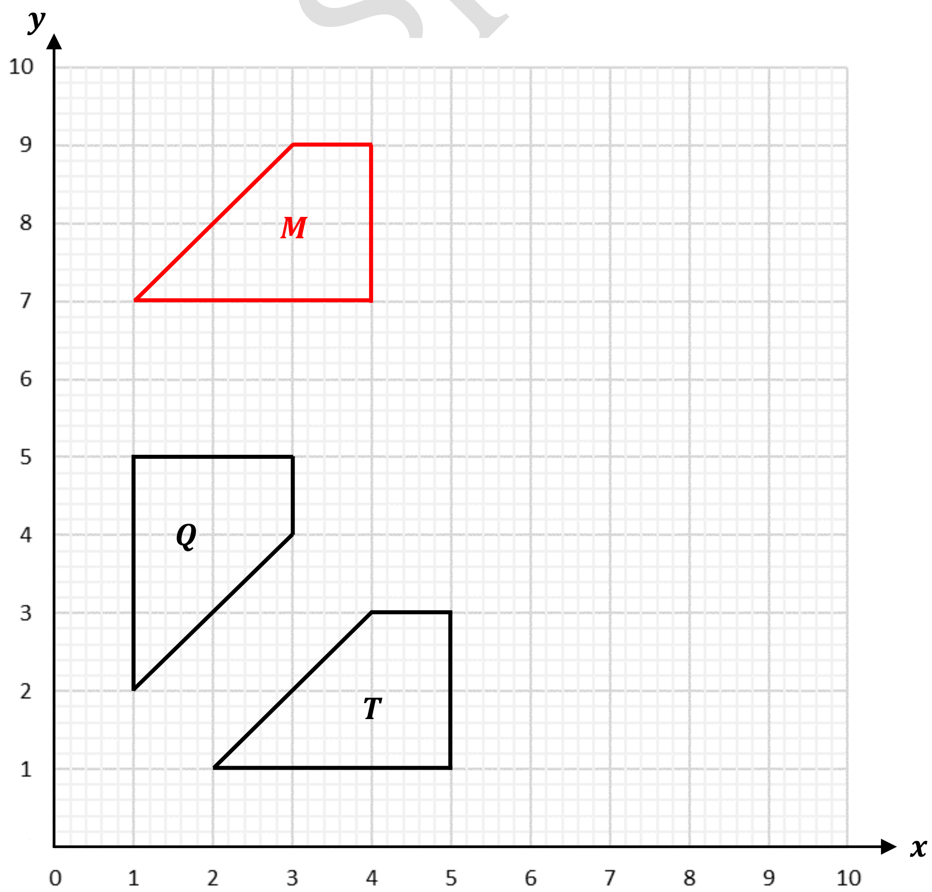
Since Angle x and Angle y are corresponding angles, they are equal.

Angle x = Angle y

$\quad\quad = 104°$

$\therefore$ The value of $x = 104°$.

(c) The diagram below shows a shape, T, and its image, Q, after a transformation.



(i) Describe fully the **single** transformation that maps Shape T onto Shape Q. [2]

The single transformation that maps Shape T onto Shape Q is a reflection in the line $y = x$.

(ii) On the diagram above, draw the image of Shape T after it undergoes a translation by the vector $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$. Label this image M. [1]

See diagram above.

Total: 9 marks

4. (a) A rectangle, $PQRS$, has a diagonal, PR, where P is the point $(-3, 10)$ and R is the point $(4, -4)$.

(i) Calculate the length of the line PR. [2]

Points are $P(-3, 10)$ and $R(4, -4)$.

Length of $PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(4 - (-3)\right)^2 + (-4 - 10)^2}$

$= \sqrt{(7)^2 + (-14)^2}$

$= \sqrt{49 + 196}$

$= 7\sqrt{5}$

$= 15.7$ units (to 3 significant figures)

$\therefore$ The length of the line PR is 15.7 units.

(ii) Determine the equation of the line PR. [3]

Points are $P(-3, 10)$ and $R(4, -4)$.

Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-4 - 10}{4 - (-3)}$

$= \frac{-14}{7}$

$= -2$

Substituting $m = -2$ and point $(4, -4)$ into $y - y_1 = m(x - x_1)$ gives:

$$y - (-4) = -2(x - 4)$$

$$y + 4 = -2x + 8$$

$$y = -2x + 8 - 4$$

$$y = -2x + 4$$

$\therefore$ The equation of the line PR is: $y = -2x + 4$

(b) Two functions, f and g, are defined as follows.

