# Solutions to CSEC Maths P2 June 2016

## Question 1(a)(i)

$$\frac{3\frac{3}{8}-2\frac{1}{4}}{1\frac{1}{2}} = \frac{\frac{27}{8}-\frac{9}{4}}{\frac{3}{2}}$$

$$= \frac{\frac{27-(9\times2)}{8}}{\frac{3}{2}}$$

$$= \frac{\frac{9}{8}}{\frac{3}{2}}$$

$$= \frac{9}{8} \times \frac{2}{3}$$

$$= \frac{3}{4} \times \frac{1}{1}$$

$$= \frac{3}{4}$$

## Question 1(a)(ii)

$(2.86+0.75) + 0.481^2 = 3.61 + 0.2316$

$= 3.8416$

$= 3.84$ (to 2 dp)

## Question 1(b)(i)

Discount = 5% of cash payment

$= 0.05 \times 1399$

$= \$69.95$

Selling Price $= \$1399 - \$69.95$

$= \$1329.05$

## Question 1(b)(ii)

Profit, if cash is paid $= \$1329.05 - \$1064.00$

$= \$265.05$

Percentage profit, if cash is paid $= \frac{Profit}{Cost\ price} \times 100$

$= \frac{\$265.05}{\$1064} \times 100$

$= 24.91\%$

If cash was not paid, then a discount is not applied.

$\therefore$ Profit $= \$1399 - \$1064$

$= \$335$

Profit percentage $= \frac{Profit}{Cost\ price} \times 100$

$= \frac{\$335}{\$1064} \times 100$

$= 31.5\%$

## Question 1(c)

Carton Size of 350 $ml = \$4.20$

$1\ ml = \frac{\$4.20}{350}$

$= \$0.012$

Carton Size of 450 $ml = \$5.35$

$1\ ml = \frac{\$5.35}{450}$

$= \$0.0119$

Carton Size of 500 $ml = \$5.80$

$$1\ ml = \frac{\$5.80}{500}$$

$$= \$0.0116$$

The most cost-effective carton would be the 500 $ml$, as it costs least per $ml$.

## Question 2(a)(i)

$4a^2 - 16$

$= 4(a^2 - 4)$

$= 4(a + 2)(a - 2)$

## Question 2(a)(ii)

$3y^2 + 2y - 8$

$= 3y^2 + 6y - 4y - 8$

$= 3y(y + 2) - 4(y + 2)$

$= (3y - 4)(y + 2)$

## Question 2(b)

$2x + y = 3$ → Equation 1

$5x - 2y = 12$ → Equation 2

Multiply Equation 2 by 2 to get:

$4x + 2y = 6$ → Equation 3

$5x - 2y = 12$ → Equation 2

Equation 3 + Equation 2 gives:

$9x = 18$

$x = \frac{18}{9}$

$x = 2$

Substituting $x = 2$ into Equation 1 gives:

$2(2) + y = 3$

$4 + y = 3$

$y = 3 - 4$

$y = -1$

$\therefore x = 2$ and $y = -1$

## Question 2(c)

Since $y$ is directly proportional to $x$, then

$y \propto x$

$y = kx$

Substituting $x = 6$ and $y = 3$ into $y = kx$ gives:

$3 = k(6)$

$k = \frac{3}{6}$

$k = \frac{1}{2}$

So, we have $y = \frac{1}{2}x$.

When $x = 10$ and $y = u$,

$u = \frac{1}{2}(10)$

$u = 5$

When $x = t$ and $y = 9$

$9 = \frac{1}{2}t$

$t = 9 \times 2$

$t = 18$

$\therefore u = 5$ and $t = 18$

## Question 3(a)(i)

Total number of students in class $= 18 - x + x + 14 - x + 5$

$$= 37 - x$$

## Question 3(a)(ii)

$(H \cup F)' = 5$ which is not equal to 0

$\therefore H \cup F \neq U$

Hence, the statement given is false.

$H \cap F' = 18 - 7$

$= 11$

So, $H \cap F' \neq 0$

$H \cap F' \neq \emptyset$

Hence, the statement given is false.

## Question 3(a)(iii)

There are a total of 30 students in the class.

