Solutions to CSEC Maths P2 June 2013

Question 1(a)(i)

$$\text{Numerator} = 1\frac{4}{5} - \frac{1}{3}$$

$$= \frac{9}{5} - \frac{1}{3}$$

$$= \frac{27-5}{15}$$

$$= \frac{22}{15}$$

$$\text{Denominator} = 2\frac{2}{5}$$

$$= \frac{12}{5}$$

$$\therefore \text{Numerator} \div \text{Denominator} = \frac{22}{15} \div \frac{12}{5}$$

$$= \frac{22}{15} \times \frac{5}{12}$$

$$= \frac{110}{180}$$

$$= \frac{11}{18}$$

Question 1(a)(ii)

$$\sqrt{1.5625} + (0.32)^2 = 1.25 + 0.1024$$

$$= 1.3524 \qquad \text{(in exact form)}$$

Question 1(b)

If 350 ml is sold for \$4.20,

Then 1 ml will cost the buyer $\frac{\$4.20}{350} = \$0.012$ per ml

If 450 ml is sold for \$5.13,

Then 1 ml will cost the buyer $\frac{\$5.13}{450} = \$0.0114$ per ml

Hence, the better buy is the 450 ml carton costing \$5.13/

Question 1(c)(i)

Interest for the first year $= 8\%\ of\ \$9600$

$= \frac{8}{100} \times\ \$9600$

$= \$768$

Question 1(c)(ii)

At the end of the first year,

Amount owed $=$ The sum borrowed $+$ The interest acquired

$= \$9600 + \$768$

$= \$10\ 368$

At the beginning of the second year,

Amount owed $=$ Total amount owed at the end of the first year – Amount repaid

$= \$10\ 368 - \$4\ 368$

$= \$6000$

Question 1(c)(iii)

Required to calculate the interest on the remaining balance for the second year.

Amount of interest on the remaining balance $= 8\%\ of\ \$6\ 000$

$= \frac{8}{100} \times \$6\ 000$

$= \$480$

Question 2(a)(i)

$2x^3 - 8x$

$= 2x(x^2 - 4)$

$= 2x(x - 2)(x + 2)$ (difference of two squares)

Question 2(a)(ii)

$3x^2 - 5x - 2$

$= 3x^2 - 6x + x - 2$

$= 3x(x - 2) + 1(x - 2)$

$= (3x + 1)(x - 2)$

Question 2(b)(i)

$F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C$

$\frac{9}{5}C = F - 32$

$9C = 5(F - 32)$

$C = \frac{5(F-32)}{9}$

Question 2(b)(ii)

When $F = 113$,

$C = \frac{5(113-32)}{9}$

$= \frac{5(81)}{9}$

$= 45$

Question 2(c)(i)(a)

The number of tickets sold $= 500$

The number of tickets sold at \$6 each $= x$

$\therefore$ The remaining tickets sold at \$10 each $= 500 - x$

Question 2(c)(i)(b)

The total cost of x tickets at \$6 each $= 6x$

The cost of $(500 - x)$ tickets at \$10 each $= 10(500 - x)$

Thus, the total amount of money collected $= 6x + 10(500 - x)$

$= 6x + 5000 - 10x$

$= 5000 - 4x$

Question 2(c)(ii)

$5000 - 4x = 4108$

$4x = 5000 - 4108$

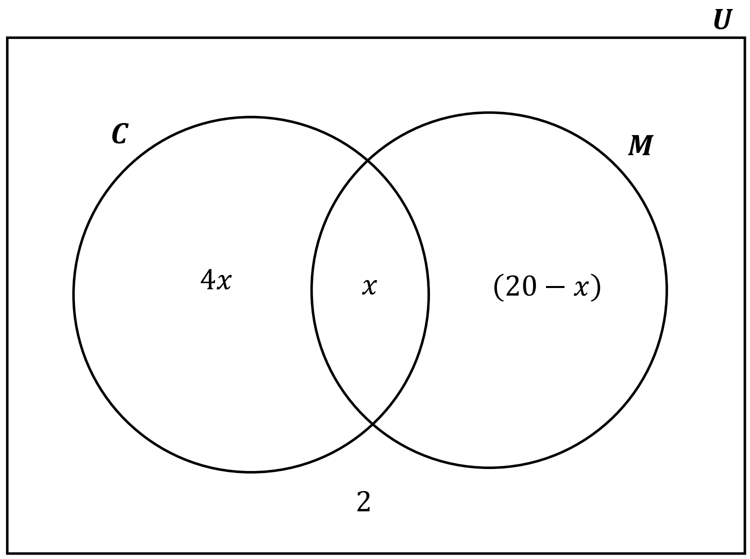
$4x = 892$

$x = \frac{892}{4}$

$x = 223$

$\therefore$ 223 tickets were sold for \$6 each.

