Solutions to CSEC Maths P2 June 2011

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Question 1(a)(i)

Required to calculate: $\frac{2\frac{1}{4}+1\frac{1}{8}}{4\frac{1}{2}}$

$$\text{Numerator} = 2\frac{1}{4} + 1\frac{1}{8}$$

$$= \frac{9}{4} + \frac{9}{8}$$

$$= \frac{18+9}{8}$$

$$= \frac{27}{8}$$

$$\text{Denominator} = 4\frac{1}{2}$$

$$= \frac{9}{2}$$

$$\therefore \text{Numerator} \div \text{Denominator} = \frac{27}{8} \div \frac{9}{2}$$

$$= \frac{27}{8} \times \frac{2}{9}$$

$$= \frac{3}{4} \qquad \text{(in exact form)}$$

Question 1(a)(ii)

Required to calculate: $3.96 \times 0.25 - \sqrt{0.0256}$

$$3.96 \times 0.25 - \sqrt{0.0256} = 0.99 \times 0.16$$

$$= 0.83 \qquad \text{(in exact form)}$$

Question 1(b)

Required to calculate the values of W, X, Y and Z.

$W = 6\frac{1}{2} \times \$2.40$

$= \$15.60$

$X = \frac{\$52.80}{4}$

$= \$13.20$

$Y = \frac{\$14.10}{2.35}$

$= 6$

$Z = \frac{VAT}{Sub\ Total} \times 100$

$= \frac{9.90}{82.50} \times 100$

$= 12$

Question 2(a)

Required to write $\frac{x-2}{3} + \frac{x+1}{4}$ as a fraction in its lowest terms.

$\frac{x-2}{3} + \frac{x+1}{4}$

$= \frac{4(x-2)+3(x+1)}{12}$

$= \frac{4x-8+3x+3}{12}$

$= \frac{7x-5}{12}$ (as a fraction in its lowest terms)

Question 2(b)

Required to calculate $3 * 4$.

$a * b = (a+b)^2 - 2ab$

$3 * 4 = (3+4)^2 - 2(3)(4)$

$= (7)^2 - 24$

$= 49 - 24$

$= 25$

Question 2(c)(i)

Required to factorise $xy^3 + x^2y$.

$xy^3 + x^2y$

$= xy(y^2 + x)$

Question 2(c)(ii)

Required to factorise $2mh - 2nh - 3mk + 3nk$.

$$2mh - 2nh - 3mk + 3nk$$

$$= 2h(m - n) - 3k(m - n)$$

$$= (m - n)(2h - 3k)$$

Question 2(d)

Required to find the values of a and b.

Now,

$$y \propto x$$

$$\therefore y = kx$$

When $x = 2$ and $y = 12$,

$$12 = k(2)$$

$$k = \frac{12}{2}$$

$$k = 6$$

Hence, the equation can now be expressed as $y = 6x$.

When $x = 5$ and $y = a$,

$$a = 6(5)$$

$$a = 30$$

When $x = b$ and $y = 48$,

$48 = 6b$

$b = \frac{48}{6}$

$b = 8$

$\therefore a = 30$ and $b = 8$

Question 3(a)(i)

Required to find the number of students who do not study either Art or Music.

From the Venn diagram, $(M \cup A)' = 4$.

Question 3(a)(ii)

Required to find the value of x.

$n(U) = (x + 5) + 8 + x + 4$

$= 2x + 17$

The total number of students is 35.

Therefore,

$2x + 17 = 35$

$2x = 35 - 17$

$2x = 18$

$x = \frac{18}{2}$

$x = 9$

Question 3(a)(iii)

Required to find the number of students studying Music only.