$f(x) = 3x + 1$ and $g(x) = x^2$

Find, in its simplest form, an expression for

(i) $f(x - 2)$ [2]

$$f(x) = 3x + 1$$

$$f(x - 2) = 3(x - 2) + 1$$

$$= 3x - 6 + 1$$

$$= 3x - 5$$

(ii) $g(3x + 2) + 10$ [2]

$$g(3x + 2) + 10 = (3x + 2)^2 + 10$$

$$= 9x^2 + 12x + 4 + 10$$

$$= 9x^2 + 12x + 14$$

Total: 9 marks

5. (a) Mr. Morgan administered a spelling test to his class. The table below shows the number of words out of 10 that each students spelt correctly.

Number of Words	5	6	7	8	9	10
Frequency	8	4	2	2	3	4

(i) For the data set shown above, state the

(a) mode [1]

The mode is 5 words spelt correctly.

(b) median [1]

The median is 6 words spelt correctly.

(ii) Calculate the mean number of words spelt correctly. [2]

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$= \frac{(5\times8)+(6\times4)+(7\times2)+(8\times2)+(9\times3)+(10\times4)}{8+4+2+2+3+4}$$

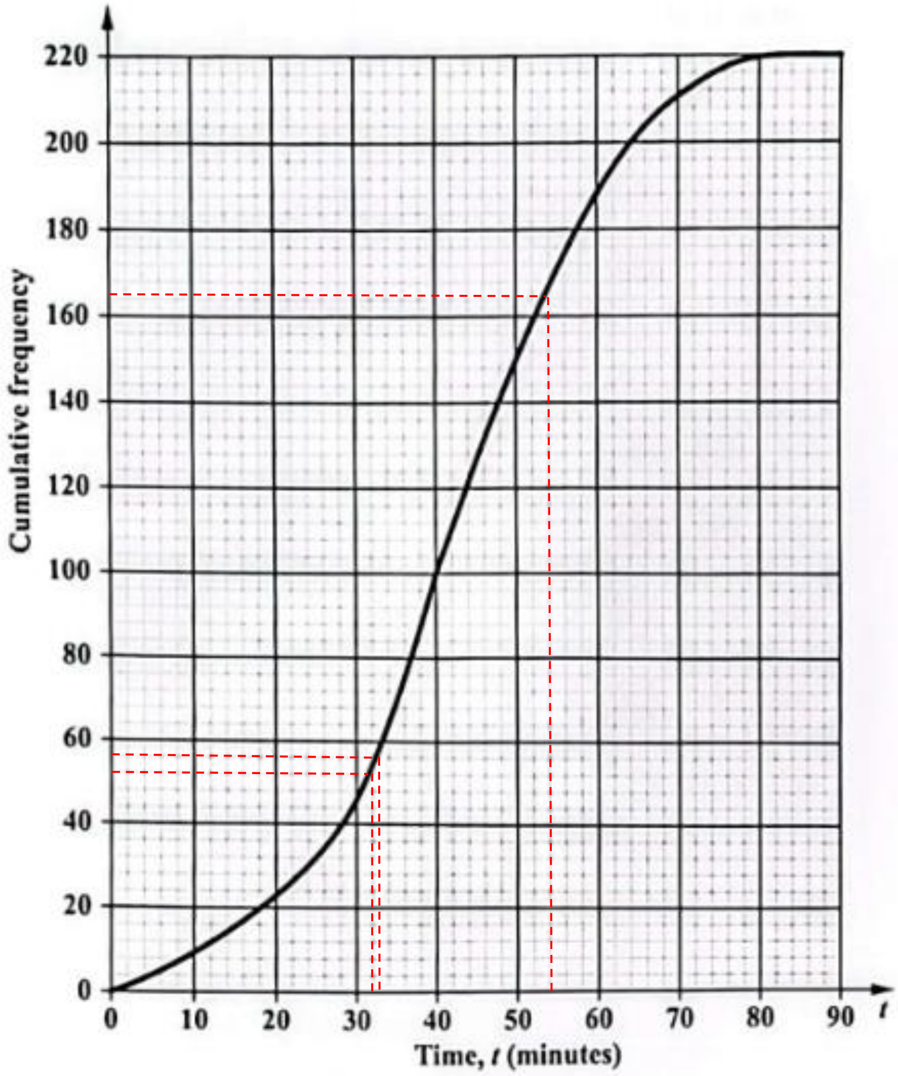
$$= \frac{40+24+14+16+27+40}{23}$$

$$= \frac{161}{23}$$

$$= 7 \text{ words}$$

$\therefore$ The mean number of words spelt correctly is 7 words.

(b) The attendance officer at a particular school recorded the time, t, in minutes, taken by each student in a group to travel to school. The data collected is shown on the cumulative frequency curve below.



Using the cumulative frequency curve, find an estimate of

(i) the number of students who took at MOST 32 minutes to travel to school [1]

From graph, the number of students who took at MOST 32 minutes to travel to school is 52 students.

