Solutions to CSEC Maths P2 June 2014

Question 1(a)(i)

$$5.25 \div 0.015 = \frac{5.25}{0.015}$$

$$= \frac{5250}{15}$$

$$= 350 \quad \text{(in exact form)}$$

Question 1(a)(ii)

$\sqrt{6.5025} = 2.55$ (by calculator)

Question 1(a)(iii)

$3.142 \times 2.236^2 = 15.7$ (to 3 significant figures)

Question 1(b)(i)

1 bucket of cement requires 6 buckets of gravel

4 buckets of cement would require $4 \times 6 = 24$ buckets of gravel

Question 1(b)(ii)(a)

1 bucket of sand will be mixed with $\frac{1}{4}$ bucket of cement

20 buckets of sand will be mixed with $20 \times \frac{1}{4} = 5$ buckets of cement

Question 1(b)(ii)(b)

1 bucket of cement is to be mixed with 6 buckets of gravel

5 buckets of cement will be mixed with $5 \times 6 = 30$ buckets of gravel

Question 1(c)(i)

Using the hire purchase plan,

$$\text{Cost of the laptop} = \text{Deposit} + 10\ \text{Monthly Instalments}$$
$$= \$350 + (10 \times \$120)$$
$$= \$350 + \$1200$$
$$= \$1550$$

Question 1(c)(ii)

$$\text{Amount saved} = \text{Hire Purchase Price} - \text{Cash Price}$$
$$= \$1550 - \$1299$$
$$= \$251$$

Question 2(a)

$$\frac{x-2}{3}+\frac{x+1}{4}=\frac{4(x-2)+3(x+1)}{12}$$

$$=\frac{4x-8+3x+3}{12}$$

$$=\frac{7x-5}{12}$$

Question 2(b)(i)

Let the unknown number be x.

An equation to represent the statement is:

$$4+x=\frac{1}{2}x+10$$

$$x-\frac{1}{2}x-10+4=0$$

$$\frac{1}{2}x-6=0 \qquad \text{(simplified form)}$$

Question 2(b)(ii)

Let the unknown number be y.

An equation to represent the statement is:

$$y^2-6=2y+9$$

$$y^2-6-2y-9=0$$

$$y^2-2x-15=0 \qquad \text{(simplified form)}$$

Question 2(c)(i)

A formula for y in terms of x is:

$y=3x+5$

Question 2(c)(ii)

If $x = 4$,

$y = 3(4) + 5$

$= 12 + 5$

$= 17$

Question 2(c)(iii)

If $y = 8$,

$8 = 3x + 5$

$8 - 5 = 3x$

$3 = 3x$

$x = \frac{3}{3}$

$x = 1$

Question 2(c)(iv)

$y = 3x + 5$

$3x = y - 5$

$x = \frac{y-5}{3}$

Question 2(d)

The pair of simultaneous equations are:

$2x + 3y = 9 \quad \rightarrow$ Equation 1

$3x - y = 8 \quad \rightarrow$ Equation 2

Multiplying Equation 2 gives,

$9x - 3y = 24$ → Equation 3

$2x + 3y = 9$ → Equation 1

Adding Equation 1 and Equation 3 gives,

$11x = 33$

$x = \frac{33}{11}$

$x = 3$

Substituting $x = 3$ into Equation 1 gives,

$2(3) + 3y = 9$

$6 + 3y = 9$

$3y = 9 - 6$

$3y = 3$

$y = \frac{3}{3}$

$y = 1$

$\therefore x = 3$ and $y = 1$

Question 3(a)(i)