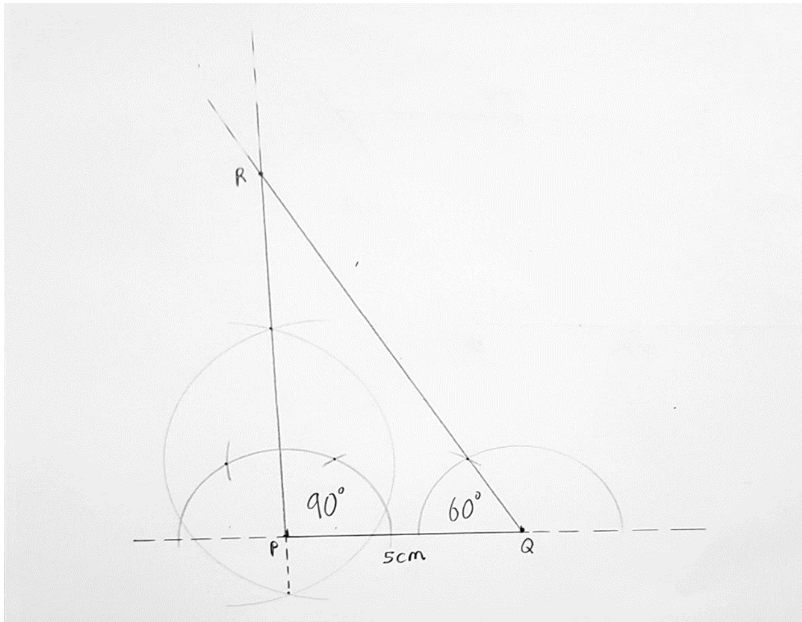
$\therefore 37 - x = 30$

$x = 37 - 30$

$x = 7$

The number of students who study both History and French is, $n(H \cap F) = 7$.

## Question 3(b)(i)




## Question 3(b)(ii)

The length of $PR = 8.7\ cm$.

The measure of $\angle PRQ = 30°$.

## Question 4(a)

External volume of the box $= l \times w \times h$

$= 30 \times 20 \times 15$

$= 9000\ cm^3$

## Question 4(b)(i)

Internal length of the box $= 30 - (2 \times 1.5)$

$= 30 - 3$

$= 27\ cm$

## Question 4(b)(ii)

Internal width of the box $= 20 - (2 \times 1.5)$

$= 20 - 3$

$= 17\ cm$

## Question 4(b)(iii)

Internal depth of the box $= 15 - (1 \times 1.5)$

$= 15 - 1.5$

$= 13.5\ cm$

## Question 4(c)

Internal volume of the box $= l \times w \times h$

$= 27 \times 17 \times 13.5$

$= 6196.5\ cm^3$

## Question 4(d)

Volume of silver = External volume of the box – Internal volume of the box

$= 9000 - 6196.5$

$= 2803.5\ cm^3$

Density of silver is $10.5\ gcm^{-3}$.

Mass of box $= 2803.5 \times 10.5\ g$

$= 29436.75\ g$

$= \frac{29436.75}{1000}\ kg$

$= 29.44\ kg$ (to 2 decimal places)

Question 5(a)(i)

Angles in a straight line $= 180°$

$x = 180° - 128°$

$= 52°$

Question 5(a)(ii)

Angle $y$ and $128°$ are corresponding angles.

$y = 128°$

Question 5(a)(iii)

The sum of angles in a triangle is equal to $180°$.