(ii) the inter-quartile range [2]

Assuming $n = 220$ students,

$$Q_3 \text{ occurs at } = \frac{3}{4}(n+1)$$

$$= \frac{3}{4}(220+1)$$

$$= \frac{3}{4}(221)$$

$$= 165.75^{\text{th}} \text{ value}$$

So, the value of Q_3 is 54 minutes.

$$Q_1 \text{ occurs at } = \frac{1}{4}(n+1)$$

$$= \frac{1}{4}(220+1)$$

$$= \frac{1}{4}(221)$$

$$= 55.25^{\text{th}} \text{ value}$$

So, the value of Q_1 is 33 minutes.

Now,

$$IQR = Q_3 - Q_1$$

$$= 54 - 33$$

$$= 21 \text{ minutes}$$

$\therefore$ The inter-quartile range is 21 minutes.

(c) The letters in the word "POSITIVITY" are each written on separate cards and placed in a bag. Dacia picks 2 of these cards, at random, **with replacement.**

P	O	S	I	T	I	V	I	T	Y

Find the probability that she picks the letter "I" then the letter "V". [2]

Probability she picks "I" $= \frac{\textit{Number of desired outcomes}}{\textit{Total number of outcomes}}$

$= \frac{3}{10}$

Probability she picks "V" $= \frac{\textit{Number of desired outcomes}}{\textit{Total number of outcomes}}$

$= \frac{1}{10}$

Now, the events are independent.

So, we have,

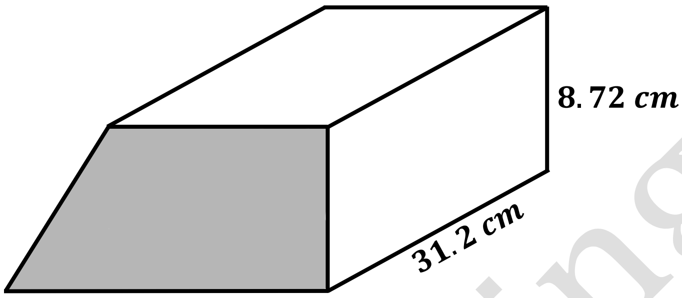
Probability that she picks "I" then "V" $= \frac{3}{10} \times \frac{1}{10}$

$= \frac{3}{100}$

Total: 9 marks

6. $\left[\text{In this question, take } \pi = \frac{22}{7}.\right]$

(a) The diagram below shows a gold bar in the shape of a trapezoidal prism. Its volume is 2 886 cm^3. The length and height of the prism are indicated on the diagram.



(i) Calculate the area of the **shaded** cross-section of the trapezoidal prism. [1]

$$\text{Volume of prism} = \text{Cross-sectional area} \times \text{length}$$

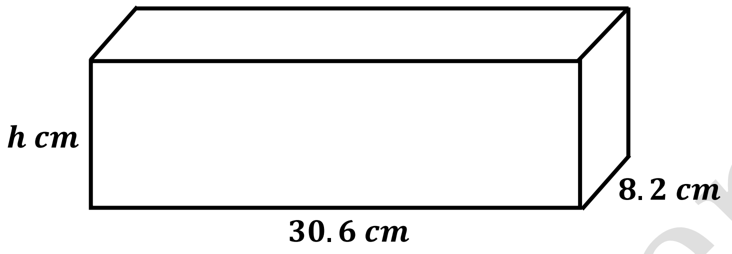
$$\text{Cross-sectional area} = \frac{\textit{Volume of prism}}{\textit{length}}$$

$$= \frac{2886}{31.2}$$

$$= 92.5\ cm$$

$\therefore$ The area of the shaded cross-section is 92.5 cm.

(ii) The cuboid-shaped gold bar shown below has the same volume as the trapezoidal prism-shaped gold bar displayed at (a).



Calculate the height, h, of the cuboid-shaped gold bar. [2]

$\text{Volume} = \text{length} \times \text{width} \times \text{height}$

$2886 = 30.6 \times 8.2 \times h$

$h = \frac{2886}{30.6 \times 8.2}$

$h = \frac{2886}{250.92}$

$h = 11.5\ cm$ (to 3 significant figures)

$\therefore$ The height, h, of the cuboid-shaped gold bar is 11.5 cm.

THE STUDENT HUB

(b) The trapezoidal gold bar is melted down and all the gold is used to make SIX identical spheres.

Calculate, for EACH sphere of gold, its

(i) radius [3]

[The volume, V, of a sphere with radius, r is $V = \frac{4}{3}\pi r^3$.]

Volume of six spheres $= 2886\ cm^3$

Volume of one sphere $= \frac{2886}{6}$

$= 481\ cm^3$

Now,

$V = \frac{4}{3}\pi r^3$

$481 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$r^3 = 481 \times \frac{3}{4} \times \frac{7}{22}$

$r^3 = \frac{10101}{88}$

$r = \sqrt[3]{\frac{10101}{88}}$

$r = 4.86\ cm$ (to 3 significant figures)

$\therefore$ The radius of each sphere of gold is $4.86\ cm$.

