

Solutions to CSEC Maths P2 June 2012

Question 1(a)

Required to calculate: $\frac{3\frac{1}{5} - \frac{2}{3}}{2\frac{4}{5}}$

$$\begin{aligned}\text{Numerator} &= 3\frac{1}{5} - \frac{2}{3} \\ &= \frac{16}{5} - \frac{2}{3} \\ &= \frac{48-10}{15} \\ &= \frac{38}{15}\end{aligned}$$

$$\begin{aligned}\text{Denominator} &= 2\frac{4}{5} \\ &= \frac{14}{5}\end{aligned}$$

$$\begin{aligned}\therefore \text{Numerator} \div \text{Denominator} &= \frac{38}{15} \div \frac{14}{5} \\ &= \frac{38}{15} \times \frac{5}{14} \\ &= \frac{19}{21} \quad (\text{in lowest terms})\end{aligned}$$

Question 1(b)(i)

Selling Price = \$44.00

Cost Price = \$55.00

Since the selling price is less than the cost price, a loss has occurred.

Loss = Cost Price - Selling Price

$$= \$55.00 - \$44.00$$

$$= \$11.00$$

$$\begin{aligned}\text{Percentage Loss} &= \frac{\text{Loss}}{\text{Cost Price}} \times 100 \\ &= \frac{11}{55} \times 100 \\ &= \frac{1}{5} \times 100 \\ &= 20\%\end{aligned}$$

Question 1(b)(ii)

A 25% profit implies that the selling price is 125% of the cost price.

$$125\% = \$100$$

$$1\% = \frac{100}{125}$$

$$100\% = \frac{100}{125} \times 100$$

$$= \$80$$

∴ The cost price is \$80.

Question 1(c)(i)

$$\text{EC } \$0.40 = \text{TT } \$1.00$$

$$\begin{aligned}\text{EC } \$1 &= \frac{\text{TT } \$1.00}{0.40} \\ &= \text{TT } \$2.50\end{aligned}$$

$$\therefore \text{EC } \$1 = \text{TT } \$2.00$$

Question 1(c)(ii)

$$\text{US } \$1.00 = \text{EC } \$2.70$$

$$\begin{aligned}\text{US } \$80 &= \text{EC } \$2.70 \times 80 \\ &= \text{EC } \$216\end{aligned}$$

$$\therefore \text{US } \$80 = \text{EC } \$216$$

Question 1(c)(iii)

$$\text{TT } \$1.00 = \text{EC } \$0.40$$

$$\begin{aligned}\text{TT } \$648 &= \text{EC } \$0.40 \times 648 \\ &= \text{EC } \$259.20\end{aligned}$$

Now,

$$\text{US } \$1.00 = \text{EC } \$2.70$$

$$\text{EC } \$2.70 = \text{US } \$1.00$$

$$\text{EC } \$1.00 = \frac{\text{US } \$1.00}{2.70}$$

$$\begin{aligned}\text{EC } \$259.20 &= \frac{\text{US } \$1.00}{2.70} \times 259.20 \\ &= \text{US } \$96.00\end{aligned}$$

$$\therefore \text{TT } \$648 = \text{US } \$96.00$$

Question 2(a)(i)

Required to factorise $2x^3y + 6x^2y^2$.

$$\begin{aligned} & 2x^3y + 6x^2y^2 \\ &= 2x^2y(x + 3y) \end{aligned}$$

Question 2(a)(ii)

Required to factorise $9x^2 - 4$.

$$\begin{aligned} & 9x^2 - 4 \\ &= (3x - 2)(3x + 4) \end{aligned}$$

Question 2(a)(iii)

Required to factorise $4x^2 + 8xy - xy - 2y^2$.

$$\begin{aligned} & 4x^2 + 8xy - xy - 2y^2 \\ &= 4x(x + 2y) - y(x + 2y) \\ &= (x + 2y)(4x - y) \end{aligned}$$

Question 2(b)

Required to solve $\frac{2x-3}{3} + \frac{5-x}{2} = 3$

$$\begin{aligned} & \frac{2x-3}{3} + \frac{5-x}{2} = 3 \\ & \frac{2(2x-3)+3(5-x)}{6} = 3 \\ & \frac{4x-6+15-3x}{6} = 3 \\ & \frac{x+9}{6} = 3 \end{aligned}$$

Multiplying by 6 throughout gives:

$$x + 9 = 6 \times 3$$

$$x + 9 = 18$$

$$x = 18 - 9$$

$$x = 9$$

Question 2(c)

Required to solve $3x - 2y = 10$ and $2x + 5y = 13$ simultaneously.

$$3x - 2y = 10 \quad \rightarrow \text{Equation 1}$$

$$2x + 5y = 13 \quad \rightarrow \text{Equation 2}$$

Multiplying Equation 1 by 2 gives:

$$6x - 4y = 20 \quad \rightarrow \text{Equation 3}$$

Multiplying Equation 2 by 3 gives:

$$6x + 15y = 39 \quad \rightarrow \text{Equation 4}$$

So, we have

$$6x - 4y = 20 \quad \rightarrow \text{Equation 3}$$

$$6x + 15y = 39 \quad \rightarrow \text{Equation 4}$$

Equation 3 - Equation 4 gives,

$$-19y = -19$$

$$y = \frac{-19}{-19}$$

$$y = 1$$

Substituting $y = 1$ into Equation 1 gives,

$$3x - 2(1) = 10$$

$$3x - 2 = 10$$

$$3x = 10 + 2$$

$$3x = 12$$

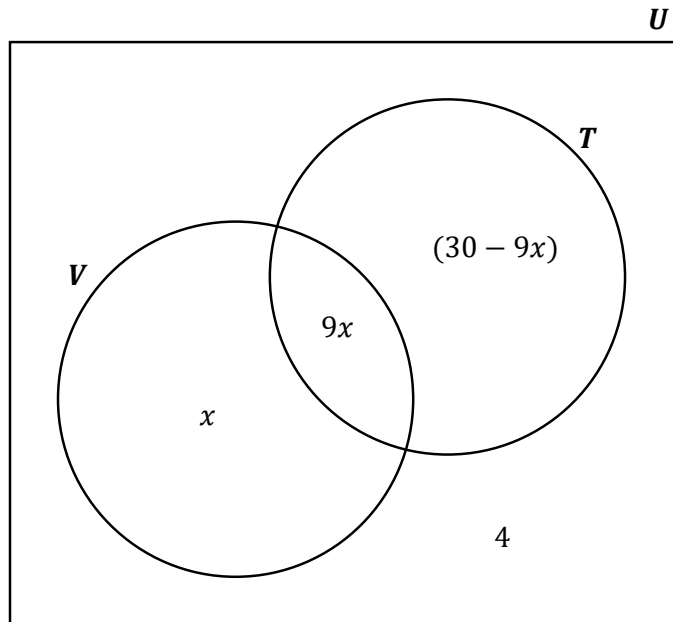
$$x = \frac{12}{3}$$

$$x = 4$$

$\therefore x = 4$ and $y = 1$.

Question 3(a)(i)

Required to copy and complete the Venn Diagram



Question 3(a)(ii)(a)

Required to write an expression in x for the total number of students in the survey.

The number of students who play volleyball only = x

The number of students who play tennis only = $30 - 9x$

The number of students who play both tennis and volleyball = $9x$

The number of students who do not play either tennis or volleyball = 4

Therefore,

Number of students in the survey = $x + (30 - 9x) + 9x + 4$

$$= x + 30 - 9x + 9x + 4$$

$$= x + 4$$

Question 3(a)(ii)(b)

Required to write an equation in x for the total number of students in the survey and to solve for x .

The total number of students = 36

The expression for the total number of students = $x + 34$

Hence, the equation is

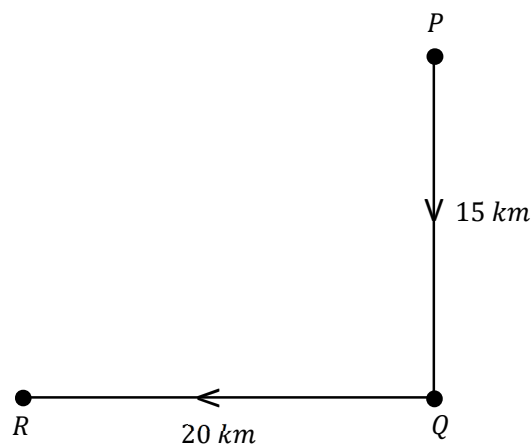
$$x + 34 = 36$$

$$x = 36 - 34$$

$$x = 2$$

Question 3(b)(i)

Required to copy the diagram and label it to show points Q and R and distances 20 km and 15 km .



Question 3(b)(ii)

Required to calculate the shortest distance of the ship from the port to where the journey started.

Using Pythagoras' Theorem,

$$\begin{aligned}PR^2 &= PQ^2 + RQ^2 \\ &= (15)^2 + (20)^2 \\ &= 225 + 400 \\ &= 625\end{aligned}$$

$$\begin{aligned}PR &= \sqrt{625} \\ &= 25 \text{ km}\end{aligned}$$

Question 3(b)(iii)

Required to calculate the measure of angle QPR , giving the answer to the nearest degree.

$$\begin{aligned}\sin Q\hat{P}R &= \frac{\text{opp}}{\text{hyp}} \\ &= \frac{20}{25}\end{aligned}$$

$$\begin{aligned}\therefore Q\hat{P}R &= \sin^{-1}\left(\frac{20}{25}\right) \\ &= 53.1^\circ \\ &= 53^\circ \quad (\text{to the nearest degree})\end{aligned}$$

Question 4(a)(i)

Required to calculate the length of the arc ABC .

$$\begin{aligned}\text{Length of the arc } ABC &= \frac{270^\circ}{360^\circ} \times 2\pi r \\ &= \frac{3}{4} \times 2 \times \frac{22}{7} \times 3.5 \\ &= 16.5 \text{ cm}\end{aligned}$$

Question 4(a)(ii)

Required to calculate the perimeter of the sector $OABC$.

$$\begin{aligned}\text{Perimeter of the sector } OABC \\ &= \text{Length of } AO + \text{Arc length } ABC + \text{Length of radius } CO \\ &= 16.5 + 3.5 + 3.5 \\ &= 23.5 \text{ cm}\end{aligned}$$

Question 4(a)(iii)

Required to calculate the area of the sector $OABC$

$$\begin{aligned}\text{Area of the sector } OABC &= \frac{270^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{3}{4} \times \frac{22}{7} \times (3.5)^2 \\ &= 28.88 \text{ cm}^2 \quad (\text{to 2 decimal places})\end{aligned}$$

Question 4(b)(i)

Required to calculate the volume of the prism.

Volume of the prism = Cross-sectional Area \times Height

$$= 28.875 \times 20$$

$$= 577.5 \text{ cm}^3$$

Question 4(b)(ii)

Required to calculate the mass of the prism, to the nearest *kg*.

1 cm^3 of tin weighs 7.3 *g*

577.5 cm^3 of tin weighs = 7.3 \times 577.5

$$= 4\,215.75 \text{ g}$$

Now,

$$1000 \text{ g} = 1 \text{ kg}$$

$$1 \text{ g} = \frac{1}{1000} \text{ kg}$$

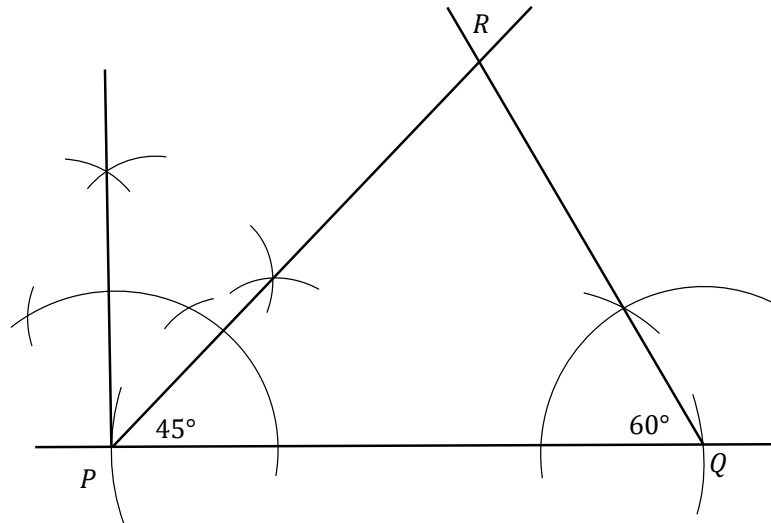
$$4\,215.75 \text{ g} = \frac{1}{1000} \times 4\,215.75$$

$$= 4.21575 \text{ kg}$$

$$= 4.2 \text{ kg} \quad (\text{to the nearest } \textit{kg})$$

Question 5(a)(i)