$w + 52° + 80° = 180°$

$w = 180° - 52° - 80°$

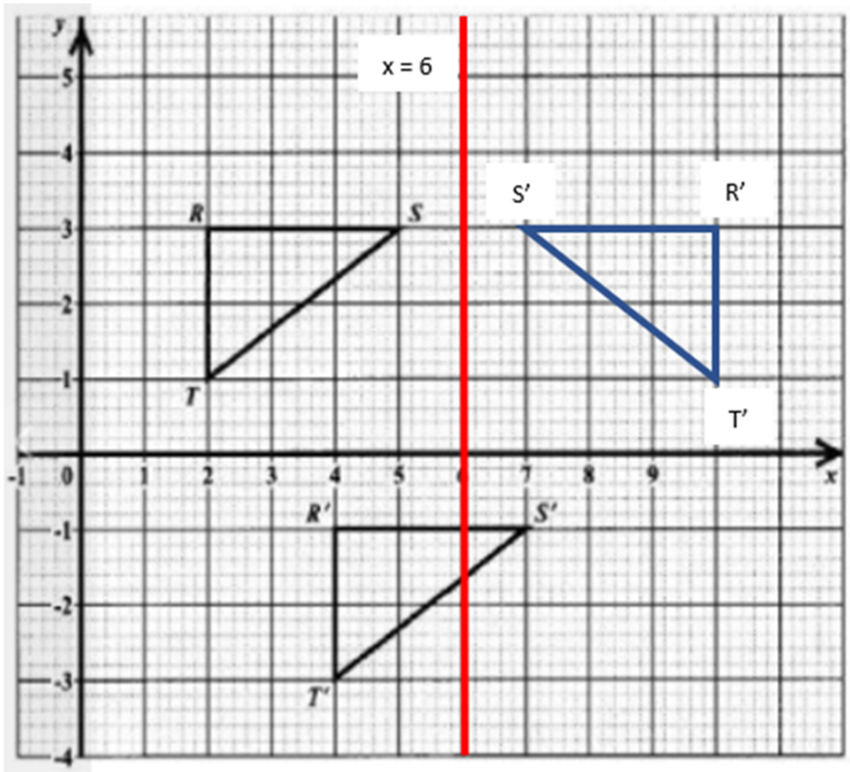
$w = 48°$

Question 5(b)(i)

This was a translation through the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

The transition can be defined as $T = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

Question 5(b)(i)



## Question 6(a)(i)

A line passing through the origin would have a $y$-intercept ($c$) value of 0.

Since the gradient is the same, then the equation of the line would be:

$x + y = 0$

## Question 6(a)(ii)

Coordinates at the ends of the line $AB$: $(0, 3)$ and $(3, 0)$

Midpoint of $AB = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{0+3}{2}, \frac{3+0}{2}\right)$

$= \left(\frac{3}{2}, \frac{3}{2}\right)$

Gradient of $AB = -1$

The product of the gradients of perpendicular lines $= -1$.

Gradient, $m = \frac{-1}{-1}$

$= 1$

Substituting $m = 1$ and point $\left(\frac{3}{2}, \frac{3}{2}\right)$ into the equation of a line:

$\frac{3}{2} = (1)\left(\frac{3}{2}\right) + c$

$c = \frac{3}{2} - \frac{3}{2}$

$c = 0$

$\therefore$ Equation of line is $y = x$.

Question 6(b)(i)

$y = x^2 - 2x - 3$

When $x = 1$,

$y = (1)^2 - 2(1) - 3$

$y = 1 - 2 - 3$

$y = -4$

When $x = 3$,

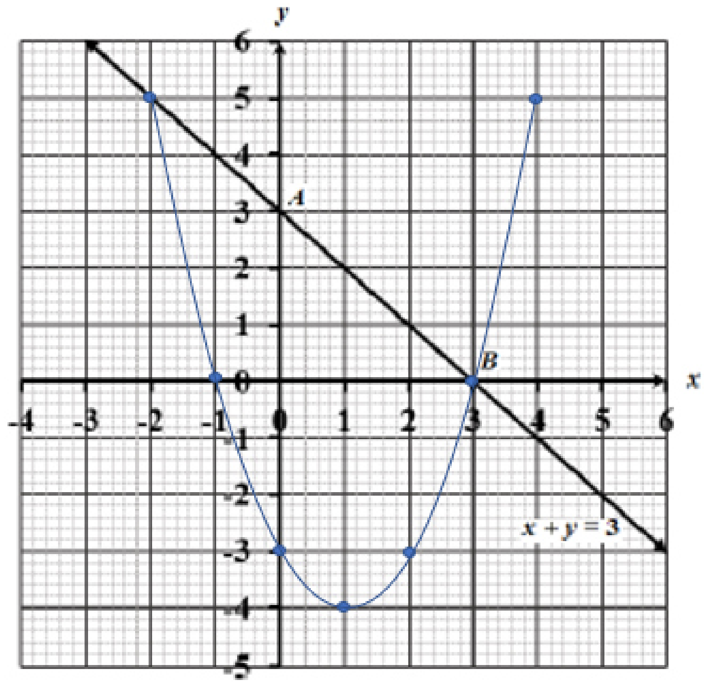
$y = (3)^2 - 2(3) - 3$

$y = 9 - 6 - 3$

$y = 0$

| $\boldsymbol{x}$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{y}$ | 5 | 0 | $-3$ | $-4$ | $-3$ | 0 | 5 |

Question 6(b)(ii)




Question 6(a)(iii)

The minimum value of $y = x^2 - 2x - 3$ occurs when $x = 1$.