(ii) surface area [1]

[The surface area, A, of a sphere with radius r, $A = 4\pi r^2$.]

Surface area, $A = 4\pi r^2$

$= 4 \times \frac{22}{7} \times (4.86)^2$

$= 297\ cm^3$ (to 3 significant figures)

$\therefore$ The surface area, A, of the sphere is $297\ cm^3$.

(iii) mass, **to the nearest kilogram**, given that the density of gold is $19.3\ g/cm^3$ [2]

$\left[\text{Density} = \frac{\text{Mass}}{\text{Volume}}\right]$

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

$\text{Mass} = \text{Density} \times \text{Volume}$

$= 19.3 \times 481$

$= 9283.3\ g$

Now,

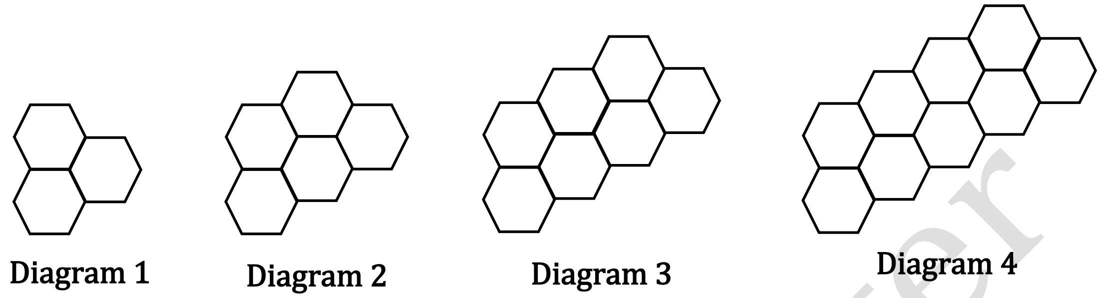
$\text{Mass} = 9283.3 \div 1000$

$= 9\ kg$ (to the nearest kilogram)

$\therefore$ The mass of each sphere is $9\ kg$.

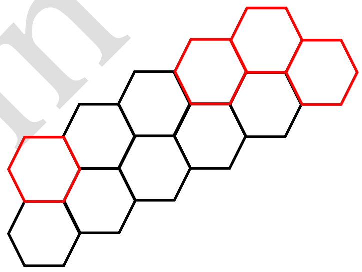
Total: 9 marks

7. The diagram below shows the first four diagrams in a sequence of regular hexagons. Each regular hexagon is made using sticks of unit length.



(a) Complete the diagram below to represent Diagram 5 in the sequence of regular hexagons. [2]

The completed diagram is as follows:



(b) The number of regular hexagons, H, the number of sticks, S, and the perimeter of each figure, P, follow a pattern. The values for H, S and P, for the first 4 diagrams are shown in the table below. Study the pattern of numbers in each row of the table and answer the questions that follow.

Complete the rows marked (i), (ii) and (iii) in the table below.

	Diagram Number (D)	Number of Hexagons (H)	Number of Sticks (S)	Perimeter (P)	
	1	3	15	12	
	2	5	23	16	
	3	7	31	20	
	4	9	39	24	
(i)	5	11	47	28	[2]
	⋮	⋮	⋮	⋮	
(ii)	23	47	191	100	[2]
	⋮	⋮	⋮	⋮	
(iii)	n	$2n+1$	$8n+7$	$4n+8$	[3]

Consider the nth figure.

$H = 2n + 1$

$S = 8n + 7$

$P = 4n + 8$

(i) When $n = 5$,

$H = 2(5) + 1$

$= 10 + 1$

$= 11$

$S = 8(5) + 7$

$= 40 + 7$

$= 47$

(ii) Consider $H = 47$.

$47 = 2n + 1$

$47 - 1 = 2n$

$46 = 2n$

$\frac{46}{2} = n$

$23 = n$

When $n = 23$,

$P = 4(23) + 8$

$= 92 + 8$

$= 100$

(c) Skyla says that she can make one of these figures with a perimeter of

EXACTLY 1 005. Explain why she is **incorrect**. [1]

$P = 4n + 8$

To be correct, $4n + 8 = 1005$ and $n \in \mathbb{N}$.

$4n + 8 = 1005$

$4n = 1005 - 8$

$4n = 997$

$n = \frac{997}{4}$

The value $n = \frac{997}{4}$ is not a natural number, so it is not a possible figure.

Hence, Skyla is incorrect.

Total: 10 marks

SECTION II

Answer ALL questions.

ALL working MUST be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The functions f and g are defined as follows.