Question 3(a)(i)



Question 3(a)(ii)

Total number of students $= 2 + 4x + x + (20 - x)$

$= 2 + 4x + x + 20 - x$

$= 4x + 22$

Question 3(a)(iii)

The total number of students in the class is 30.

So,

$4x + 22 = 30$

$4x = 30 - 22$

$4x = 8$

$x = \frac{8}{4}$

$x = 2$

Therefore, the number of students who uses cameras only $= 4x$

$= 4(2)$

$= 8$

Question 3(b)(i)

Consider the triangle ABC.

Using Pythagoras' Theorem,

$AB^2 + BC^2 = AC^2$

$\therefore BC^2 = AC^2 - AB^2$

$= (10)^2 - (8)^2$

$= 100 - 64$

$= 36$

$BC = \sqrt{36}$

$= 6\ m$

Question 3(b)(ii)

The two triangles, ADE and ABC, have the same size of angles and are equiangular.

However, none of their sides are equal.

So, the two triangles are equiangular but not congruent.

Therefore, the two triangles ADE and ABC are similar since they possess the same ship but differ in size.

Question 3(b)(iii)

Note that if two triangles are similar, then the ratio of their corresponding sides are equal.

$AD = 8 - 3.2$

$= 4.8\ m$

Hence,

$\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{4.8}{8} = \frac{DE}{6}$

$DE = \frac{6 \times 4.8}{8}$

$= 3.6\ m$

Question 4(a)(i)

By measurement with a ruler, $DE = 5\ cm$.

Question 4(a)(ii)

By measurement with a protractor, $E\hat{C}D = 37°$.

Question 4(a)(iii)

By measurement with a ruler, $ED = EC = 5\ cm$ and $CD = 8\ cm$.

Therefore,

The perimeter of triangle $CDE = 5 + 5 + 8$

$= 18\ cm$

Question 4(a)(iv)

By measurement with a ruler, $EG = 3\ cm$.

Since G is the midpoint of CD and ΔCDE is isosceles, then EG is perpendicular to CD.

Therefore,

Area of $\Delta CDE = \frac{b \times h}{2}$

$= \frac{8 \times 3}{2}$

$= \frac{24}{2}$

$= 12\ cm^2$

Question 4(b)(i)

Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-4}{3-(-1)}$

$= \frac{-2}{4}$

$= -\frac{1}{2}$

Question 4(b)(ii)

Let the midpoint of AB be M.

Using the midpoint formula, we get

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-1+3}{2}, \frac{4+2}{2}\right)$

$= \left(\frac{2}{2}, \frac{6}{2}\right)$

$= (1,3)$

Question 4(b)(iii)

The gradient of the perpendicular bisector $= \frac{-1}{-\frac{1}{2}}$

$= -1 \div -\frac{1}{2}$

$= -1 \times -2$

$= 2$

Thus, $m = 2$.

Substituting $m = 2$ and the midpoint $(1, 3)$ into $y = mx + c$, we get

$3 = (2)(1) + c$

$3 = 2 + c$

$c = 3 - 2$

$c = 1$

$\therefore$ Equation of the perpendicular bisector AB is $y = 2x + 1$.

Question 5(a)(i)

$A \propto R^2$

$\therefore A = kR^2$ where k is the constant of proportion

Question 5(a)(ii)

Substituting $A = 36$ and $R = 3$ gives,

$36 = k(3)^2$

$36 = 9k$

$k = \frac{36}{9}$

$k = 4$

Question 5(a)(iii)

$A = 4R^2$

When $R = 5$,

$A = 4(5)^2$

$= 4 \times 25$

$= 100$

When $A = 196$,

$196 = 4R^2$

$R^2 = \frac{196}{4}$

$R^2 = 49$

$R = \sqrt{49}$

$R = 7$

The completed table is shown below.

