Solutions to CSEC Maths P2 June 2015

Question 1(a)(i)

$$2\frac{2}{5} - 1\frac{1}{3} + 3\frac{1}{2} = \frac{12}{5} - \frac{4}{3} + \frac{7}{2}$$

$$= \frac{72}{30} - \frac{40}{30} + \frac{105}{30}$$

$$= \frac{72-40+105}{30}$$

$$= \frac{137}{30}$$

$$= 4\frac{17}{30} \qquad \text{(in exact form)}$$

Question 1(a)(ii)

$$(4.14 \div 5.75) + (1.62)^2 = 0.72 + 2.6244$$

$$= 3.3444 \qquad \text{(in exact form)}$$

Question 1(a)(iii)

$$2 \times 3.142 \times 1.25 = 7.855 \qquad \text{(in exact form)}$$

Question 1(a)(iv)

$$\sqrt{2.89} \times \tan 45° = 1.7 \qquad \text{(in exact form)}$$

Question 1(b)(i)

Using the information provided in the table,

3 kg of sugar cost \$10.80

$\therefore$ 1 kg of sugar costs $= \frac{\$10.80}{3}$

$= \$3.60$

Hence, $X = \$3.60$.

Question 1(b)(ii)

1 kg of flour costs \$1.60

$\therefore$ The cost of 1 kg of rice $= \$1.60 + \$0.80$

$= \$2.40$

The cost of 4 kg of rice $= \$2.40 \times 4$

$= \$9.60$

Hence, $Z = \$9.60$.

Question 1(b)(iii)

The sum of Mrs. Rowe's bill before tax $= \$10.80 + \$9.60 + \$3.20$

$= \$23.60$

Tax $= 10\%\ of\ \$23.60$

$= \frac{10}{100} \times \$23.60$

$= \$2.36$

Hence, Mrs. Rowe's total bill $= \$23.60 + \$2.36$

$= \$25.96$

Question 2(a)(i)

$a - b + c = (4) - (2) + (-1)$

$= 4 - 2 - 1$

$= 1$

Question 2(a)(ii)

$2a^b = 2(4)^2$

$2a^b = 2 \times 16$

$2a^b = 32$

Question 2(b)(i)

The amount of juice left in the bottle

= The initial amount of juice – The amount of juice poured out

$= 500 - p$

$\therefore$ The expression is $(500 - p)\ ml$.

Question 2(b)(i)

The amount of juice in q glasses with r ml each $= qr$

The amount of juice left in the bottle

= The initial amount of juice – The amount of juice poured out

$= 500 - qr$

$\therefore$ The expression is $(500 - qr)\ ml$.

Question 2(c)

$\frac{2k}{3}+\frac{2-k}{5}$

$=\frac{5(2k)+3(2-k)}{15}$

$=\frac{10k+6-3k}{15}$

$=\frac{7k+6}{15}$

Question 2(d)(i)

Let the cost of one mango be represented by x and the cost of one pear be represented by y.

Using the information provided can be represented by the pair of simultaneous equations:

$$4x+2y=24$$

$$2x+3y=16$$

Question 2(d)(ii)

x represents the cost of 1 mango

y represents the cost of 1 pear

Question 2(e)(i)

a^3-12a

$=a(a^2-12)$

Question 2(e)(ii)

$$2x^2 - 5x + 3$$

$$= 2x^2 - 2x - 3x + 3$$

$$= 2x(x - 1) - 3(x - 1)$$

$$= (2x - 3)(x - 1)$$

Question 3(a)(i)

From the given information, we know that 12 student played neither the guitar nor the violin.

Question 3(a)(ii)

The total number of students in the class $= 2x + x + 4 + 12$

$$= 3x + 16$$

Question 3(a)(iii)

The total number of students in the class as an equation is $3x + 16 = 40$.

Question 3(a)(iv)

To determine the number of students that play the guitar we must first determine the unknown variable x.