Required to construct Triangle PQR with $PQ = 8\text{ cm}$, $\angle PQR = 60^\circ$ and $\angle QPR = 45^\circ$



Question 5(a)(ii)

Required to measure and state the length of RQ .

By measurement using a ruler, the length of $RQ = 6\text{ cm}$.

Question 5(b)(i)

Required to find the gradient of the line.

The points are $S(6, 6)$ and $T(0, -2)$.

$$\begin{aligned}\text{Gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - (-2)}{6 - 0} \\ &= \frac{8}{6} \\ &= \frac{4}{3}\end{aligned}$$

Question 5(b)(ii)

Required to find the equation of the line.

Since $(0, -2)$ lies on the line, this implies that $c = -2$.

In part (b)(i), we found that $m = \frac{4}{3}$.

Therefore, the equation of the line is $y = \frac{4}{3}x - 2$.

Question 5(b)(iii)

Required to find the midpoint of the line segment TS .

Point $T(0, -2)$ and Point $S(6, 6)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) \\ &= \left(\frac{6+0}{2}, \frac{6+(-2)}{2} \right) \\ &= \left(\frac{6}{2}, \frac{4}{2} \right) \\ &= (3, 2) \end{aligned}$$

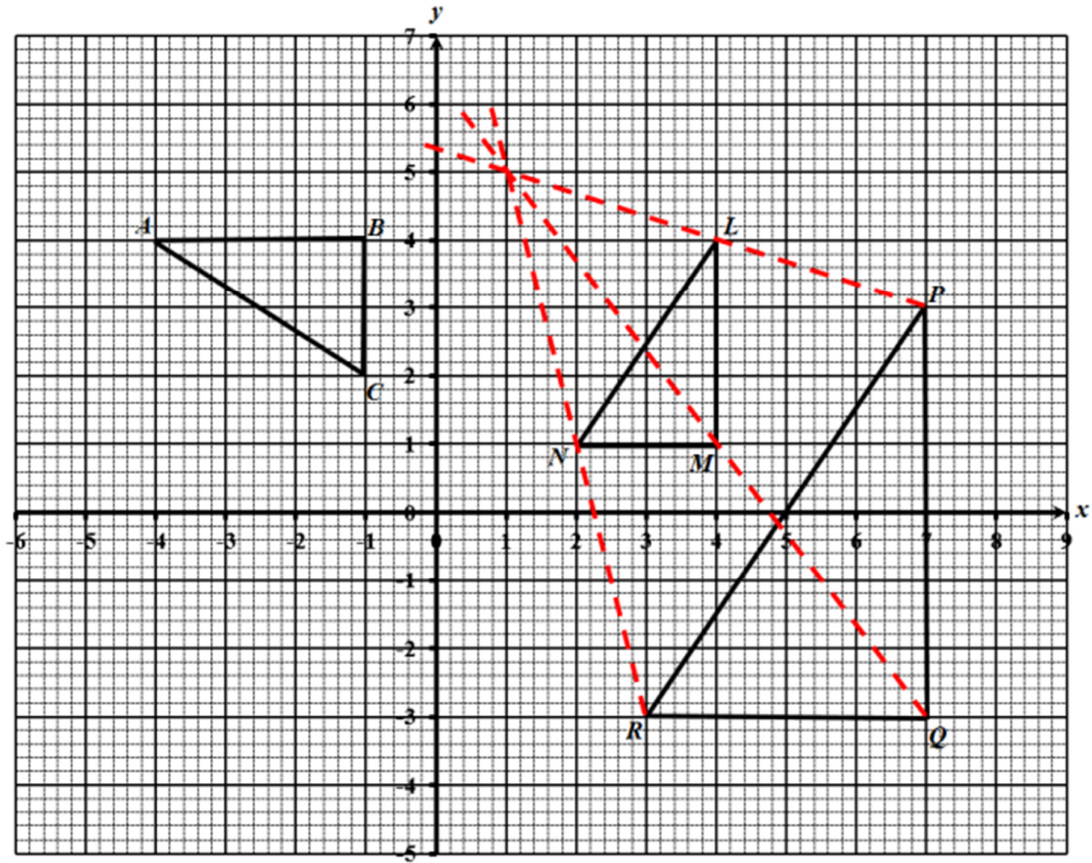
Question 5(b)(iv)

Required to find the length of line segment TS .

$$\begin{aligned} \text{Length of line segment } TS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 0)^2 + (6 - (-2))^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \text{ units} \end{aligned}$$

Question 6(a)

Required to locate the centre of enlargement.



The center of enlargement is (1, 5).

Question 6(b)

Required to state the scale factor.

$$\text{Scale factor} = \frac{\text{Length of } PQ}{\text{Length of } LM}$$

$$= \frac{6}{3}$$

$$= 2$$

Question 6(c)

Required to determine the value of $\frac{\text{Area of } PQR}{\text{Area of } LMN}$.