$f(x) = \frac{2x-1}{3}$ and $g(x) = 5 - x^2$

(i) Determine the value of

(a) $g(2)$ [1]

$g(x) = 5 - x^2$

$g(2) = 5 - (2)^2$

$= 5 - 4$

$= 1$

(b) $f^{-1}(3)$ [2]

We are given that $f(x) = \frac{2x-1}{3}$.

Now,

$3 = \frac{2x-1}{3}$

$2x - 1 = 3 \times 3$

$$2x - 1 = 9$$

$$2x = 9 + 1$$

$$2x = 10$$

$$x = \frac{10}{2}$$

$$x = 5$$

$$\therefore f^{-1}(3) = 5$$

(ii) Derive an expression, in its simplest form, for $fg(x)$. [2]

$$fg(x) = f[g(x)]$$

$$= f(5 - x^2)$$

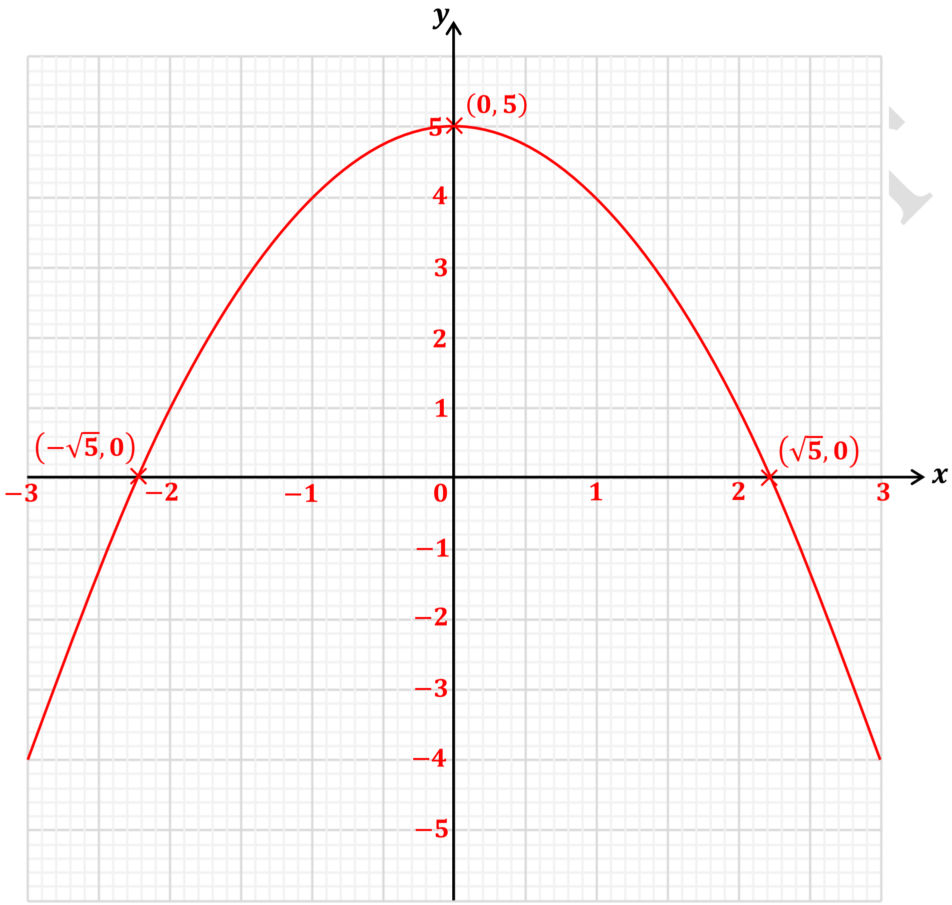
$$= \frac{2(5-x^2)-1}{3}$$

$$= \frac{10-2x^2-1}{3}$$

$$= \frac{9-2x^2}{3}$$

$$\therefore fg(x) = \frac{9-2x^2}{3}$$

(iii) Sketch the graph of the function $g(x)$ in the space provided below. On your sketch, indicate the maximum/minimum point and the roots of the function. [3]



$g(x) = 5 - x^2$

Let $y = 5 - x^2$.

When $x = 0$,

$y = 5 - (0)^2$

$= 5 - 0$

$= 5$

So, the curve cuts the y-axis at the point $(0, 5)$.