$3x + 16 = 40$

$3x = 40 - 16$

$3x = 24$

$x = \frac{24}{3}$

$x = 8$

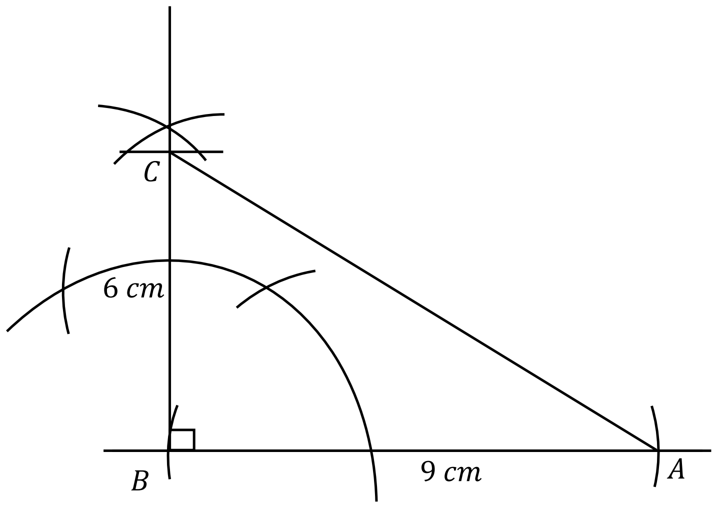
The number of students who play the guitar $= 2x + x$

$$= 3x$$

Since $x = 8$, then $3(8) = 24$.

Hence, 24 students played the guitar.

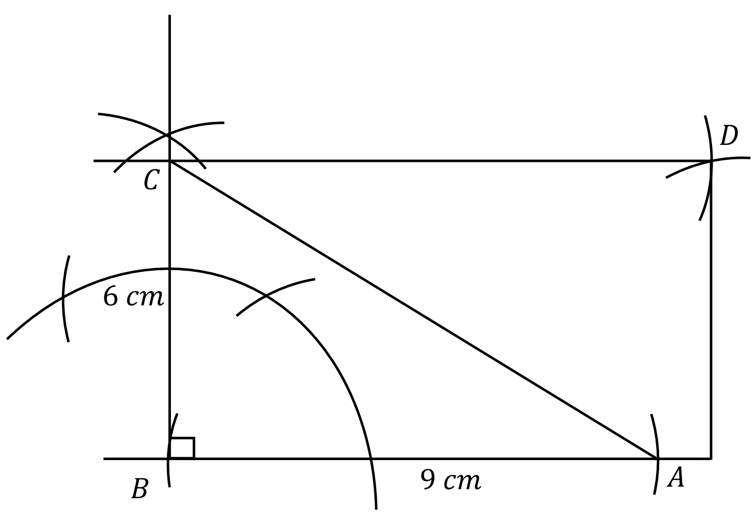
Question 3(b)(i)



Question 3(b)(ii)

The measure of $\angle BAC = 34°$.

Question 3(b)(iii)



Question 4(a)