Since $\frac{RQ}{NM} = \frac{2}{1}$, then

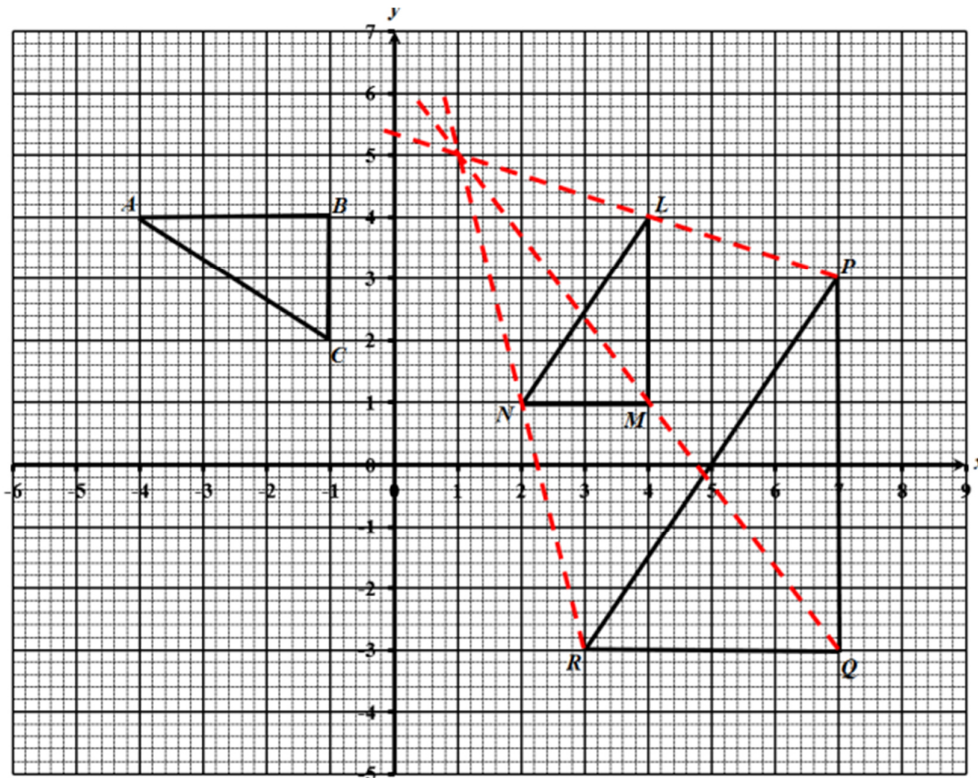
$$\frac{\text{Area of } PQR}{\text{Area of } LMN} = \left(\frac{2}{1}\right)^2$$

$$= (2)^2$$

$$= 4$$

Question 6(d)

Required to draw and label triangle ABC .



Question 6(e)

Required to describe fully the transformation which maps triangle LMN onto triangle ABC .

The transformation which maps ΔLMN into ΔABC is a rotation of 90° in an anti-clockwise direction, about the origin, $(0,0)$.

Question 7(a)

Required to copy and complete the table to show cumulative frequency.

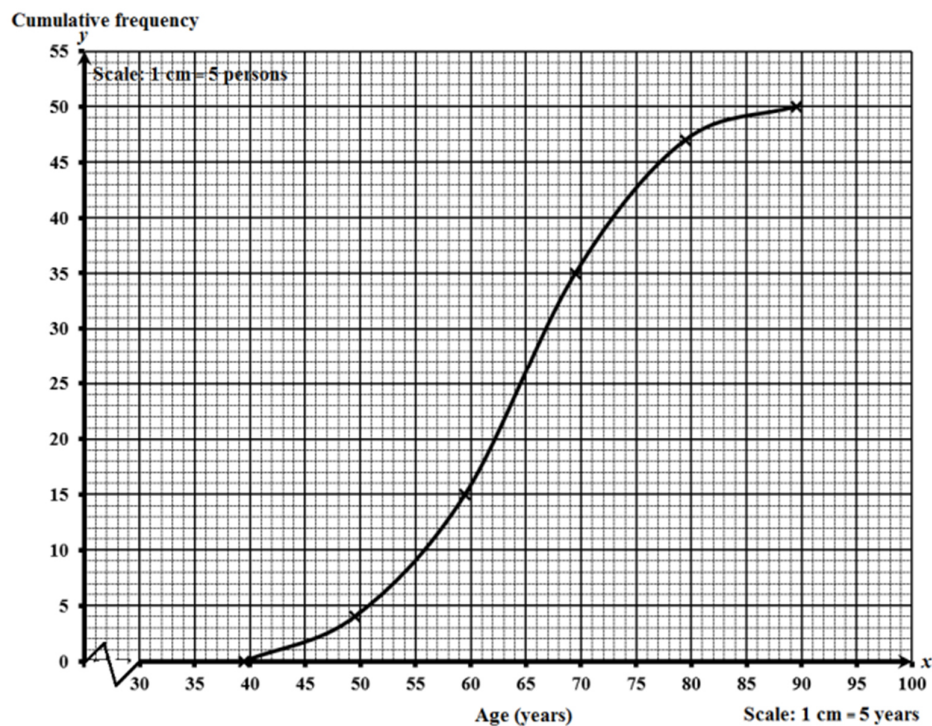
The completed table is shown below.

Age (x)	Upper Class Boundary (UCB)	Number of Persons	Cumulative Frequency
40-49	49.5	4	4
50-59	59.5	11	15
60-69	69.5	20	35
70-79	79.5	12	47
80-89	89.5	3	50

Question 7(b)