$n(M) = x + 5$

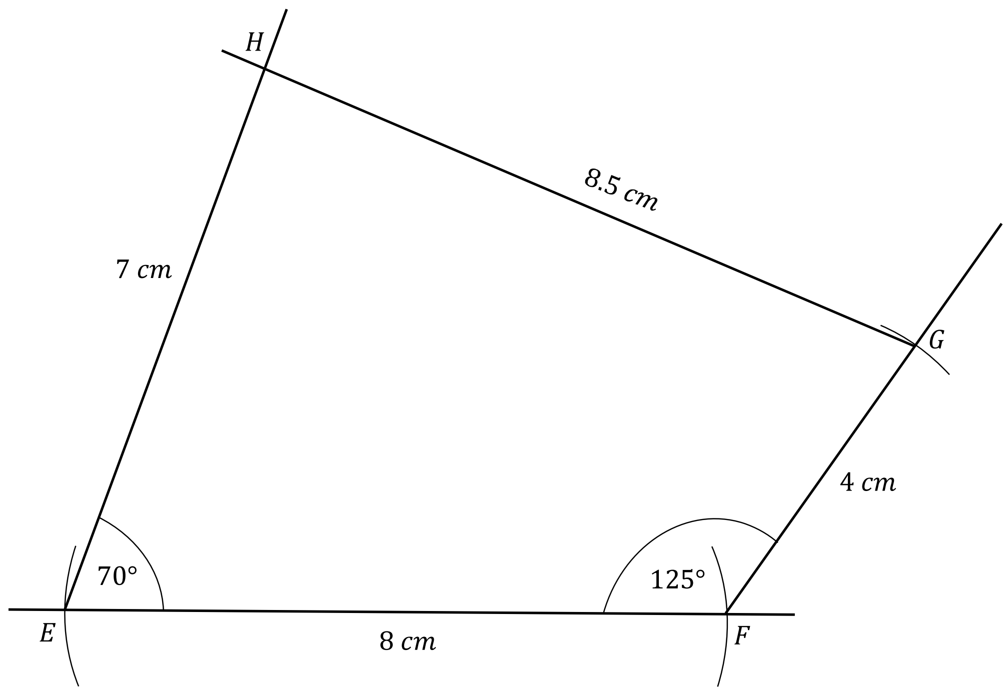
$= 9 + 5$

$= 14$

Question 3(b)(i)

Required to construct a quadrilateral.

$EF = 8\ cm$, angle $EFG = 125°$, $FG = 4\ cm$, angle $HEF = 70°$, $EH = 7\ cm$



Question 3(b)(ii)

Required to find the length of GH.

By measurement using a ruler, $GH = 8.5\ cm$.

Question 4(a)(i)

Required to find x given that $5 - 2x < 9$.

$5 - 2x < 9$

$-2x < 9 - 5$

$-2x < 4$

$x > \frac{4}{-2}$

$x > -2$

Question 4(a)(ii)

Required to find the smallest value of x.

Now $x > -2$ and $x \in Z$.

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

$\therefore x_{min} = -1$

The smallest integer that satisfies the inequality is the next integer that is greater than -2 which is -1.

Question 4(b)(i)(a)

Required to find the length, l, of one side.

Area of the square $= l^2$

So, we have

$l^2 = 121$

$l = \sqrt{121}$

$l = 11\ cm$

Question 4(b)(i)(b)

Required to find the perimeter of the square.

$$\text{Perimeter of the square} = 4 \times l$$
$$= 4 \times 11$$
$$= 44\ cm$$

$\therefore$ The perimeter of the square is $44\ cm$.

Question 4(b)(ii)(a)

Required to find the radius of the circle.

Since the circumference of the circle is the same as the perimeter of the square, then

Circumference of circle $= 44\ cm$

So, we have

$$2\pi r = 44$$
$$2 \times \frac{22}{7} \times r = 44$$
$$\frac{44}{7} \times r = 44$$
$$r = 44 \div \frac{44}{7}$$
$$r = \frac{44}{1} \times \frac{7}{44}$$
$$r = 7\ cm$$

$\therefore$ The radius of the circle, $r = 7\ cm$.

Question 4(b)(ii)(b)

Required to find the area of the circle.

Area of circle $= \pi r^2$

$= \frac{22}{7} \times (7)^2$

$= 154\ cm^2$

Question 5(a)(i)

Required to find the value of the scale factor, k.

$$\text{Scale Factor} = \frac{Image\ Length}{Object\ Length}$$

$$= \frac{N'M'}{NM}$$

$$= \frac{16}{8}$$

$$= 2$$

Question 5(a)(ii)

Required to calculate the length of OM.

Using Pythagoras' Theorem,

$$OM^2 = ON^2 + NM^2$$

$$= (6)^2 + (8)^2$$

$$= 36 + 64$$

$$= 100$$

$$OM = \sqrt{100}$$

$$= 10\ cm$$

Question 5(a)(iii)

Required to calculate the length of OM'.

The scale factor of enlargement is 2.