When $x = -1$,

$y = (-1)^2 - 2(-1) - 3$

$= 1 + 2 - 3$

$= 0$

When $x = 3$,

$y = (3)^2 - 2(3) - 3$

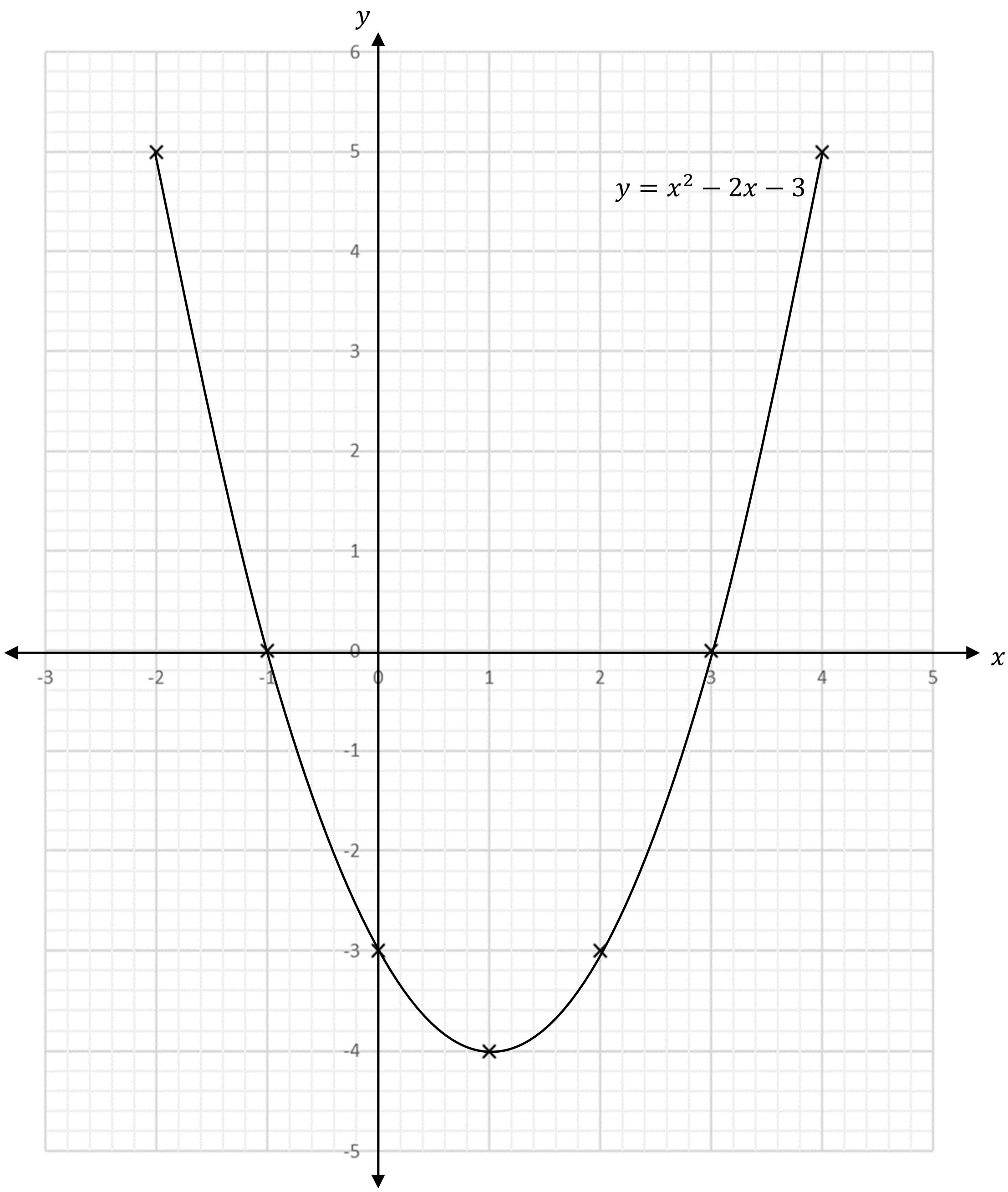
$= 9 - 6 - 3$

$= 0$

The completed table is provided below:

x	-2	-1	0	1	2	3	4
y	5	$\underline{0}$	-3	-4	-3	$\underline{0}$	5

Question 4(b) and (c)



Question 4(d)(i)

The values of x for which $x^2 - 2x - 3 = 0$ are $\underline{\ -1\ }$ and $\underline{\ 3\ }$.

Question 4(d)(ii)

The minimum value of $x^2 - 2x - 3$ is $\underline{\ -4\ }$.

Question 4(d)(iii)

The equation of the line of symmetry of the graph of $y = x^2 - 2x - 3$ is $\underline{\ x = 1\ }$.

Question 5(a)(i)

Distance covered in 1 hour $= 54\ km$

$\therefore$ The distance covered in $2\frac{1}{4}$ hours $= 54 \times 2\frac{1}{4}$

$$= 121\frac{1}{2}\ km$$

Question 5(a)(ii)

$1\ kmh^{-1} = \frac{5}{18}\ ms^{-1}$

$\therefore 54\ kmh^{-1} = \frac{5}{18} \times 54$

$$= 15\ ms^{-1}$$

Therefore, the time taken to cover $315\ m = \frac{Distance}{Speed}$

$$= \frac{315}{15}$$

$$= 21\ s$$

Question 5(b)(i)

$1\ mm = 1\ m$

$1\ mm = 1000\ mm$

Therefore, the scale is $1 : 1000$.