$U = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\}$

$n(U) = 16$

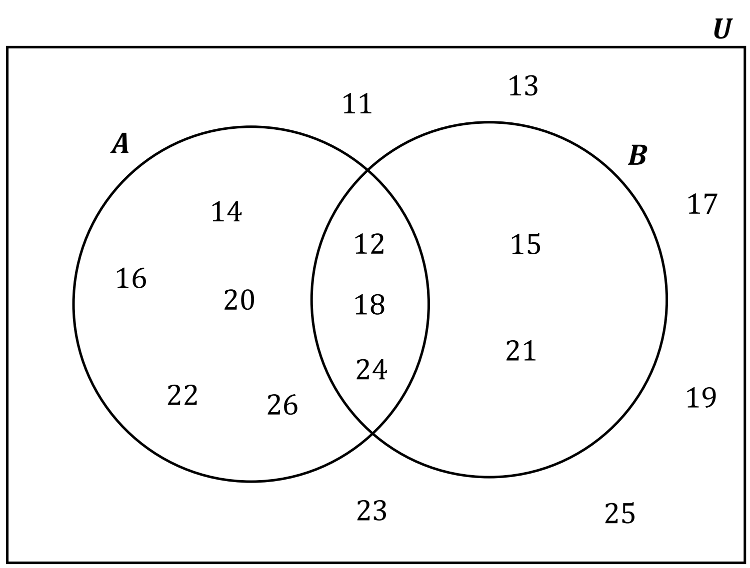
Question 3(a)(ii)

$A = \{12, 14, 16, 18, 20, 22, 24, 26\}$

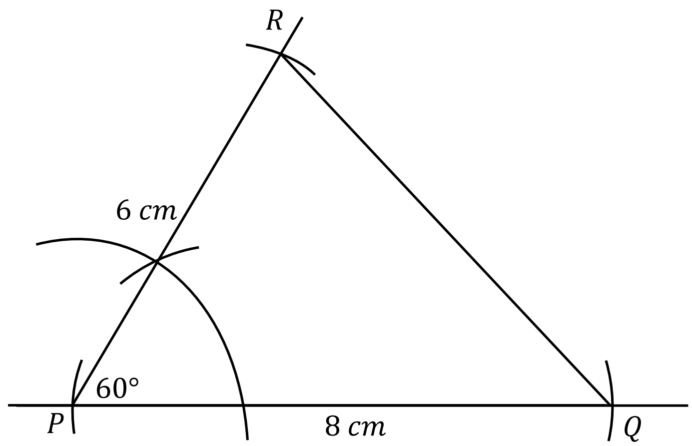
Question 3(a)(iii)

$B = \{12, 15, 18, 21, 24\}$

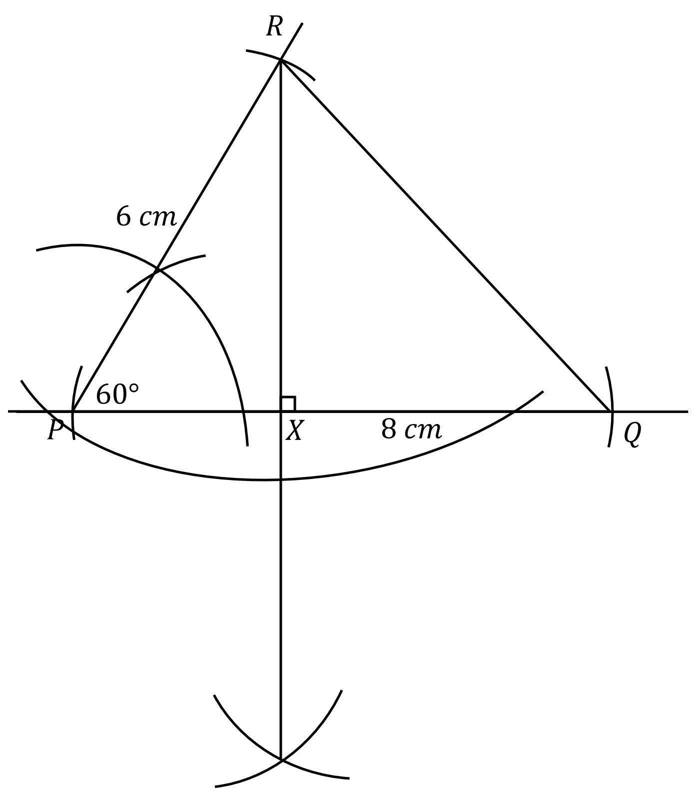
Question 3(a)(iv)



Question 3(b)(i)(a)



Question 3(b)(i)(b)



Question 3(b)(ii)

The size of $\angle QRX = 44°$.