So,

$\frac{OM'}{OM} = 2$

$\frac{OM'}{10} = \frac{2}{1}$

$OM' = 2 \times 10$

$= 20\ cm$

Question 5(b)(i)

Required to find the length of QS, to 3 significant figures.

Consider ΔPQS.

$\sin 15° = \frac{QS}{12.6}$

$QS = 12.6 \times \sin 15°$

$QS = 3.26\ m$ (to 3 significant figures)

Question 5(b)(ii)

Required to find $R\hat{Q}S$, to 3 significant figures.

Consider ΔRQS.

$\cos R\hat{Q}S = \frac{3.261}{8.4}$

$R\hat{Q}S = \cos^{-1}\left(\frac{3.261}{8.4}\right)$

$R\hat{Q}S = 67.2°$ (to 3 significant figures)

Question 5(b)(iii)

Required to find the area of ΔPQR.

The sum of angles in a triangle add up to 180°.

$P\hat{Q}S = 180° - (90° + 15°)$

$= 180° - 105°$

$= 75°$

$\therefore P\hat{Q}R = R\hat{Q}S + P\hat{Q}S$

$= 67.2° + 75°$

$= 142.21°$

Hence,

Area of $\Delta PQR = \frac{1}{2}(QP)(QR)\sin P\hat{Q}R$

$= \frac{1}{2}(12.6)(8.4)\sin 142.21°$

$= 32.4\ cm^2$ (to 3 significant figures)

Question 6(a)(i)

Required to calculate $g\left(\frac{1}{2}\right)$.

$g(x) = \frac{x-2}{3}$

$g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-2}{3}$

$= \frac{-\frac{3}{2}}{3}$

$= -\frac{3}{2} \div 3$

$= -\frac{3}{2} \times \frac{1}{3}$

$= -\frac{1}{2}$

Question 6(a)(ii)

Required to calculate an expression for $gf(x)$.

$gf(x) = g[f(x)]$

$= g(6x + 8)$

$= \frac{(6x+8)-2}{3}$

$= \frac{6x+8-2}{3}$

$= \frac{6x+6}{3}$

$= \frac{3(2x+2)}{3}$

$= 2x + 2$

$\therefore gf(x) = 2x + 2$

Question 6(a)(iii)

Required to calculate $f^{-1}(x)$.

$f(x) = 6x + 8$

Let $y = f(x)$.

$y = 6x + 8$

Interchange variables x and y.

$x = 6y + 8$

Make y the subject.

$x - 8 = 6y$

$\frac{x-8}{6} = y$

$y = \frac{x-8}{6}$

$\therefore f^{-1}(x) = \frac{x-8}{6}$

Question 6(b)(i)

Required to find the coordinates of A and B.

From the graph,

The coordinate of A is $(-2, 3)$.

The coordinate of B is $(4, 6)$.

Question 6(b)(ii)

Required to find the gradient of AB.

Points are $A(-2,3)$ and $B(4,6)$.

Gradient of $AB = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{6-3}{4-(-2)}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Question 6(b)(iii)

Required to find the equation of the line passing through A and B.

Substituting $m = \frac{1}{2}$ and point $A(-2,3)$ into $y - y_1 = m(x - x_1)$ gives,

$y - 3 = \frac{1}{2}\left(x - (-2)\right)$

$y - 3 = \frac{1}{2}(x + 2)$

$y - 3 = \frac{1}{2}x + 1$

$y = \frac{1}{2}x + 1 + 3$

$y = \frac{1}{2}x + 4$

∴ The equation of the line passing through A and B is $y = \frac{1}{2}x + 4$.

Question 7(a)

Required to copy and complete the table to show cumulative frequency for the distribution.