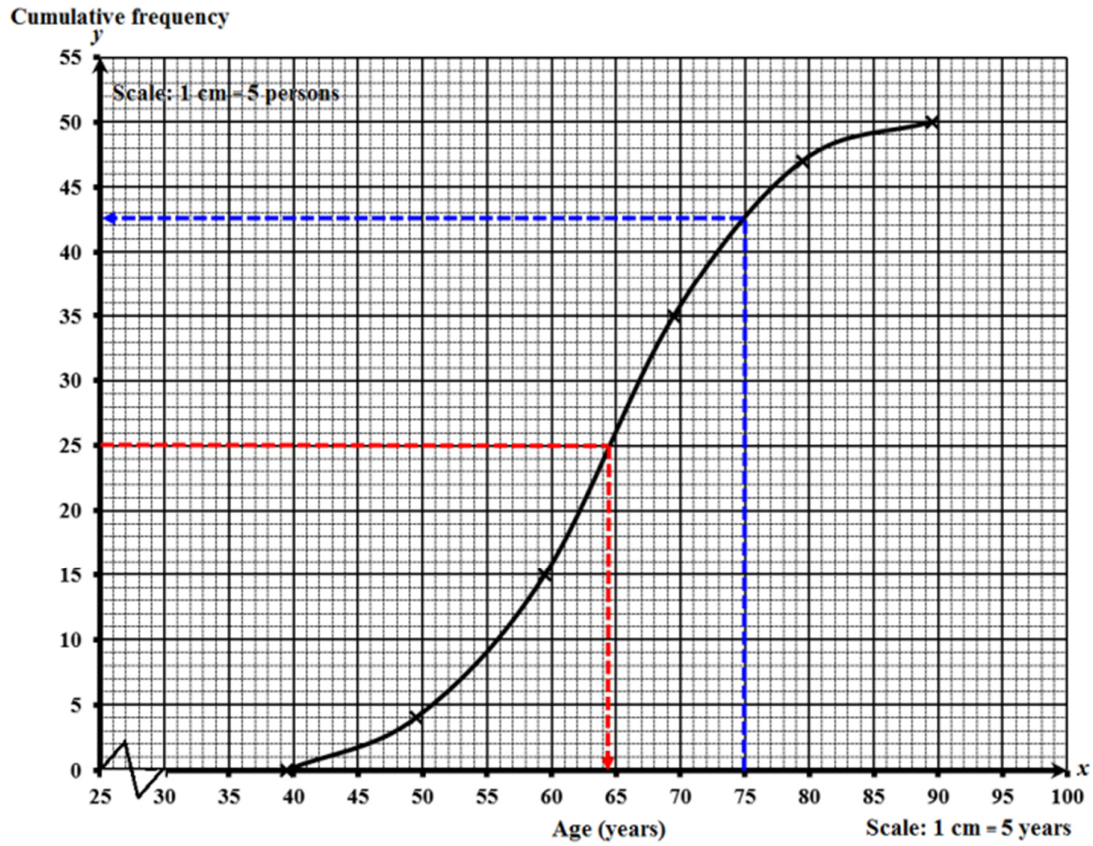
Required to draw the cumulative frequency curve to represent the data.

The cumulative frequency graph for the data is shown below.



Question 7(c)(i)

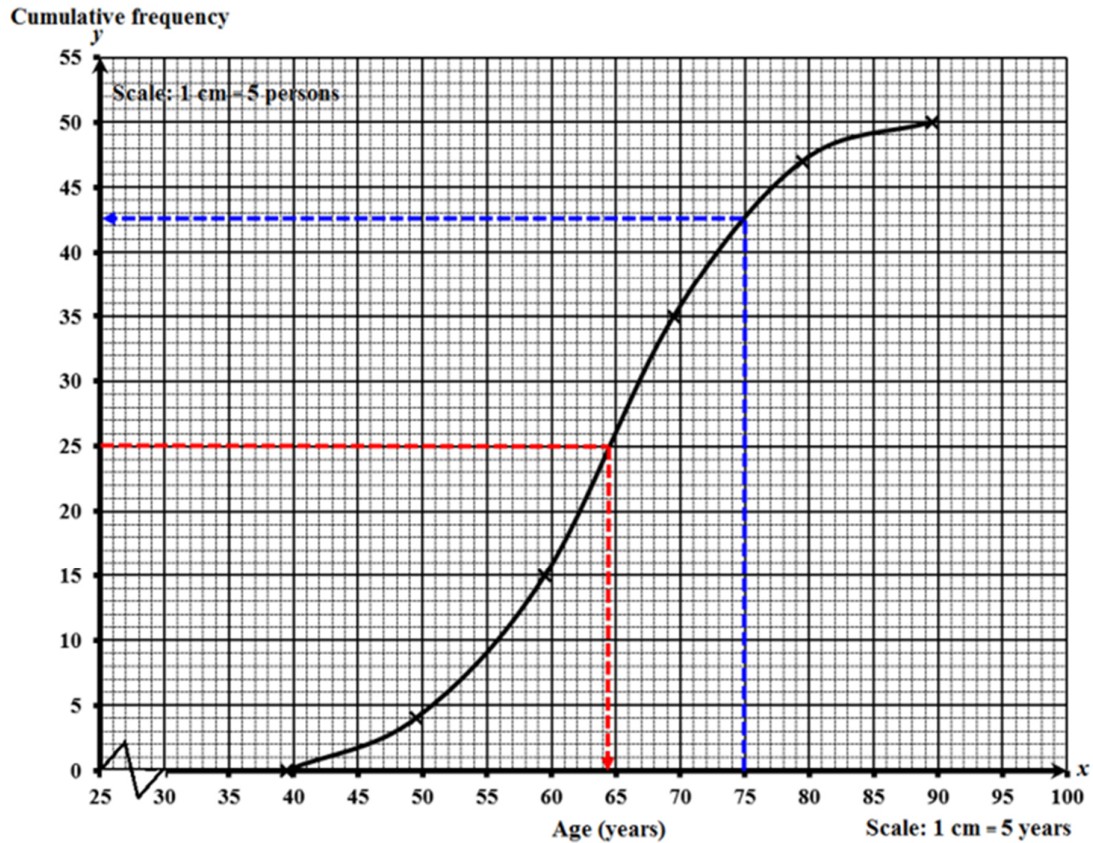
Required to estimate the median age for the data.



The median age for the data is 64.5 years.

Question 7(c)(ii)

Required to estimate the probability that a person who visited the clinic was 75 years or younger.