Hence, $x = 1000$.

Question 5(b)(ii)

$1\ m = 100\ cm$

If $2\ cm = 6\ m$

Then $2\ cm = 600\ cm$

$1\ cm = 300\ cm$

Therefore, the scale is $1 : 300$.

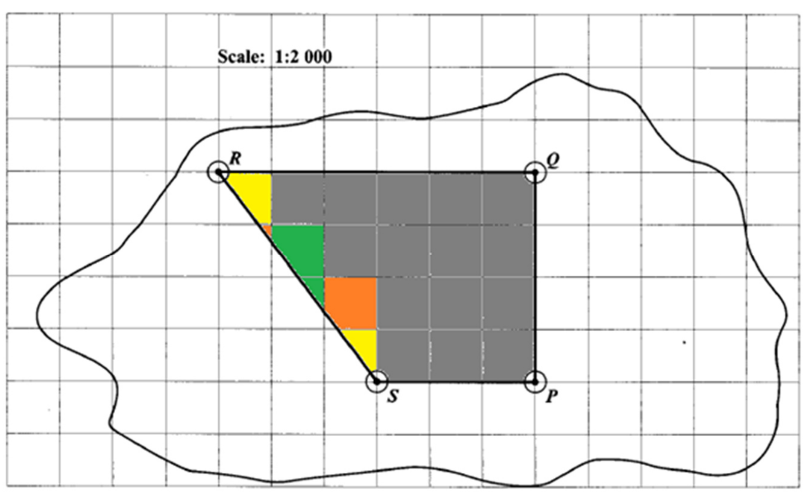
Hence, $x = 300$.

Question 5(c)(i)

The distance from Q to $R = 6 \times 1$

$= 6\ cm$

Question 5(c)(ii)



As shown the in illustration below there are approximately eighteen (18) 1 cm^2 tiles. All of the whole tiles are coloured in grey. The corresponding pieces of broken 1 cm^2 tiles that forms a full 1 cm^2 tile is appropriately colour coded. Therefore, the area, in square centimetres of the plane $PQRS$ on the map is 18 cm^2.

Question 5(c)(iii)

The area of $PQRS = 18\ cm^2$.

Since $1\ cm = 2000\ cm$,

Then $1\ cm^2 = 2000 \times 2000\ cm^2$

$\therefore$ The actual area of $PQRS = 18 \times 2000 \times 2000\ cm^2$

Since $1\ m = 100\ cm$,

Then $1\ m^2 = 100 \times 100\ cm^2$

Hence, actual area of the $PQRS$ in square metres $= \frac{18 \times 2000 \times 2000}{100 \times 100}\ m^2$

$= 7200\ m^2$

Question 6(a)(i)

Radius $= \frac{Diameter}{2}$

$= \frac{8}{2}$

$= 4\ m$

Volume of water in Tank B $= \pi r^2 h$

$= 3.14 \times (4)^2 \times 5$

$= 251.2\ m^3$

Question 6(a)(ii)

Area of the circular base $= \pi r^2$

So, $\pi r^2 = 314$

$3.14 \times r^2 = 314$

$r^2 = \frac{314}{3.14}$

$r^2 = 100$

$r = \sqrt{100}$

$r = \pm 10$

Since $r > 0$, then $r = 10$.

$\therefore$ The radius of Tank A is $10\ m$.