When $y = 0$,

$5 - x^2 = 0$

$x^2 = 5$

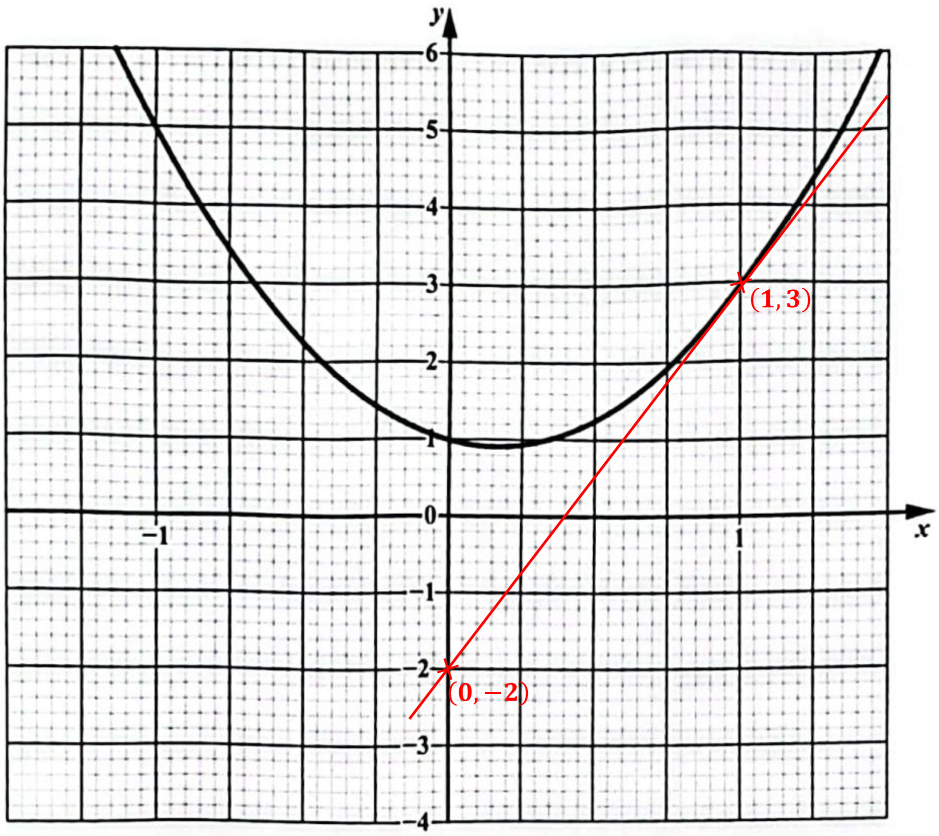
$x = \pm\sqrt{5}$

$x = \pm 2.24$

So, the roots of the curve are at $(-2.24, 0)$ and $(2.24, 0)$.

The maximum point of the curve is at the point $(0, 5)$.

(b) The graph below shows a quadratic function.



(i) On the grid above, draw the tangent to the curve at $x = 1$. [1]

See grid above.

(ii) Use the tangent drawn to estimate the gradient of the curve at $x = 1$. [2]

Points are $(0, -2)$ and $(1, 3)$.

Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-(-2)}{1-0}$

$= \frac{5}{1}$

$= 5$

$\therefore$ The gradient of the curve at $x = 1$ is $m = 5$.

(iii) Write down the equation of the tangent in the form $y = mx + c$. [1]

From graph, $c = -2$.

Substituting $m = 5$ and $c = -2$ into $y = mx + c$ gives:

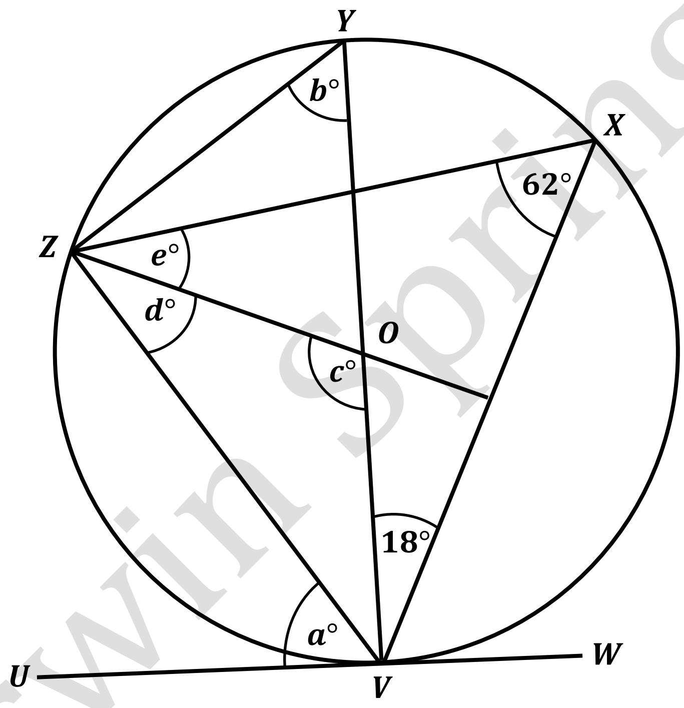
$y = 5x - 2$

$\therefore$ The equation of the tangent is: $y = 5x - 2$

Total: 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) V, X, Y and Z lie on the circumference of a circle shown below, centre O, with diameter VY. UW is a tangent to the circle at V. Angle $VXZ = 62°$ and Angle $XVY = 18°$.