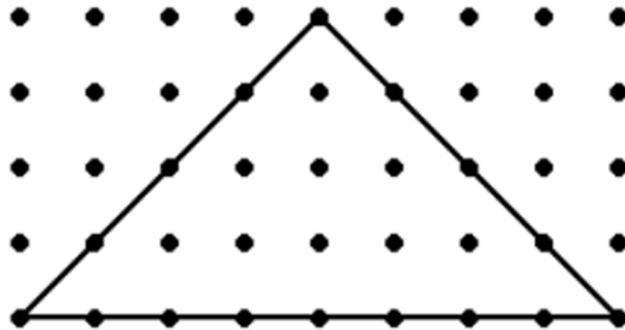


$$\begin{aligned}
 P(\text{person is 75 years or younger}) &= \frac{\text{Number of persons 75 years or younger}}{\text{Total number of persons}} \\
 &= \frac{42.5}{50} \\
 &= \frac{17}{20}
 \end{aligned}$$

Question 8(a)

Required to draw the fourth figure in the sequence of figures.

The fourth diagram in the sequence is shown below:



Question 8(b)

Required to copy and complete the missing values in the table.

The completed table is shown below.

Figure	Area of Triangle	No. of Pins on Base
1	1	$(2 \times 1) + 1 = 3$
2	4	$(2 \times 2) + 1 = 5$
3	6	$(2 \times 3) + 1 = 7$
4	<u>16</u>	<u>$(2 \times 4) + 1 = 9$</u>
<u>10</u>	100	<u>$(2 \times 10) + 1 = 21$</u>
20	<u>400</u>	<u>$(2 \times 20) + 1 = 41$</u>
n	<u>n^2</u>	<u>$2n + 1$</u>

Consider the n th figure.

$$\text{Area of triangle} = n^2$$

$$\text{No. of Pins on Base} = 2n + 1$$

When $n = 4$,

$$\text{Area of triangle} = 4^2$$

$$= 16$$

When $n = 4$,

$$\text{No. of Pins on Base} = 2(4) + 1$$

$$= 8 + 1$$

$$= 9$$

If the area of triangle = 100,

Then,

$$n^2 = 100$$

$$n = \sqrt{100}$$

$$= 10$$

When $n = 10$,

$$\text{No. of Pins on Base} = 2(10) + 1$$

$$= 20 + 1$$

$$= 21$$

When $n = 20$,

$$\text{Area of triangle} = 20^2$$

$$= 400$$

When $n = 20$,

$$\text{No. of Pins on Base} = 2(20) + 1$$

$$= 40 + 1$$

$$= 41$$

Question 9(a)(i)

Required to solve $y = 8 - x$ and $2x^2 + xy = -16$ simultaneously.

$$y = 8 - x \quad \rightarrow \text{Equation 1}$$

$$2x^2 + xy = -16 \quad \rightarrow \text{Equation 2}$$

Substituting Equation 2 into Equation 1 gives,

$$2x^2 + x(8 - x) = -16$$

$$2x^2 + 8x - x^2 = -16$$

$$x^2 + 8x + 16 = 0$$

$$x^2 + 4x + 4x + 16 = 0$$

$$x(x + 4) + 4(x + 4) = 0$$

$$(x + 4)(x + 4) = 0$$

We have

$$x + 4 = 0$$

$$x = -4$$

Substituting $x = -4$ into Equation 1 gives,

$$y = 8 - (-4)$$

$$= 8 + 4$$

$$= 12$$

$$\therefore x = -4 \text{ and } y = 12$$

Question 9(a)(ii)

Required to state with reason whether or not $y = 8 - x$ is a tangent to the curve with equation $2x^2 + xy = -16$.

When $y = 8 - x$ and $2x^2 + xy = -16$ were solve simultaneously, only one solution was obtained. Therefore, the straight line does not intersect the curve, but touches it at that one point where $x = -4$.

Hence, $y = 8 - x$ is a tangent to the curve with equation $2x^2 + xy = -16$ at the point $(-4, 12)$.

Question 9(b)(i)

Required to write the three inequalities for the constraints given.

Constraint: The number of orchids must be at least half the number of roses.

Inequality: $y \geq \frac{1}{2}x$

Constraint: There must be at least 2 roses.

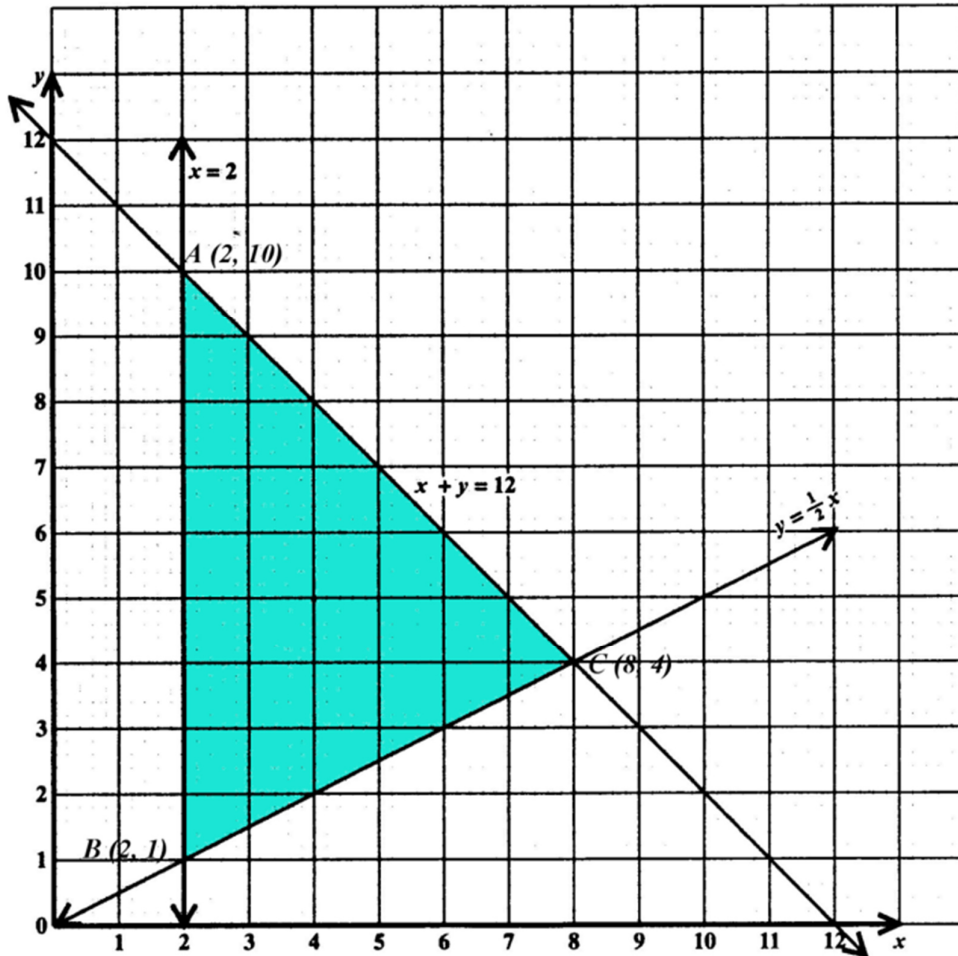
Inequality: $x \geq 2$

Constraint: There must be no more than 12 flowers in the bouquet.