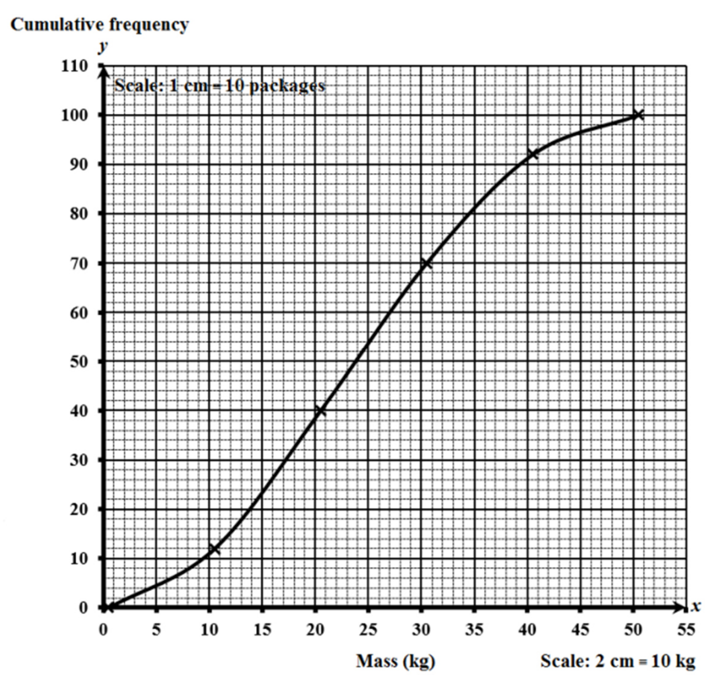
The completed table is shown below.

Mass, m (kg)	Upper Class Boundary (UCB)	Number of Packages (Frequency)	Cumulative Frequency
1-10	10.5	12	12
11-20	20.5	28	40
21-30	30.5	30	70
31-40	40.5	22	92
41-50	50.5	8	100

Question 7(b)

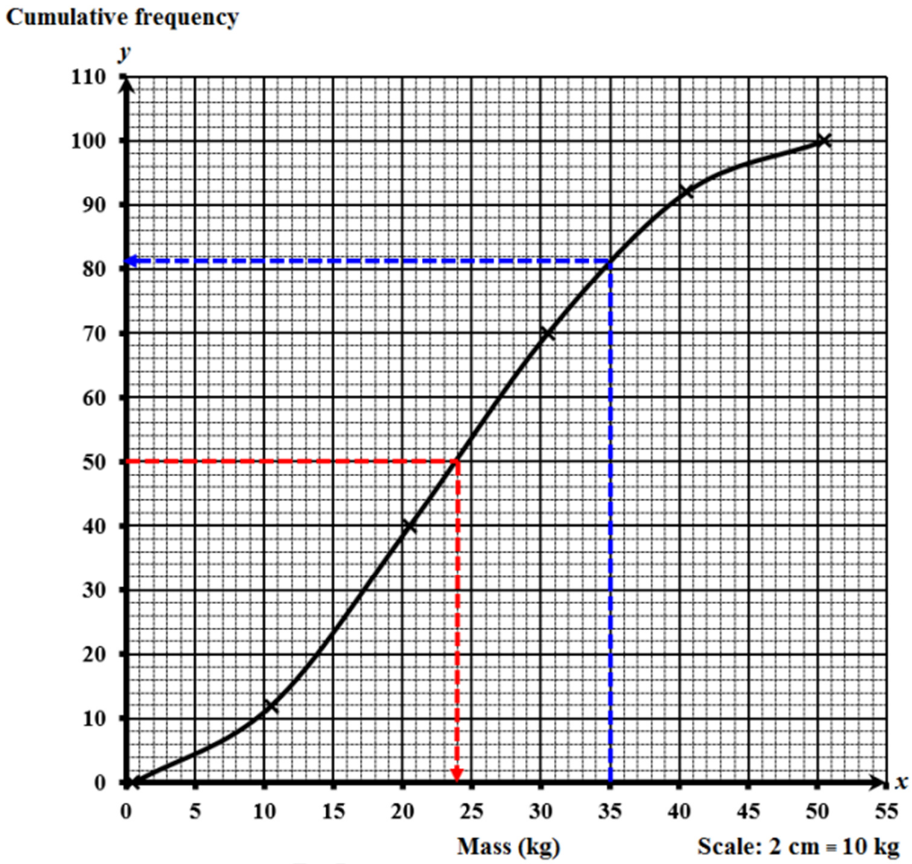
Required to draw the cumulative frequency curve for the data.

The cumulative frequency graph for the data is shown below.