(i) State a theorem that justifies the values of EACH of the following angles.

(a) Angle $b = 62°$ [1]

Angles in the same segment, standing on the same chord, are equal.

Hence, Angle $b = 62°$.

(b) Angle $c = 124°$ [1]

The angle at the centre of the circle is twice the angle at the circumference.

Angle $c = 2 \times 62°$

$= 124°$

(c) Angle $OVW = 90°$ [1]

The angle between a tangent and a radius is 90°.

Hence, Angle $OVW = 90°$.

(ii) Find the values of Angles a, d and e. Show ALL working where appropriate. [3]

$\angle a =$

The angle between a tangent UW and a chord VZ is equal to the angle in the alternate segment.

$\therefore \angle a = 62°$

$\angle d =$

Since OZ and OV are both radii of the same circle, then $OZ = OV$.

So, Triangle OVZ is an isosceles triangle and the base angles are equal.

Hence,

$\angle d = \frac{180° - 124°}{2}$

$= \frac{56°}{2}$

$= 28°$

$\angle e =$

The angle between a tangent and a radius is 90°.

Angle $ZVY = 90° - 62°$

$= 28°$

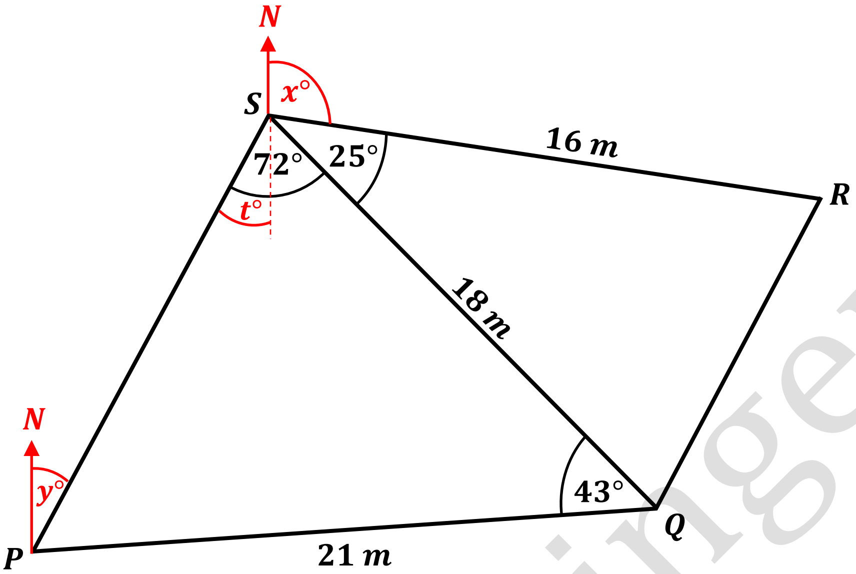
All angles in a triangle add up to 180°.

$\angle e = 180° - (62° + 18° + 28° + 28°)$

$= 180° - 136°$

$= 44°$

(b) The diagram below shows a quadrilateral $PQRS$ formed by joining two triangles, PQS and QRS.



(i) Calculate the length of QR. [3]

Using the cosine rule,

$(QR)^2 = (RS)^2 + (QS)^2 - 2(RS)(QS)\cos Q\hat{R}S$

$(QR)^2 = (16)^2 + (18)^2 - 2(16)(18)\cos 25°$

$(QR)^2 = 57.96671467$

$QR = \sqrt{57.96671467}$

$QR = 7.61\ m$ (to 3 significant figures)

$\therefore$ The length of QR is $7.61\ m$.

(ii) The bearing of P from S is 205°. Determine the bearing of

(a) R from S [1]

$$x = 205° - (25° + 72°)$$
$$= 205° - 97°$$
$$= 108°$$

$\therefore$ The bearing of R from S is 108°.

(b) S from P [2]

$$t = 205° - 180°$$
$$= 25°$$

Since angle y and angle t are alternate angles, they are equal.

Hence, angle $y = 25°$.

$\therefore$ The bearing of S from P is 25°.

Total: 12 marks

VECTORS AND MATRICES

10. (a) The determinant of the matrix $\begin{pmatrix} 6 & 2v \\ -5 & -v \end{pmatrix}$ is 24.

Calculate the value of v. [2]

Let $A = \begin{pmatrix} 6 & 2v \\ -5 & -v \end{pmatrix}$.

The matrix is in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

where $a = 6, b = 2v, c = -5$ and $d = -v$.

We are given that $\det(A) = 24$.

$\det(A) = ad - bc$

$24 = (6)(-v) - (2v)(-5)$

$24 = -6v + 10v$

$24 = 4v$

$\frac{24}{4} = v$

$6 = v$

$\therefore$ The value of $v = 6$.