A	36	100	196
R	3	5	7

Question 5(b)(i)

$g(2) = 4(2) + 5$

$= 8 + 5$

$= 13$

$\therefore fg(2) = f(13)$

$= \frac{2(13)+1}{3}$

$= \frac{26+1}{3}$

$= \frac{27}{3}$

$= 9$

Question 5(b)(ii)

$f(x) = \frac{2x+1}{3}$

Let $y = f(x)$.

$y = \frac{2x+1}{3}$

Interchange the variables x and y.

$x = \frac{2y+1}{3}$

Make y the subject.

$3x = 2y + 1$

$2y = 3x - 1$

$y = \frac{3x-1}{2}$

$\therefore f^{-1}(x) = \frac{3x-1}{2}$

Now,

$f^{-1}(3) = \frac{3(3)-1}{2}$

$= \frac{9-1}{2}$

$= \frac{8}{2}$

$= 4$

Question 6(a)(i)

$54\ kmh^{-1} = 54 \times \frac{1000}{3600}$

$= 15\ ms^{-1}$

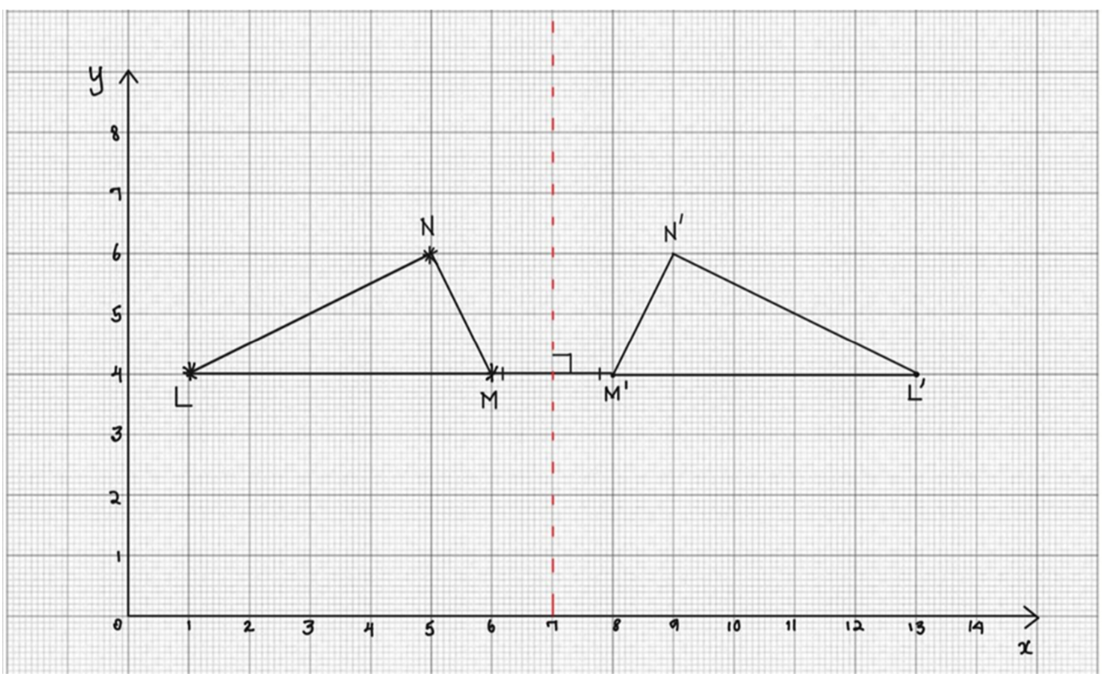
Question 6(a)(ii)

Distance = Speed × Time

$= 15 \times 20$

$= 300\ m$

Question 6(b)(i)