Question 4(a)(i)

1 cm on the map represents 50 000 cm on the island

Now,

$100\ 000\ cm = 1\ km$

$$1\ cm = \frac{1}{100\ 000}$$

$$50\ 000\ cm = \frac{1}{100\ 000} \times 50\ 000$$

$$= \frac{1}{2}\ km$$

Question 4(a)(ii)

$(50\ 000 \times 50\ 000)cm^2 = 2\ 500\ 000\ 000\ cm^2$

$$= 250\ 000\ cm^2 \qquad (1\ km^2 = 100\ 000\ m^2)$$

$$= \frac{1}{4}\ km^2$$

$\therefore$ An area of 1 cm^2 on the map actually represents an area of $\frac{1}{4}\ km^2$.

Question 4(a)(iii)

1 cm on the map $= 50\ 000\ cm$

$$= \frac{50\ 000}{100\ 000}\ km$$

$$= \frac{1}{2}\ km$$

Question 4(b)(i)

The distance L to M is $LM = 8\ cm$.

Question 4(b)(ii)

$1\ cm = \frac{1}{2}\ km$

The actual distance $LM = 8 \times \frac{1}{2}$

$= 8 \times \frac{1}{2}$

Question 4(c)(i)

The total number of squares is approximately 15.

Hence,

Estimated area of the forest reserve on the map $= 15 \times 1$

$= 15\ cm^2$

Question 4(c)(ii)

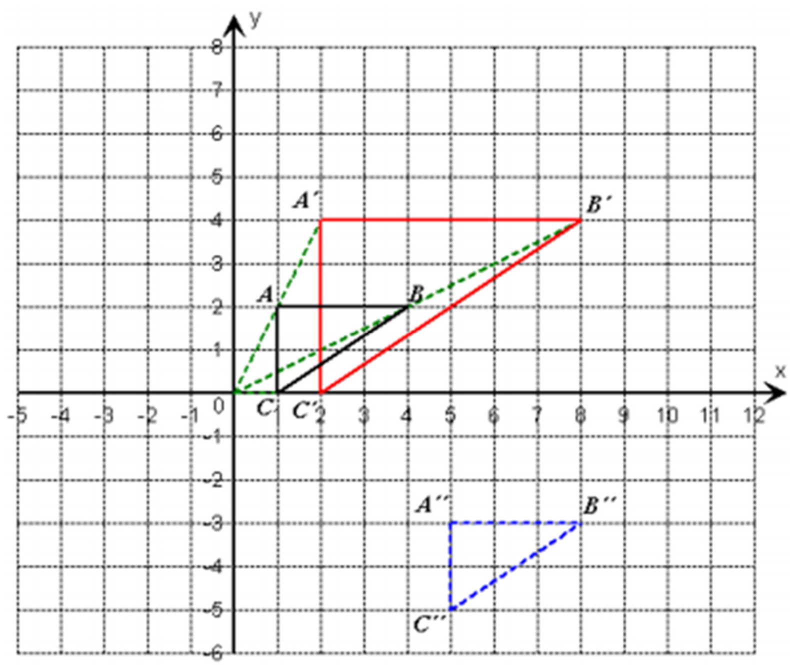
Firstly, $1\ cm^2 = \frac{1}{2} \times \frac{1}{2}\ km^2$.

Actual area of the forest reserve on the map $= 15 \times \left(\frac{1}{2} \times \frac{1}{2}\right)\ km^2$

$= 3\frac{3}{4}\ km^2$

Question 5(a)(i)

The diagram is shown below.



Question 5(a)(ii)

The triangle ABC is mapped onto triangle $A"B"C"$ by a shift of 4 units horizontally to the right and 5 units vertically downwards and is a translation.

$\therefore$ M is a translation, represented by the matrix, $M = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

Question 5(b)(i)

The angle of elevation, shown by the angle $F\hat{P}T = 40°$.

Question 5(b)(ii)

Required to determine the length of FP.

$\tan 40° = \frac{80}{FP}$

$\therefore FP = \frac{80}{\tan 40°}$

$= 95.34\ m$

Question 5(b)(iii)

Required to determine $T\hat{Q}F$.

$\tan T\hat{Q}F = \frac{80}{95.34+118}$

$\tan T\hat{Q}F = 0.375$

$T\hat{Q}F = \tan^{-1}(0.375)$

$= 20.6°$ (to 1 decimal place)

Question 6(a)(i)

$N(x, 9)$

$y = x^2$

$\therefore 9 = x^2$

$x = \sqrt{9}$

$x = \pm 3$

Since $x > 0, x = 3$.