(b) The matrices L and M are defined as follows.

$$L = \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix}, \qquad M = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Evaluate EACH of the following.

(i) The matrix product LM [2]

$$LM = \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} (9 \times 2) + (5 \times -4) \\ (3 \times 2) + (2 \times -4) \end{pmatrix}$$

$$= \begin{pmatrix} 18 + (-20) \\ 6 + (-8) \end{pmatrix}$$

$$= \begin{pmatrix} 18 - 20 \\ 6 - 8 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

(ii) L^{-1}, the inverse of L [2]

$$L = \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix}$$

$$\det(L) = ad - bc \qquad\qquad adj = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= (9)(2) - (5)(3) \qquad\qquad = \begin{pmatrix} 2 & -5 \\ -3 & 9 \end{pmatrix}$$

$$= 18 - 15$$

$$= 3$$

Now,

$$L^{-1} = \frac{1}{\det(L)} \times adj$$

$$= \frac{1}{3}\begin{pmatrix} 2 & -5 \\ -3 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{3} & -\frac{5}{3} \\ -\frac{3}{3} & \frac{9}{3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{3} & -\frac{5}{3} \\ -1 & 3 \end{pmatrix}$$

(c) $\overrightarrow{PQ} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$.

If P is the point $(-2, 3)$, determine the coordinates of Q. [2]

P is the point $(-2, 3)$.

Then, $\overrightarrow{OP} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

Using the triangle law,

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\overrightarrow{OQ} = \overrightarrow{PQ} + \overrightarrow{OP}$

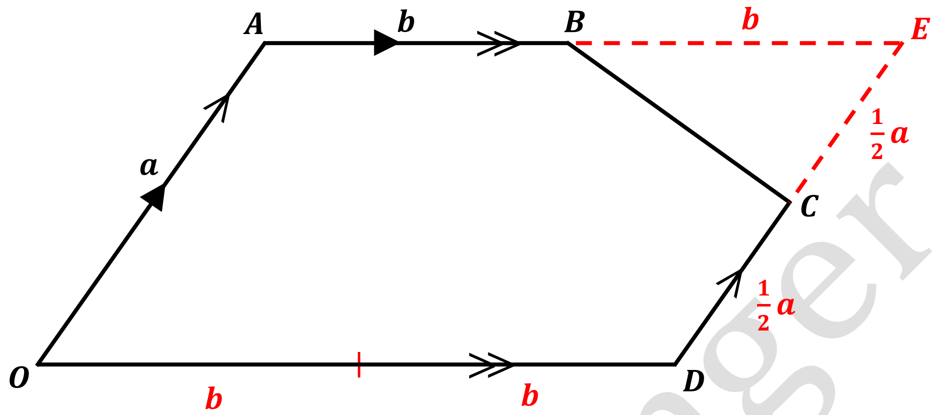
$\overrightarrow{OQ} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$\overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\therefore$ The coordinates of Q are $(3, -1)$.

(d) In the pentagon $OABCD$, OA is parallel to DC and AB is parallel to OD.

$OD = 2AB$ and $OA = 2DC$. $\overrightarrow{OA} = a$ and $\overrightarrow{AB} = b$.



Find, in terms of a and b, in its simplest form,

(i) $\overrightarrow{AD}$ [1]

$\overrightarrow{OD} = 2\overrightarrow{AB}$

$= 2\boldsymbol{b}$

Using the triangle law,

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= -\overrightarrow{OA} + \overrightarrow{OD}$

$= -\boldsymbol{a} + 2\boldsymbol{b}$

(ii) $\overrightarrow{BC}$ [2]

$2\overrightarrow{CD} = \overrightarrow{OA}$

$\overrightarrow{CD} = \frac{1}{2}\overrightarrow{OA}$

$\overrightarrow{CD} = \frac{1}{2}\boldsymbol{a}$

Using the triangle law,

$$\overrightarrow{BC} = \overrightarrow{BE} + \overrightarrow{EC}$$

$$= \boldsymbol{b} - \frac{1}{2}\boldsymbol{a}$$

$$= -\frac{1}{2}\boldsymbol{a} + \boldsymbol{b}$$

(iii) State the conclusion about $\left|\overrightarrow{AD}\right|$ and $\left|\overrightarrow{BC}\right|$ that can be drawn from your responses in (i) and (ii). [1]

$$\overrightarrow{AD} = -\boldsymbol{a} + 2\boldsymbol{b}$$

$$\overrightarrow{BC} = -\frac{1}{2}\boldsymbol{a} + \boldsymbol{b}$$

We can deduce that $\left|\overrightarrow{AD}\right| = 2\left|\overrightarrow{BC}\right|$.

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.