Question 7(c)(i)

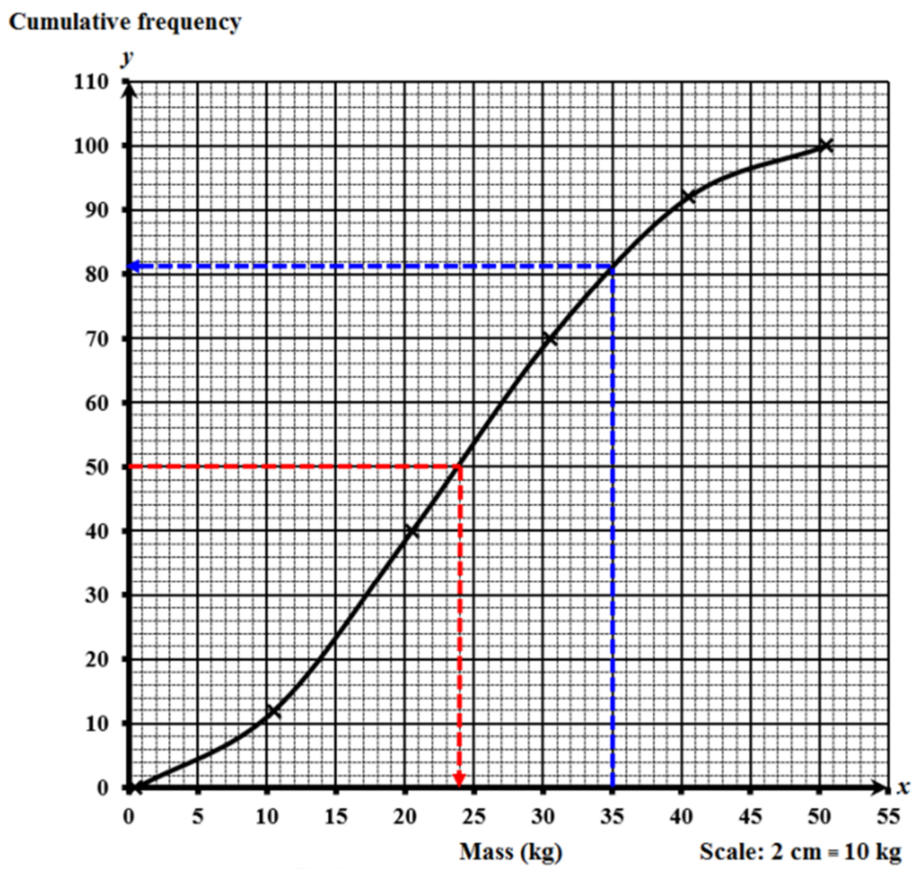
Required to find the median mass.



The median mass for the data is 24 kg.

Question 7(c)(ii)

Required to find the probability that the mass of a package is less than 35 kg.



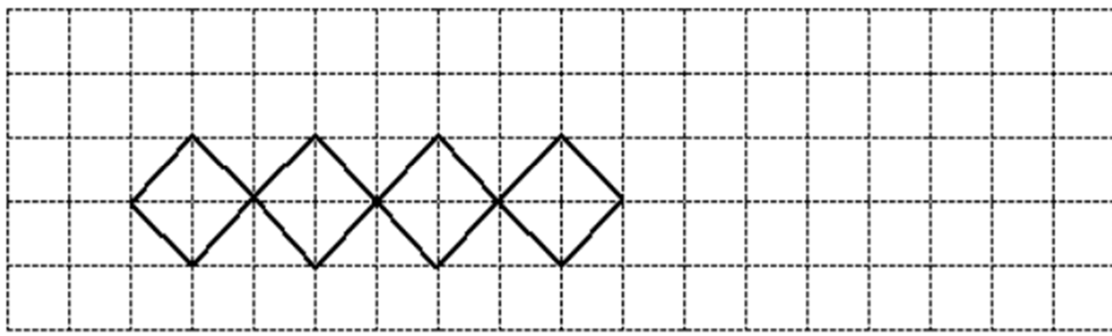
$$P(mass\ is\ less\ than\ 35\ kg) = \frac{Number\ of\ packages\ less\ than\ 35\ kg}{Total\ number\ of\ packages}$$

$$= \frac{81}{100} \quad \text{or} \quad 0.81 \quad \text{or} \quad 81\%$$

Question 8(a)

Required to draw the fourth diagram in the sequence.

The fourth diagram in the sequence is shown below:



Question 8(b)(i)

Required to find how many sticks are in the sixth diagram.

Number of sticks $= 6 \times 4$

$= 24$ sticks

Question 8(b)(ii)

Required to find how many thumb tacks are in the seventh diagram.