Question 6(a)(ii)

$M(-1, y)$

$y = x^2$

$\therefore y = (-1)^2$

$y = 1$

Question 6(b)(i)

$M = (-1, 1)$ and $N = (3, 9)$

Gradient of $MN = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9-1}{3-(-1)}$

$= \frac{8}{4}$

$= 2$

Question 6(b)(ii)

Gradient of $MN = 2$

Using point $N = (3, 9)$,

$\frac{y-9}{x-3} = 2$

$y - 9 = 2(x - 3)$

$y - 9 = 2x - 6$

$y = 2x - 6 + 9$

$y = 2x + 3$

Question 6(b)(iii)

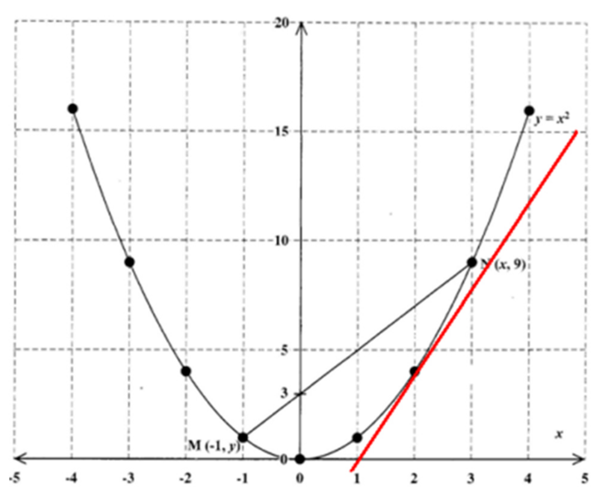
Since parallel line have the same gradient, the required line has the same gradient as MN, which is 2.

$\therefore$ Equation of the required line is $\frac{y-0}{x-0} = 2$.

$\frac{y}{x} = 2$

$y = 2x$

Question 6(c)



Question 6(d)

Using points $(2, 4)$ and $(1, 0)$,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-0}{2-1}$$

$$= \frac{4}{1}$$

$$= 4$$

Question 7(a)

The completed table is shown below.

Number of Books (x)	Tally	Frequency (f)	$f \times x$
1	||	2	2
2	|||	3	6
3	~~||||~~	5	$5 \times 3 = 15$
4	~~||||~~ |	6	$6 \times 4 = 24$
5	~~||||~~ ||	7	$7 \times 5 = 35$
6	||||	4	$4 \times 6 = 24$
7	|||	3	$3 \times 7 = 21$

Question 7(b)

The modal number of books is 5.

Question 7(c)(i)

The total number of books, $\sum fx = 2 + 6 + 15 + 24 + 35 + 24 + 21$

$$= 127$$

Question 7(c)(ii)

The mean number of books per bag, $\bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{127}{30}$$

$$= 4.2 \qquad \text{(to 1 decimal place)}$$

Question 7(b)(iii)

Required to determine the probability that a student at random has less than 4 books in their bag.

$$P(student\ has\ less\ than\ 4\ books) = \frac{Number\ of\ students\ who\ have\ less\ than\ 4\ books}{Total\ number\ of\ students}$$

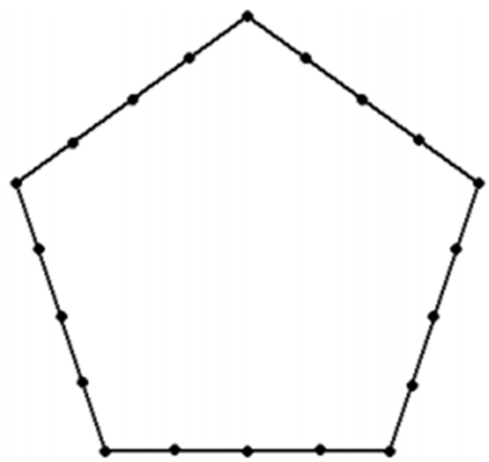
$$= \frac{2+3+5}{30}$$

$$= \frac{10}{30}$$

$$= \frac{1}{3}$$

Question 8(a)

The next figure of the sequence is shown below:



Question 8(b)

Figure (f)	Total Number of Dots	
	Formula	**Number (n)**
1	$5 \times 2 - 5$	5
2	$5 \times 3 - 5$	10
3	$5 \times 4 - 5$	15
4	Not required	Not required
5	$5 \times (5+1) - 5$	25
6	$5 \times (6+1) - 5$	30

Question 8(c)

The number of dots, n, in the fth figure is given by

$5f = n$

Question 8(d)

Required to determine the figure in the sequence containing 145 dots.