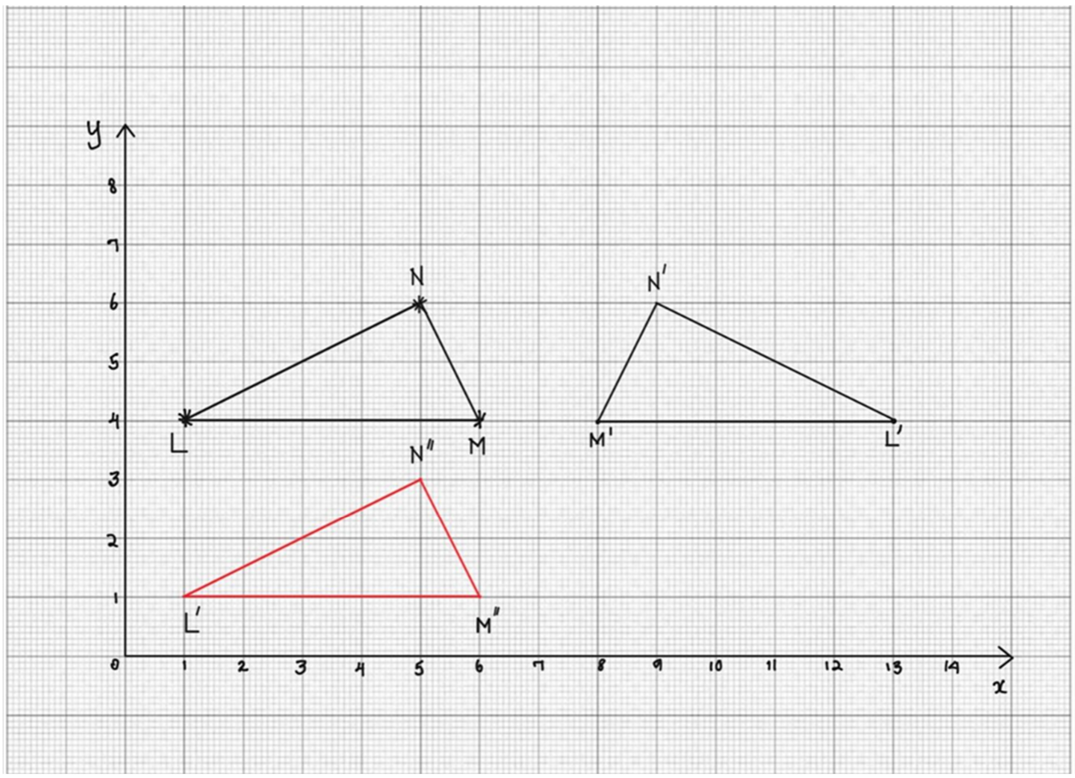


Note that the image and the object are congruent. Also, the image is laterally inverted.

Therefore, the transformation is a reflection in the line $x = 7$.

Question 6(b)(ii)

Required to draw $\Delta L''M''N''$.



Question 6(b)(iii)

Translate $\Delta L''M''N''$ by the vector $-\begin{pmatrix} 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ which will result in ΔLMN.

Then reflect the image ΔLMN in the line $x = 7$ which will result in the image $\Delta L'M'N'$.

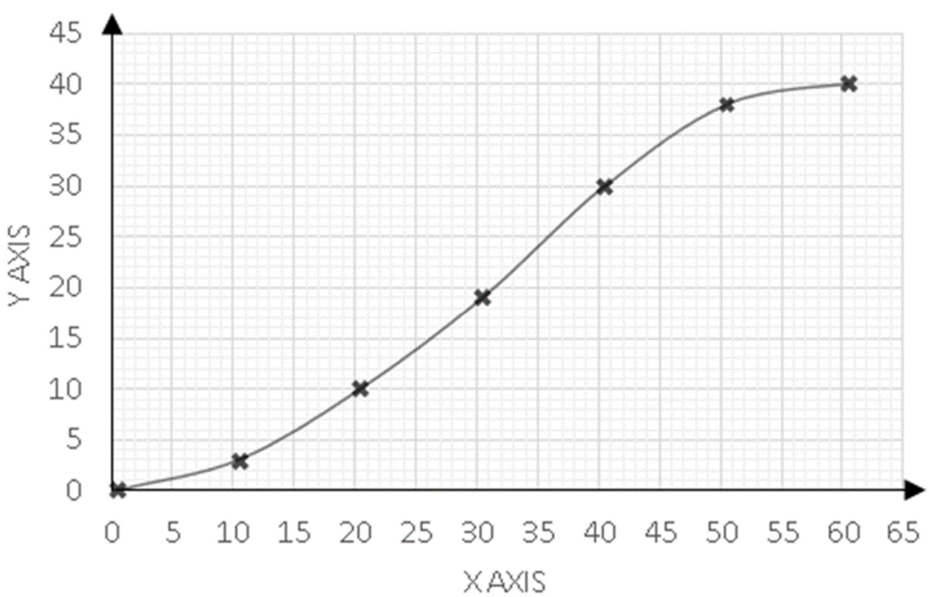
Question 7(a)

The completed table is shown below.