Question 6(a)(iii)

Volume of water in Tank B $= 251.2\ m^3$

Since Tank A holds 8 times as much water as Tank B,

Then Volume of water in Tank A $= 251.2 \times 8$

$= 2009.6\ m^3$

Area of the base of Tank A $= \pi r^2$

$= 3.14 \times (10)^2$

$= 314\ m^2$

$\therefore$ The height of Tank A $= \frac{2009.6}{314}$

$= 6.4\ m$

Question 6(b)(i)

Scale factor $= \frac{Image\ length}{Object\ length}$

$= \frac{P'R'}{PR}$

$= \frac{6}{3}$

$= 2$

The scale factor has a magnitude of 2 but the scale factor is -2.

$\therefore$ The size of the scale factor is __2__ .

Question 6(b)(ii)

The scale factor is negative because the image is on the opposite side of the object and is flipped upside down.

Question 6(b)(iii)

The length of $PQ = \sqrt{13}$ units

The length of $P'Q' = 2 \times PQ$

$= 2\sqrt{13}$ units

The length of PQ is $\sqrt{13}$ units, therefore, the length of $P'Q'$ is $2\sqrt{13}$ units.

Question 6(b)(iv)

Area of $\Delta PQR = \frac{QR \times RP}{2}$

$= \frac{2 \times 3}{2}$

$= 3\ units^2$

The area of triangle PQR is 3 square units.

Question 6(b)(v)

Area of $\Delta P'Q'R' = 2^2 \times$ Area of ΔPQR

$= 2^2 \times 3$

$= 12\ units^2$

The area of $P'Q'R'$ is 4 times the area of triangle PQR which is 12 square units.

Question 7(a)

The completed table is shown below.

Month	July	August	September	October	November
Sales in $ Thousands	13	20	36	25	15

Question 7(b)(i)

Increase in sales from July to August $= \$20\,000 - \$13\,000$

$= \$7\,000$

Increase in sales from August to September $= \$36\,000 - \$20\,000$

$= \$16\,000$

$\therefore$ Between August to September, there was the greatest increase in sales.

Question 7(b)(ii)

Between July to August, there was the smallest increase in sales.

Question 7(b)(iii)

The gradient of the line over two consecutive months gives an indication of the rate of sales.

Question 7(c)

Mean monthly sales $= \frac{\sum x}{n}$

$$= \frac{\$13\,000 + \$20\,000 + \$36\,000 + \$25\,000 + \$15\,000}{5}$$

$$= \frac{\$109\,000}{5}$$

$$= \$21\,800$$

Question 7(d)(i)

Total sales from July to December = \$130 000

Total sales from July to November = \$109 000

∴ Sales for the month of December

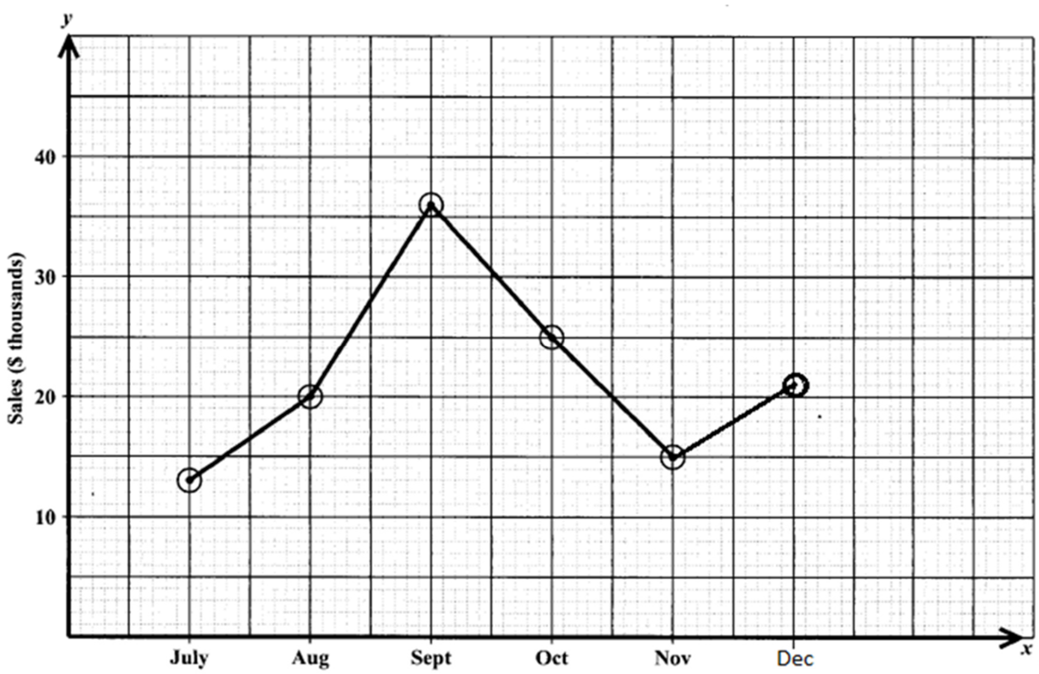
= Total sales from July to December − Total sales from July to November