Number of sticks $= 7(4)$

The rule connecting t and s gives

$1 + \left(\frac{3}{4} \times 7(4)\right) = 22$

Hence, the number of thumb tacks are in the seventh diagram is 22.

Question 8(c)

Required to copy and complete the missing values in the table.

The completed table is shown below.

No. of Sticks s	Rule Connecting t and s	No. of Thumb Tacks t
4	$1+\left(\frac{3}{4}\times 4\right)$	4
8	$1+\left(\frac{3}{4}\times 8\right)$	7
12	$1+\left(\frac{3}{4}\times 12\right)$	10
52	$\underline{1+\left(\frac{3}{4}\times 52\right)}$	$\underline{40}$
$\underline{72}$	$\underline{1+\left(\frac{3}{4}\times 72\right)}$	55

When $s=52$,

$$t=1+\left(\frac{3}{4}\times 52\right)$$

$$=1+39$$

$$=40$$

When $t=55$,

$$1+\left(\frac{3}{4}\times s\right)=55$$

$$\frac{3}{4}s=55-1$$

$$\frac{3}{4}s=54$$

$$s=54\times\frac{4}{3}$$

$$s=72$$

Question 8(d)

Required write in terms of s and t, to show how t is related to s.

The required equation is: $t = 1 + \left(\frac{3}{4} \times s\right)$.

Question 9(a)

Required to solve $y = x^2 - x + 3$ and $y = 6 - 3x$ simultaneously.

$y = x^2 - x + 3 \quad \rightarrow$ Equation 1

$y = 6 - 3x \quad \rightarrow$ Equation 2

Equating Equation 1 and Equation 2 gives,

$$x^2 - x + 3 = 6 - 3x$$

$$x^2 - x + 3x + 3 - 6 = 0$$

$$x^2 + 2x - 3 = 0$$

$$x^2 + 3x - x - 3 = 0$$

$$x(x + 3) - 1(x + 3) = 0$$

$$(x - 1)(x + 3) = 0$$

Either $x - 1 = 0$ or $x + 3 = 0$

$x = 1$ $\qquad$ $x = -3$

When $x = 1$,

$$y = 6 - 3(1)$$

$$= 6 - 3$$

$$= 3$$

When $x = -3$,

$$y = 6 - 3(-3)$$

$$= 6 + 9$$

$$= 15$$

$\therefore x = 1, y - 3$ or $x = -3, y = 15$

Question 9(b)(i)

Required to express $4x^2 - 8x - 2$ in the form $a(x+h)^2 + k$.

$4x^2 - 8x - 2$

$= 4(x^2 - 2x) - 2$

$= 4(x^2 - 2x + 1) - 2 - 4(1)$

$= 4(x-1)^2 - 2 - 4$

$= 4(x-1)^2 - 6$

which is of the form $a(x+h)^2 + k$ where $a = 4, h = -1$ and $k = -6$.

Question 9(b)(ii)

Required to find the value of x for which $f(x)$ is a minimum.

Value of x for which $f(x)$ is a minimum $= -h$

$= -(-1)$

$= 1$

Question 9(c)(i)

Required to find the value of x.

Gradient $= \frac{12-0}{x-0}$

Since the gradient is equal to 0.6 ms^{-2}, then we have,

$0.6 = \frac{12}{x}$

$x = \frac{12}{0.6}$

$x = 20\ s$

Question 9(c)(ii)

Required to find the gradient of the graph during the second stage and to explain, in one sentence, what the car is doing in this stage.

In the second stage of the journey, the velocity is constant as the gradient is 0. Hence, the car is moving at a constant speed of 12 ms^{-1}.

Question 9(c)(iii)

Required to find the distance travelled during the third stage.

Distance travelled = Area under the graph

$$= \frac{1}{2}(60 - 25) \times 12$$

$$= 210\ m$$

Question 10(a)(i)

Required to calculate $X\hat{Y}Z$.

The opposite angles of the cyclic quadrilateral $WXYZ$ are supplementary.

$X\hat{Y}Z = 180° - 64°$

$= 116°$

Question 10(a)(ii)

Required to calculate $Y\hat{X}Z$.

The angle made by the tangent to a circle and a chord, angle VYZ at the point of contact is equal to the angle in the alternate segment, angle YXZ.

So, $Y\hat{X}Z = 23°$.

Question 10(a)(iii)

Required to calculate $O\hat{X}Z$.

Since OX and OZ are radii of the same circle, then $OX = OZ$.