Inequality: $x + y \leq 12$

Question 9(b)(ii)

Required to shade the region that satisfies all three inequalities.



Question 9(b)(iii)

Required to state the coordinates of the points which represent the vertices of the region showing the solution set.

The vertices of the region are: $A(2, 10)$, $B(2, 1)$ and $C(8, 4)$

Question 9(b)(iv)

Required to determine the maximum possible profit on the sale of a bouquet.

A profit of \$3 is made on each rose and \$4 on each orchid.

Let P be the profit.

Then, $P = 3x + 4y$.

We test the three points at the three vertices of the feasible region.

For the point $A(2, 10)$, $x = 2$ and $y = 10$.

$$P = 3(2) + 4(10)$$

$$= 6 + 40$$

$$= 46$$

For the point $B(2, 1)$, $x = 2$ and $y = 1$.

$$P = 3(2) + 4(1)$$

$$= 6 + 4$$

$$= 10$$

For the point $C(8, 4)$, $x = 8$ and $y = 4$.

$$P = 3(8) + 4(4)$$

$$= 24 + 16$$

$$= 40$$

\therefore The maximum profit occurs at point $A(2, 10)$ where the profit is \$46, the number of roses is 2 and the number of orchids is 10.

Question 10(a)(i)

Required to calculate the length of RS .

Consider triangle QRS . Using the sine rule,

$$\frac{QS}{\sin \hat{R}} = \frac{RS}{\sin \hat{Q}}$$

$$\frac{7}{\sin 60^\circ} = \frac{RS}{\sin 48^\circ}$$

$$RS \sin 60^\circ = 7 \sin 48^\circ$$

$$RS = \frac{7 \sin 48^\circ}{\sin 60^\circ}$$

$$RS = 6.01 \quad (\text{to 2 decimal places})$$

Question 10(a)(ii)

Required to calculate the measure of $\angle QTS$.

Consider the triangle QTS . Using the cosine rule,

$$QS^2 = QT^2 + TS^2 - 2(QT)(TS) \cos \hat{T}$$

$$(7)^2 = (8)^2 + (10)^2 - 2(8)(10) \cos \hat{T}$$

$$49 = 64 + 100 - 160 \cos \hat{T}$$

$$49 = 164 - 160 \cos \hat{T}$$

$$\cos \hat{T} = \frac{164-49}{160}$$

$$\cos \hat{T} = 0.71875$$

$$\hat{T} = \cos^{-1}(0.71875)$$

$$\hat{T} = 44.0^\circ \quad (\text{to 1 decimal place})$$

$$\therefore \angle QTS = 44^\circ$$

Question 10(b)(i)(a)

Required to calculate the measure of angle OUZ .

The sum of angles on a straight line add up to 180° .

$$\begin{aligned} Z\hat{O}U &= 180^\circ - 70^\circ \\ &= 110^\circ \end{aligned}$$

Since OZ and OU are radii of the same circle, $OZ = OU$.

Therefore, ΔZOU is an isosceles triangle. The base angles of an isosceles triangle are equal and the sum of the three interior angles of a triangle add up to 180° .

$$\begin{aligned} \therefore O\hat{U}Z &= \frac{180^\circ - 110^\circ}{2} \\ &= 35^\circ \end{aligned}$$

Question 10(b)(i)(b)

Required to calculate the measure of angle UVY .

The angle at the centre of a circle is twice the angle at the circumference of the circle from the same chord.

$$\begin{aligned} U\hat{Y}V &= \frac{70^\circ}{2} \\ &= 35^\circ \end{aligned}$$

The angle made by the tangent UW to a circle and a radius OU is a right angle.

$$\therefore O\hat{U}V = 90^\circ$$

The sum of angles in a triangle add up to 180° .

$$\begin{aligned}U\hat{V}Y &= 180^\circ - (90^\circ + 35^\circ) \\ &= 180^\circ - 125^\circ \\ &= 55^\circ\end{aligned}$$

Question 10(b)(i)(c)

Required to calculate the measure of angle UWO .

The sum of angles in a triangle add up to 180° .

$$\begin{aligned}W\hat{O}U + O\hat{U}W + U\hat{W}O &= 180^\circ \\ 70^\circ + 90^\circ + U\hat{W}O &= 180^\circ \\ U\hat{W}O &= 180^\circ - (70^\circ + 90^\circ) \\ &= 180^\circ - 160^\circ \\ &= 20^\circ\end{aligned}$$

Question 10(b)(ii)(a)

Required to name the triangle which is congruent to ΔZOU .

Since OZ and OY are radii of the same circle, $OZ = OY$.

Since OU and OX are radii of the same circle, $OZ = OX$.

Since vertically opposite angles are equal, then $Z\hat{O}U = Y\hat{O}X$.

Hence, ΔZOU is congruent to ΔYOX .

Question 10(b)(ii)(b)

Required to name the triangle which is congruent to ΔYXU .

Consider ΔYXU and ΔZUX .

Since YU and ZX are both diameter of the same circle, then $YU = ZX$.

Since the angle in a semicircle is equal to 90° , then $\widehat{YXU} = \widehat{ZUX} = 90^\circ$.

UX is a common side to both triangles.