= \$130 000 − \$109 000

= \$21 000

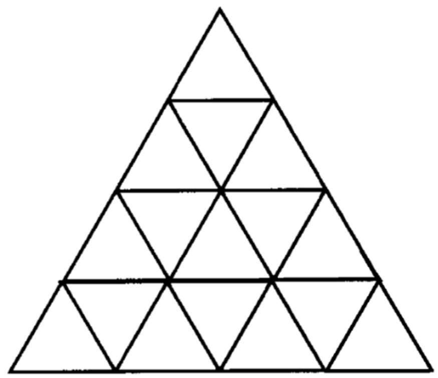
Question 7(d)(ii)

The completed line graph to show the sales in December is shown below.



Question 8(a)

Figure 4 of the sequence is shown below:



Question 8(b)

	Figure	Number of Unit Triangle	Number of Unit Sides
	1	1	$\frac{(3\times1)(1+1)}{2} = 3$
	2	4	$\frac{(3\times2)(2+1)}{2} = 9$
	3	9	$\frac{(3\times3)(3+1)}{2} = 18$
(i)	4	16	$\frac{(3\times4)(4+1)}{2} = 30$
(ii)	12	144	$\frac{(3\times12)(12+1)}{2} = 234$
(iii)	25	625	$\frac{(3\times25)(25+1)}{2} = 975$
(iv)	n	n^2	$\frac{3n(n+1)}{2}$

Question 9(a)(i)