Amount Spent (x) L.C.L—U.C.L	Lower Class Boundary & Upper Class Boundary	Number of Students Frequency (f)	Cumulative Frequency	Points to be Plotted
				(0.5,0)
1-10	$0.5 \leq x < 10.5$	3.	3	(10.5,3)
11-20	$10.5 \leq x < 20.5$	7	$7 + 3 = 5$	(20.5,10)
21-30	$20.5 \leq x < 30.5$	9	$10 + 9 = 19$	(30.5,19)
31-40	$30.5 \leq x < 40.5$	11	$19 + 11 = 30$	(40.5,30)
41-50	$40.5 \leq x < 50.5$	8	$30 + 8 = 38$	(50.5,38)
51-60	$50.5 \leq x < 60.5$	2	$38 + 2 = 40$	(60.5,40)

Question 7(b)

The cumulative frequency graph for the data is shown below.



Question 7(c)(i)

Now, $\frac{1}{2} of\ 40 = 20$.

Therefore, the median amount of money spent = \$31.50 (by read-off).

Question 7(c)(ii)

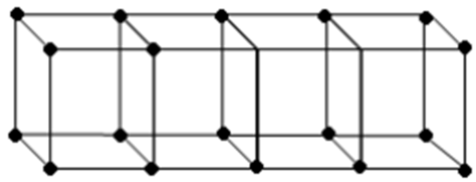
$$P(student\ spends\ less\ than\ \$23) = \frac{Number\ of\ students\ who\ spends\ less\ than\ \$23}{Total\ number\ of\ students}$$

$$= \frac{12}{40}$$

$$= \frac{3}{10}$$

Question 8(a)

The fourth diagram in the sequence is shown below:



Question 8(b)

Required to complete the missing values in the table.

	Name of Diagram (N)	No. of Wires (W)	No. of Balls (B)
	1	12	6
	2	20	12
	3	28	16
(i)	4	36	20
(ii)	20	164	84

For the Nth diagram,

$W = 8N + 4$ and $B = 4N + 4$

When $N = 4$,

$W = 8(4) + 4$ and $B = 4(4) + 4$

$= 32 + 4$ $= 16 + 4$

$= 36$ $= 20$

When $N = 20$,

$W = 8(20) + 4$ and $B = 4(20) + 4$

$= 160 + 4$ $= 80 + 4$

$= 164$ $= 84$

Question 8(c)(i)

The rule to find W is : $W = 8N + 4$

Question 8(c)(ii)

The rule to find B is : $B = 4N + 4$

Question 9(a)(i)

Number of oranges $= x$

Number of mangoes $= y$

Total number of fruits $= x + y$

Since the total number of oranges and mangoes must not exceed 6, then the inequality to represent this information is:

$x + y \leq 6$

Question 9(a)(ii)

Since Trish must buy at least 2 mangoes, then the inequality to represent this information is:

$y \geq 2$

Question 9(a)(iii)