So, both right angled triangles, YXU and ZUX have the same hypotenuse and share a common side.

Hence, ΔYXU is congruent to ΔZUX .

Question 11(a)(i)(a)

Required to express the vector \overrightarrow{BA} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

$$\begin{aligned}
 \overrightarrow{BA} &= \overrightarrow{BO} + \overrightarrow{OA} \\
 &= -\overrightarrow{OB} + \overrightarrow{OA} \\
 &= -\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\
 &= \begin{pmatrix} -3 \\ -4 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\
 &= \begin{pmatrix} -3 + 6 \\ -4 + 2 \end{pmatrix} \\
 &= \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad \text{which is of the form } \begin{pmatrix} x \\ y \end{pmatrix} \text{ where } x = 3 \text{ and } y = -2.
 \end{aligned}$$

Question 11(a)(i)(b)

Required to express the vector \overrightarrow{BC} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.

$$\begin{aligned}
 \overrightarrow{BC} &= \overrightarrow{BO} + \overrightarrow{OC} \\
 &= -\overrightarrow{OB} + \overrightarrow{OC} \\
 &= -\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ -2 \end{pmatrix} \\
 &= \begin{pmatrix} -3 \\ -4 \end{pmatrix} + \begin{pmatrix} 12 \\ -2 \end{pmatrix} \\
 &= \begin{pmatrix} -3 + 12 \\ -4 - 2 \end{pmatrix} \\
 &= \begin{pmatrix} 9 \\ -6 \end{pmatrix} \quad \text{which is of the form } \begin{pmatrix} x \\ y \end{pmatrix} \text{ where } x = 9 \text{ and } y = -6.
 \end{aligned}$$

Question 11(a)(ii)

Required to state one geometrical relationship between BA and BC .

$$\overrightarrow{BA} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\begin{aligned} \overrightarrow{BC} &= \begin{pmatrix} 9 \\ -6 \end{pmatrix} \\ &= 3 \begin{pmatrix} 3 \\ -4 \end{pmatrix} \end{aligned}$$

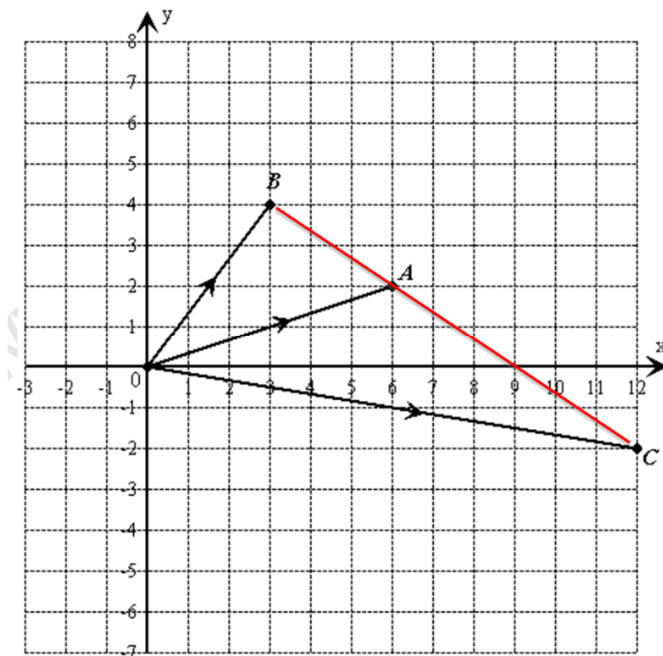
$$\text{So, } |\overrightarrow{BC}| = 3|\overrightarrow{BA}|.$$

Since \overrightarrow{BA} can be represented as a scalar multiple of \overrightarrow{BC} , and B is a common point on both vectors, then \overrightarrow{BA} and \overrightarrow{BC} are parallel.

Therefore, A , B and C are collinear and \overrightarrow{BC} is three times the length of \overrightarrow{BA} .

Question 11(a)(iii)

Required to draw a sketch to show the relative positions of A , B and C .



Question 11(b)(i)

Required to calculate the values of a and b .

$$\begin{pmatrix} a & -4 \\ 1 & b \end{pmatrix} \begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} (a \times 2) + (-4 \times 1) & (a \times -4) + (-4 \times -3) \\ (1 \times 2) + (b \times 1) & (1 \times -4) + (b \times -3) \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 2a - 4 & -4a + 12 \\ 2 + b & -4 - 3b \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Comparing the equivalent matrices and equating corresponding entries, we get:

$$2a - 4 = 2$$

and

$$2 + b = 0$$

$$2a = 4 + 2$$

$$b = -2$$

$$2a = 6$$

$$a = \frac{6}{2}$$

$$a = 3$$

$\therefore a = 3$ and $b = -2$.

Question 11(b)(ii)

Required to find the inverse of $\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}$.

$$\det(A) = ad - bc$$

$$= (2)(-3) - (-4)(1)$$

$$= -6 - (-4)$$

$$= -6 + 4$$

$$= -2$$

$$\begin{aligned} \text{adj}(A) &= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \\ &= \begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \therefore A^{-1} &= \frac{1}{\det} \times \text{adj}(A) \\ &= -\frac{1}{2} \times \begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{-3}{-2} & \frac{4}{-2} \\ \frac{-1}{-2} & \frac{2}{-2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{3}{2} & -2 \\ \frac{1}{2} & -1 \end{pmatrix} \end{aligned}$$

Question 11(b)(iii)

Required to solve for x and y in $\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$.

$$\begin{aligned} \begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 12 \\ 7 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 12 \\ 7 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= -\frac{1}{2} \begin{pmatrix} -3 & 4 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ 7 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= -\frac{1}{2} \begin{pmatrix} (-3 \times 12) + (4 \times 7) \\ (-1 \times 12) + (2 \times 7) \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= -\frac{1}{2} \begin{pmatrix} -36 + 28 \\ -12 + 14 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= -\frac{1}{2} \begin{pmatrix} -8 \\ 2 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 4 \\ -1 \end{pmatrix} \end{aligned}$$

$\therefore x = 4$ and $y = -1$.