Score for student $= 60$ marks

$$\text{Percentage score} = \frac{Student\ score}{Maximum\ score} \times 100$$

$$= \frac{60}{120} \times 100$$

$$= 50\%$$

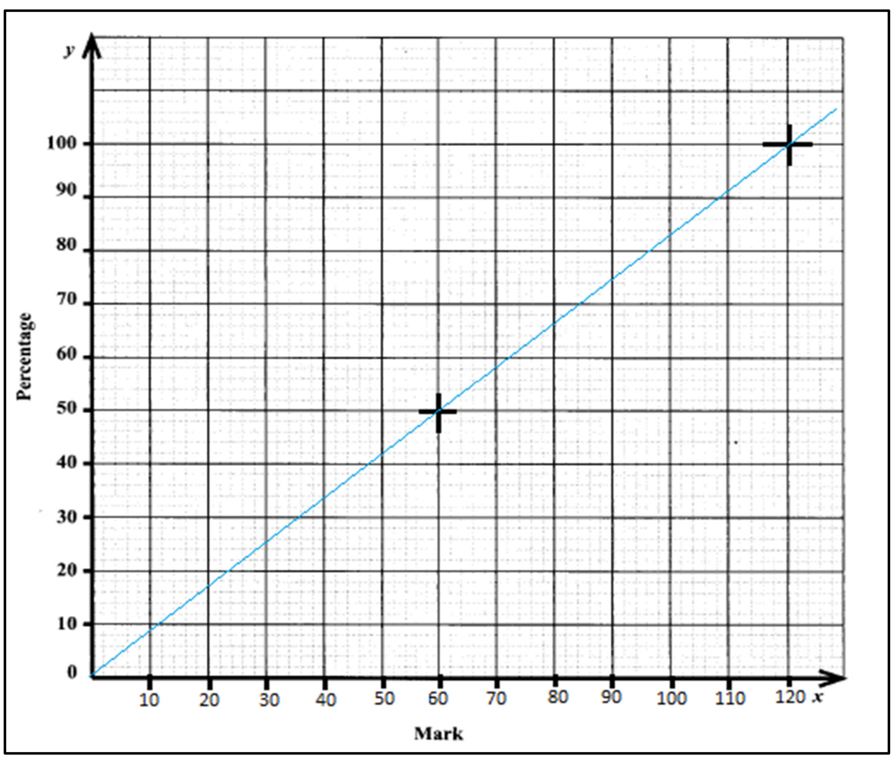
Score for student $= 120$ marks

$$\text{Percentage score} = \frac{Student\ score}{Maximum\ score} \times 100$$

$$= \frac{120}{120} \times 100$$

$$= 100\%$$

Question 9(a)(ii)



Question 9(a)(iii)

By reading off the graph, if a candidate is awarded 95 marks on the examination, the candidate's percentage score will be approximately 79.5%.

Question 9(a)(iv)

Grade A is awarded for 85% or more.

The minimum marks for a Grade A = 102.

Question 9(b)(i)

$g(x) = \frac{x^2-1}{3}$

$g(5) = \frac{(5)^2-1}{3}$

$= \frac{25-1}{3}$

$= \frac{24}{3}$

$= 8$

Question 9(b)(ii)

$f(x) = 3x + 2$

Let $y = f(x)$.

$y = 3x + 2$

Interchange variables x and y.

$x = 3y + 2$

Make y the subject of the formula.

$x - 2 = 3y$

$y = \frac{x-2}{3}$

$\therefore f^{-1}(x) = \frac{x-2}{3}$

Question 9(b)(iii)

$gf(x) = \frac{(3x+2)^2-1}{3}$

$= \frac{9x^2+12x+4-1}{3}$

$= \frac{9x^2+12x+3}{3}$

$= 3x^2 + 4x + 1$

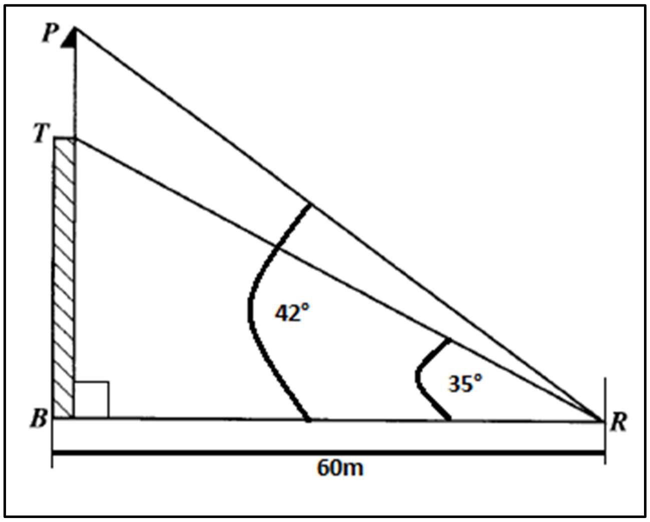
$= 3x^2 + 3x + x + 1$

$= 3x(x + 1) + 1(x + 1)$

$= (3x + 1)(x + 1)$

Question 10(a)(i)

The diagram is labelled as follows.



Question 10(a)(ii)

$\tan 35° = \frac{BT}{60}$

$BT = 60 \tan 35°$

$\tan 42° = \frac{PB}{60}$

$PB = 60 \tan 42°$

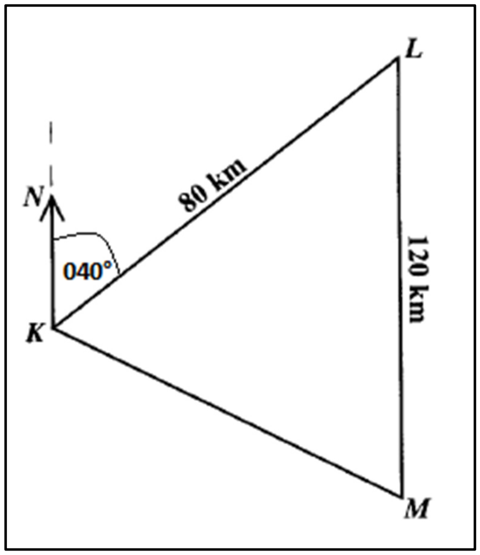
Length of the flagpole $=$ Length of $PB -$ Length of BT