$145 = 5f$

$f = \frac{145}{5}$

$f = 29$

Hence, there are 145 dots in the 29^{th} figure.

Question 9(a)(i)

When $x + 1 = 0$.

$$x = -1$$

$\therefore f(x)$ is undefined when $x = -1$

Question 9(a)(ii)

$$f(5) = \frac{2(5)+7}{5+1}$$

$$= \frac{17}{6}$$

$$gf(5) = g\left(\frac{17}{6}\right)$$

$$= 4\left(\frac{17}{6}\right) + 3$$

$$= \frac{34}{3} + 3$$

$$= \frac{43}{3}$$

Question 9(a)(iii)

$$f(x) = \frac{2x+7}{x+1}$$

Let $y = f(x)$.

$$y = \frac{2x+7}{x+1}$$

Interchange variables x and y.

$$x = \frac{2y+7}{y+1}$$

Make y the subject.

$x(y+1) = 2y + 7$

$xy + x = 2y + 7$

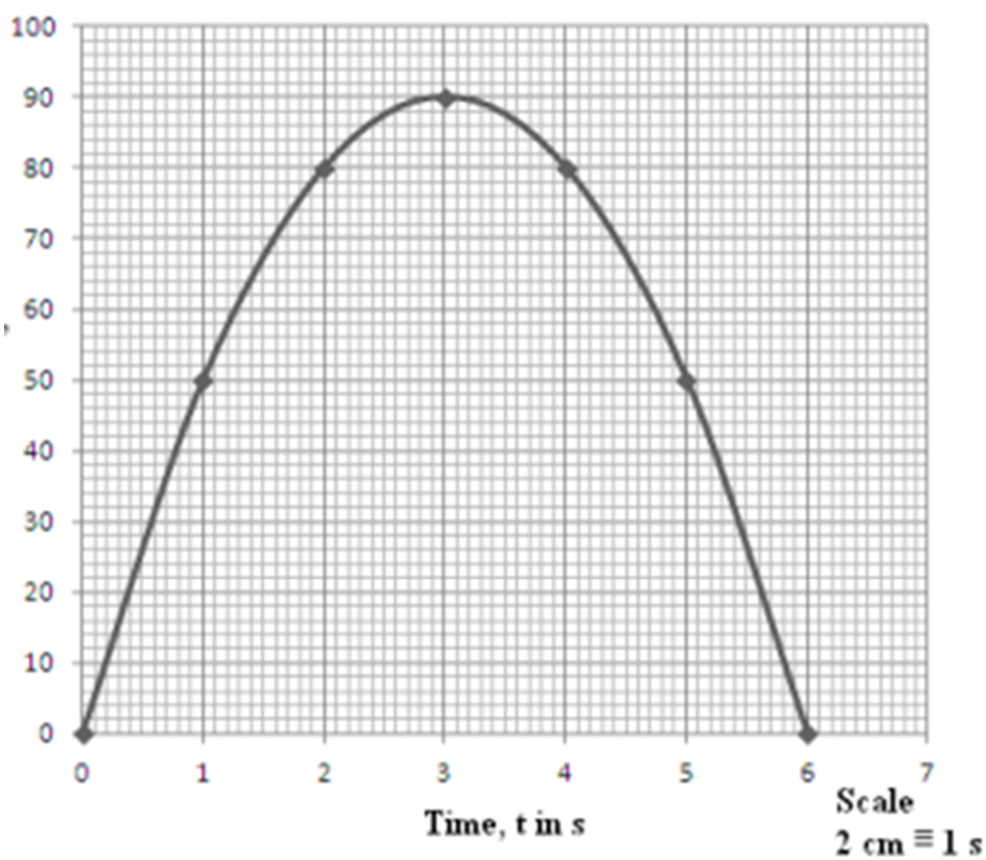
$xy - 2y = 7 - x$

$y(x-2) = 7 - x$

$y = \frac{7-x}{x-2}$

$\therefore f^{-1}(x) = \frac{7-x}{x-2}$

Question 9(b)(i)