Triangle OXZ is an isosceles triangle since the two sides are equal and the base angles are also equal.

The sum of angles in a triangle add up to 180°.

$O\hat{X}Z = \frac{(180° - 128°)}{2}$

$= \frac{52°}{2}$

$= 26°$

Question 10(b)(i)

Required to calculate the value of x.

The angles at a point in a straight line add up to 180°.

$x = 180° - (48° + 56°)$

$= 76°$

Question 10(b)(ii)

Required to calculate the length of RP.

Consider the ΔPQR. Using the cosine rule,

$RP^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos R\hat{Q}P$

$RP^2 = (220)^2 + (360)^2 - 2(220)(360)\cos 56°$

$RP^2 = 48\,400 + 129\,600 - 158\,400\cos 56°$

$RP^2 = 89\,423.84409$

$RP = \sqrt{89\,423.84409}$

$RP = 299\ km$ (to the nearest whole number)

Question 10(b)(iii)

Required to calculate the bearing of R from P.

Consider the ΔPQR. Using the sine rule,

$\frac{RQ}{\sin R\hat{P}Q} = \frac{RP}{\sin 56°}$

$\frac{360}{\sin R\hat{P}Q} = \frac{299}{\sin 56°}$

$\sin R\hat{P}Q = \frac{360 \times \sin 56°}{299}$

$R\hat{P}Q = \sin^{-1}\left(\frac{360 \times \sin 56°}{299}\right)$

$R\hat{P}Q = 86.5°$

$\therefore$ Bearing of R from $P = 132° + 86.5°$

$= 218.5°$

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Question 11(a)

Required to calculate the inverse of the matrix, $M = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$.

$\det(M) = ad - bc$

$= (3)(4) - (5)(2)$

$= 12 - 10$

$= 2$

$adj(M) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$= \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

$\therefore M^{-1} = \frac{1}{det} \times adj(A)$

$= \frac{1}{2} \times \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{2} & \frac{-5}{2} \\ \frac{-2}{2} & \frac{3}{2} \end{pmatrix}$

$= \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{pmatrix}$

Question 11(b)(i)

Required to calculate the value of a and b.

So, we have,

$$\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}\begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} (0 \times 7) + (a \times 2) \\ (b \times 7) + (0 \times 2) \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} 0 + 2a \\ 7b + 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} 2a \\ 7b \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

Comparing the equivalent matrices and equating the corresponding entries gives:

$2a = 2$ and $7b = -7$

$a = \frac{2}{2}$ $b = \frac{-7}{7}$

$a = 1$ $b = -1$

$\therefore a = 1$ and $b = -1$.

Question 11(b)(ii)

Required to describe the transformation that M represents.

$$M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix M represents a 90° clockwise rotation about the origin.

Question 11(c)(i)(a)

Required to write an expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{WY}$.

$$\overrightarrow{WY} = \overrightarrow{WV} + \overrightarrow{VY}$$

$$= \boldsymbol{a} + (-\boldsymbol{b})$$

$$= \boldsymbol{a} - \boldsymbol{b}$$

Question 11(c)(i)(b)

Required to write an expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{WS}$.

$\overrightarrow{WS} = \frac{1}{3}\overrightarrow{WY}$

$= \frac{1}{3}(\boldsymbol{a} - \boldsymbol{b})$

$= \frac{1}{3}\boldsymbol{a} - \frac{1}{3}\boldsymbol{b}$

Question 11(c)(i)(c)

Required to write an expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{SX}$.

$\overrightarrow{SX} = \overrightarrow{SW} + \overrightarrow{WX}$

$= -\frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b} + \boldsymbol{a}$

$= \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

Question 11(c)(ii)

Required to prove that R, S and X are collinear.

$\overrightarrow{RX} = \overrightarrow{RW} + \overrightarrow{WX}$

$\overrightarrow{RX} = \frac{1}{2}b + a$

$\overrightarrow{RX} = a + \frac{1}{2}b$

Now,

$\overrightarrow{SX} = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

$= \frac{2}{3}\left(\boldsymbol{a} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{b}\right)$

$= \frac{2}{3}\overrightarrow{RX}$

Since $\overrightarrow{SX}$ is related to $\overrightarrow{RX}$ by the scalar factor of $\frac{2}{3}$, they are parallel.

The vectors share a common point R, therefore, they are collinear.