$= 60 \tan 42° - 60 \tan 35°$

$= 12\ m$ (to the nearest m)

Question 10(b)(i)

The bearing of 040° is shown in the diagram below:



Question 10(b)(ii)

Since Angle KLM and Angle NKL are alternating angles,

Then $K\hat{L}M = N\hat{K}L$

$= 40°$

Question 10(b)(iii)

Applying the cosine rule to triangle KLM,

$KM^2 = (80)^2 + (120)^2 - 2(80)(120)\cos 40°$

$= 6400 + 14\,400 + 160(120)\cos 40°$

$= 6091.95$

$KM = \sqrt{6091.95}$

$= 78\ km$ (to the nearest km)

Question 10(b)(iv)

Applying the sine rule to triangle KLM,

$$\frac{120}{\sin L\widehat{K}M} = \frac{KM}{\sin 40°}$$

$$\therefore \sin L\widehat{K}M = \frac{120 \times \sin 40°}{78.0}$$

$$= 0.9889$$

$$L\widehat{K}M = \sin^{1}(0.9889)$$

$$= 81° \qquad \text{(to the nearest degree)}$$

Question 10(b)(v)

The bearing of M from $K = 40° + 81.4°$

The bearing of M from $K = 121.4°$

Question 11(a)(i)

$$AB = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \times 1) + (1 \times 0) & (1 \times 2) + (1 \times 1) \\ (2 \times 1) + (3 \times 0) & (2 \times 2) + (3 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 0 & 2 + 1 \\ 2 + 0 & 4 + 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$$

Question 11(a)(ii)

$$BA = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \times 1) + (2 \times 2) & (1 \times 1) + (2 \times 3) \\ (0 \times 1) + (1 \times 2) & (0 \times 1) + (1 \times 3) \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 4 & 1 + 6 \\ 0 + 2 & 0 + 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$

Since $AB \neq BA$, then the matrix product of A and B is not commutative.

Question 11(a)(iii)

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$\det(A) = ad - bc$$

$$= (1)(3) - (1)(2)$$

$$= 3 - 2$$

$$= 1$$

$$adj\ (A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{\det(A)} adj(A)$$

$$= \frac{1}{1}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$$

Question 11(a)(iv)

$$M = \begin{pmatrix} 2x & 2 \\ 9 & 3 \end{pmatrix}$$

$$|M| = ad - bc$$

$$= (2x)(3) - (2)(9)$$

$$= 6x - 18$$

Since $|M| = 0$,

$$6x - 18 = 0$$

$$6x = 18$$

$$x = \frac{18}{6}$$

$$x = 3$$

$\therefore$ The value of x for which $|M| = 0$ is $x = 3$.

Question 11(b)(i)

$\overrightarrow{OR} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$\left|\overrightarrow{OR}\right| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$ units

Question 11(b)(ii)

$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$

$= -\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 3 + 1 \\ -4 + 1 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ which is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 4$ and $y = -3$

$\overrightarrow{ST} = \overrightarrow{SO} + \overrightarrow{OT}$

$= -\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -1 + 5 \\ -1 - 2 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ which is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x = 4$ and $y = -3$

Question 11(b)(iii)

$\overrightarrow{RS} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\overrightarrow{ST} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\overrightarrow{ST} = 1 \times \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\therefore$ $\overrightarrow{ST}$ is a scalar multiple of $\overrightarrow{RS}$, so $\overrightarrow{ST}$ is parallel to $\overrightarrow{RS}$.

But S is a common point on both vectors.

Hence, R, S and T lie on the same straight line.