Question 9(b)(ii)(a)

Average speed $= \frac{Total\ distance\ covered}{Total\ time\ taken}$

$= \frac{80\ m}{2\ s}$

$= 40\ ms^{-1}$

Question 9(b)(ii)(b)

The tangent to the graph when $t = 3$ is a horizontal line.

The gradient of a horizontal line is 0.

Therefore, the speed of the ball when $t = 3$ is $0\ ms^{-1}$.

Question 10(a)(i)

Since OB and OE are both radii of the same circle, then $OB = OE$.

Since the base angles of an isosceles triangle are equal, then $O\hat{E}B = 20°$.

The sum of the interior angles of a triangle add up to 180°.

$\therefore B\hat{O}E = 180° - (20° + 20°)$

$= 180° - 40°$

$= 140°$

Question 10(a)(ii)

The angle made by a tangent to a circle and any chord, at the point of contact, is equal to the angle in the alternate segment of the circle.

So, $B\hat{E}D = 42°$

$\therefore O\hat{E}D = 42° - 20°$

$= 22°$

Question 10(a)(iii)

The angle subtended by a chord at the center of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc.

So,

$E\hat{D}B = \frac{1}{2}(140°)$

$= 70°$

The opposite angles of a cyclic quadrilateral are supplementary.

$\therefore B\hat{F}E = 180° - 70°$

$= 110°$

Question 10(b)(i)

The bearing of P from $Q = 66° + 54°$

$= 120°$

Question 10(b)(ii)

Applying the cosine rule,

$PR^2 = (100)^2 + (80)^2 - 2(100)(80)\cos 54°$

$PR^2 = 6995.436$

$PR = \sqrt{6995.436}$

$PR = 83.64\ km$ (to 2 decimal places)

Question 10(b)(iii)

Applying the sine rule,

$\frac{83.64}{\sin 54°} = \frac{100}{\sin Q\hat{P}R}$

$\sin Q\hat{P}R = \frac{100 \sin 54°}{83.64}$

$= 0.9673$

$\therefore Q\hat{P}R = \sin^{-1}(0.9763)$

$= 75°$ (to the nearest degree)

Question 11(a)

The matrix M is singular if $\det M = 0$.

$$\det M = (7 \times -1) - (2 \times p)$$

$$0 = -7 - 2p$$

$$2p = -7$$

$$p = -\frac{7}{2}$$

Question 11(b)

$4x - 2y = 0$

$2x + 3y = 4$

Placing the given equation in the form $AX = B$ is

$$\begin{pmatrix} 4 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

Question 11(c)(i)

OP is the position vector from O to P.

O is the fixed point from which the vector is drawn.

Question 11(c)(ii)(a)

$P(2, 4)$

$\therefore OP = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ which is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$ where $a = 2$ and $b = 4$

Question 11(c)(ii)(a)

$Q(8, 2)$

$\therefore OQ = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ which is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$ where $a = 8$ and $b = 2$

Question 11(c)(ii)(c)

$PQ = PO + OQ$

$= -\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ which is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$ where $a = 6$ and $b = -2$

Question 11(c)(iii)

Let R be the point (x, y).

So, $OR = \begin{pmatrix} x \\ y \end{pmatrix}$.

$RQ = RO + OQ$

$= -\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -x + 8 \\ -y + 2 \end{pmatrix}$

Since $OP = RQ$, then

$\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -x + 8 \\ -y + 2 \end{pmatrix}$

Comparing the equivalent matrices, we get:

$2 = -x + 8$

$x = 8 - 2$

$x = 6$

$4 = -y + 2$

$y = 2 - 4$

$y = -2$

$\therefore OP = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ and the coordinates of R will be $(6, -2)$.

Question 11(c)(iv)

The type of quadrilateral formed by $PQRO$ is a quadrilateral.