Inequality: $y \leq 2x$

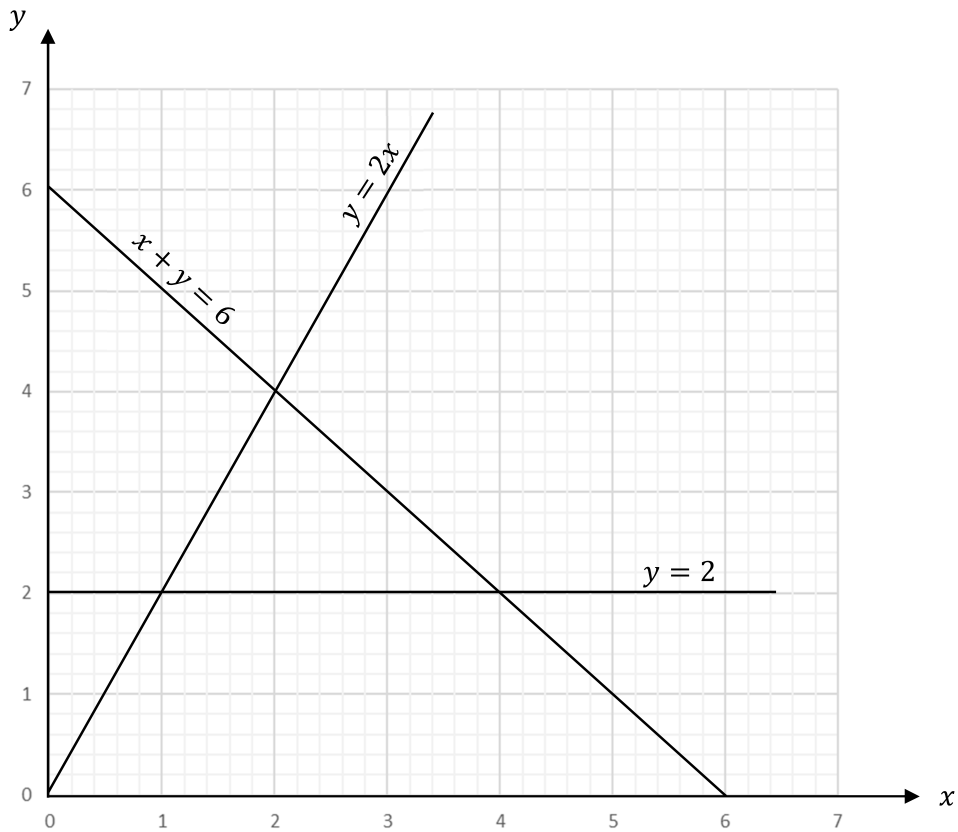
In words, the number of mangoes is less than or equal to two times the number of oranges.

Question 9(a)(iv)

Required to draw the lines associated with the following two inequalities:

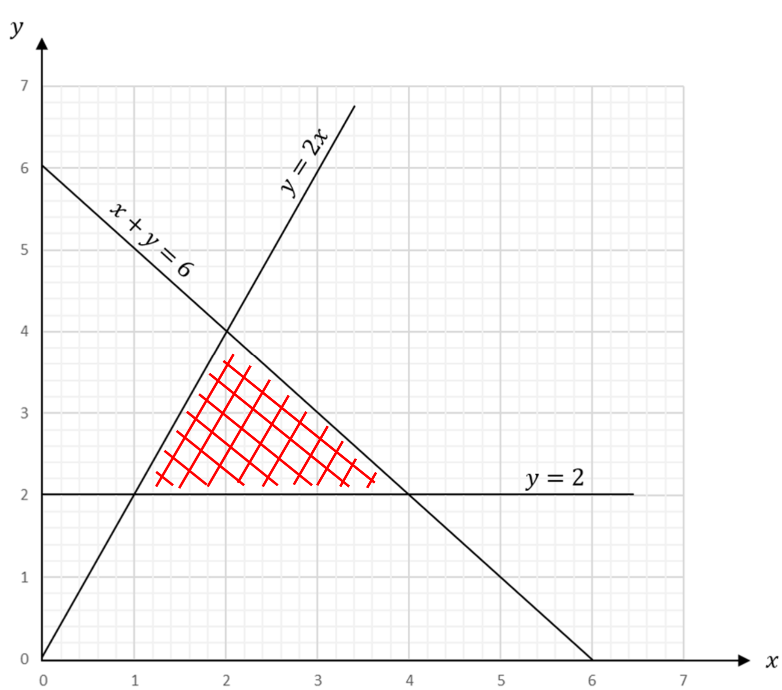
$x + y \leq 6$

$y \geq 2$



Question 9(a)(v)

Required to shade the region that satisfies all the inequalities.



Question 9(b)(i)

$3x^2 - 12x + 8 = 3(x^2 - 4x) + 8$

$= 3(x^2 - 4x + 4) + 8 - 3(4)$

$= 3(x - 2)^2 + 8 - 12$

$= 3(x - 2)^2 - 4$ which is of the form $a(x + h)^2 + k$

where $a = 3, h = -2$ and $k = -4$

Question 9(b)(ii)

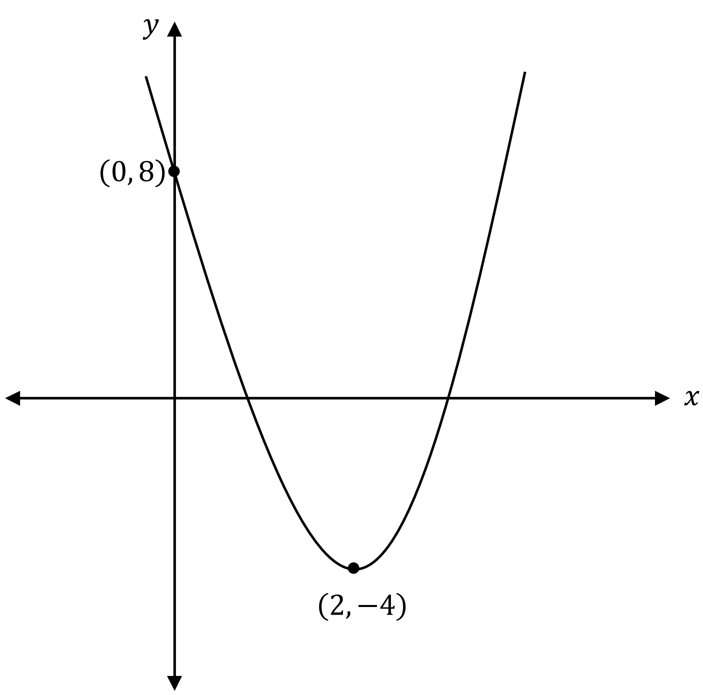
When $x = 0$,

$y = 3(0)^2 - 12(0) + 8$

$= 0 - 0 + 8$

$= 8$

The minimum point is $(2, -4)$.



Question 10(a)(i)

The angle made by a tangent to a circle and any chord, at the point of contact, is 90°.

So, $O\hat{B}E = 90°$.

OB is the radius and EBC is the tangent. The point of contact is B.

$\therefore E\hat{B}F = 90° - 35°$

$= 55°$

Question 10(a)(ii)

Since OB and OA are both radii of the same circle, then $OB = OA$.

The base angles of an isosceles triangle are equal.

$\therefore \Delta OAB$ is an isosceles triangle with $O\hat{B}A = O\hat{A}B = 40°$.

The sum of angles in a triangle add up to 180°.

$\therefore B\hat{O}A = 180° - (40° + 40°)$

$= 180° - 80°$

$= 100°$

Question 10(a)(iii)

The angle subtended by a chord at the center of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc.

So,

$A\hat{F}B = \frac{1}{2}(100°)$

$= 50°$

Question 10(a)(iv)

The sum of angles in a triangle add up to 180°.

$\therefore O\hat{A}F = 180° - (50° + 35° + 40° + 40°)$

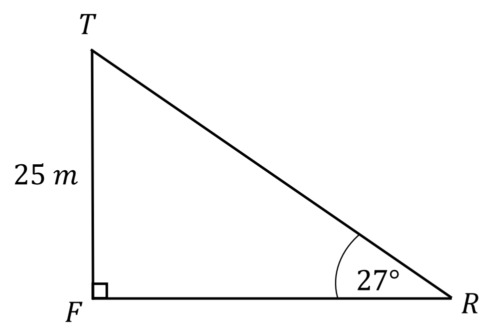
$= 180° - 165°$

$= 15°$

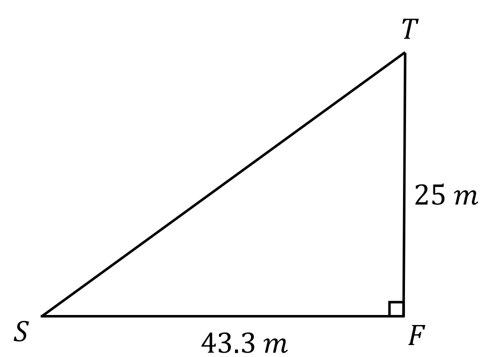
Question 10(b)(i)

The diagrams are as follows.

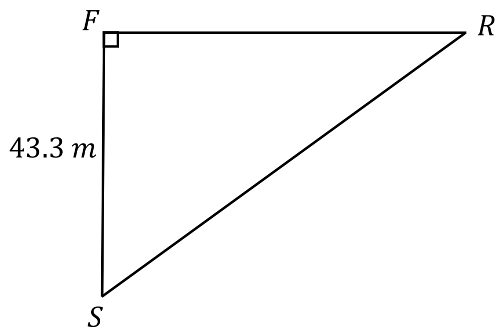
For triangle RFT:



For triangle TFS:

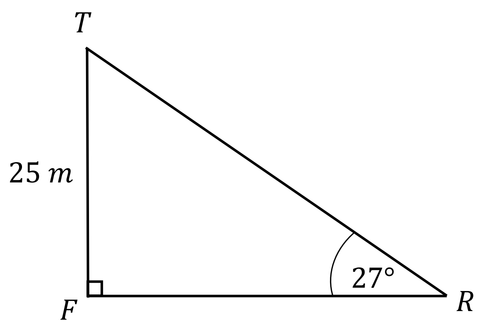


For triangle SFR:



Question 10(b)(ii)

Consider triangle RFT:



$\tan 27° = \frac{25}{RF}$

$\therefore RF = \frac{25}{\tan 27°}$

$= 49.1\ m$

Question 10(b)(iii)

Consider the triangle SFR.

Using Pythagoras' Theorem,

$SR^2 = FR^2 + FS^2$

$= (49.06)^2 + (43.3)^2$

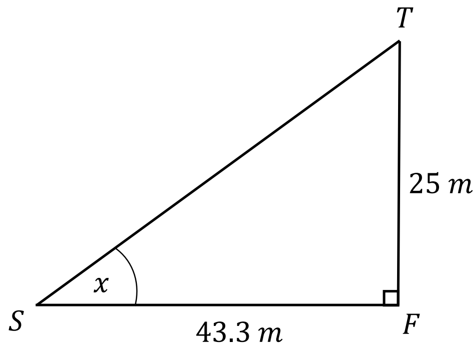
$= 4281.77$

$SR = \sqrt{4281.77}$

$= 65.4\ m$ (to 1 decimal place)

Question 10(b)(iv)

Consider triangle TFS:



$\tan x = \frac{25}{43.3}$

$x = \tan^{-1}\left(\frac{25}{43.3}\right)$

$x = 30.0°$ (to 1 decimal place)

Question 11(a)(i)(a)

Using the vector triangle law, we get

$AB = AO + OB$

$= -(2\boldsymbol{a}) + 2\boldsymbol{b}$

$= -2\boldsymbol{a} + 2\boldsymbol{b}$

Question 11(a)(i)(b)

$OP = PA = \boldsymbol{a}$

$AQ = \frac{1}{2}AB$

$= \frac{1}{2}(-2\boldsymbol{a} + 2\boldsymbol{b})$

$PQ = PA + AQ$

$= \boldsymbol{a} + \frac{1}{2}(-2\boldsymbol{a} + 2\boldsymbol{b})$

$= \boldsymbol{a} - \boldsymbol{a} + \boldsymbol{b}$

$= \boldsymbol{b}$

Question 11(a)(ii)

$OB = 2PQ$

Since PQ can be represented as a scalar multiple of AB, then OB and PQ are parallel.

Therefore, $|OB| = 2|PQ|$.

That is, OB is twice the length of PQ.

Question 11(b)(i)

$\det M = ad - bc$

$= (2)(3) - (1)(4)$

$= 6 - 4$

$= 2$

$adj(M) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$= \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

$\therefore M^{-1} = \frac{1}{\det M} \times adj(M)$

$= \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ \frac{-4}{2} & \frac{2}{2} \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$

Question 11(b)(ii)

Required to show that $M^{-1}M = I$.

$M^{-1}M = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} (3 \times 2) + (-1 \times 4) & (3 \times 1) + (-1 \times 3) \\ (-4 \times 2) + (2 \times 4) & (-4 \times 1) + (2 \times 3) \end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix} 6-4 & 3-3 \\ -8+8 & -4+6 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I$$

$\therefore M^{-1}M = I$

Question 11(b)(iii)

Required to solve for r, s, t and u.

Now,

$$M^{-1} \times M \times \begin{pmatrix} r & s \\ t & u \end{pmatrix} = M^{-1} \times \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} r & s \\ t & u \end{pmatrix} = M^{-1} \times \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} (3 \times 2) + (-1 \times 4) & (3 \times 1) + (-1 \times -1) \\ (-4 \times 2) + (2 \times 4) & (-4 \times 1) + (2 \times -1) \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 6-4 & 3+1 \\ -8+8 & -4-2 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 4 \\ 0 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{2} & \frac{4}{2} \\ \frac{0}{2} & \frac{-6}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$$

Equating the corresponding entries, we get:

$r = 1$

$s = 2$

$t = 0$

$u = -3$