The values of $x$ for which $x^2 - 2x - 3 = -x + 3$ are $x = -2$ and $x = 3$.

Question 7(a)

| Weight (kg) | Tally | Number of Bags |
|---|---|---|
| 1-10 | III | 3 |
| 11-20 | IIII | 4 |
| 21-30 | ~~IIII~~ | 5 |
| 31-40 | ~~IIII~~ I | 6 |
| 41-50 | II | 2 |

Question 7(b)(i)

The upper class boundary of the class interval 21-30 is 30.5.

Question 7(b)(ii)

Class Width = Upper Class Boundary – Lower Class Boundary

$= 30.5 - 20.5$

$= 10$

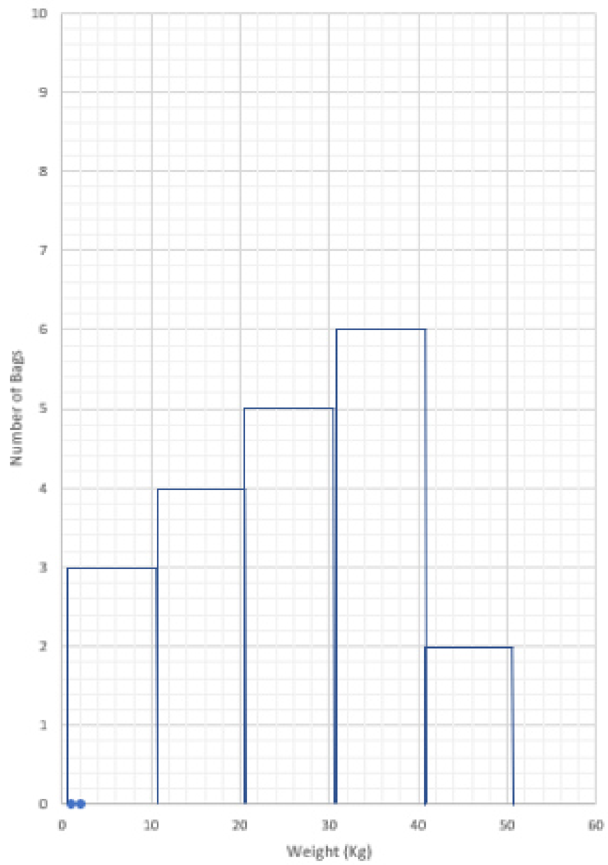
Question 7(b)(iii)

Class Midpoint $= \frac{UCB+LCB}{2}$

$= \frac{20.5+30.5}{2}$

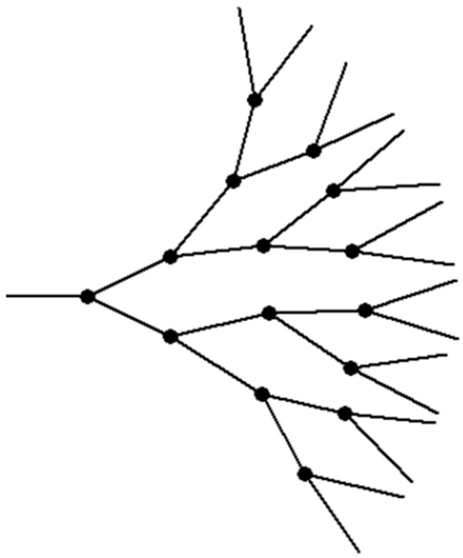
$= 25.5$

Question 7(c)



Question 8(a)

Figure 4 of the sequence is shown below:



Question 8(b)

| Figure Number ($N$) | Number of Knots ($K$) | Number of Strings ($S$) |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 3 | 7 |
| 3 | 7 | 15 |
| 4 | 15 | 31 |
| | | |
| 7 | 127 | 255 |
| 10 | 1023 | 2047 |

Consider the Figure $n$.

The number of knots $= 2^n - 1$

The number of strings $= 2^{n+1} - 1$

(i) For Figure 4, number of knots $= 2^4 - 1$

$= 16 - 1$

$= 15$

For Figure 4, number of strings $= 2^{4+1} - 1$

$= 32 - 1$

$= 31$

(ii) $255 = 2^{n+1} - 1$

With the use of a calculator, it can be deduced that $n = 7$

For Figure 7, number of knots $= 2^7 - 1$

$= 128 - 1$

$= 127$

(iii) For Figure 10, number of knots $= 2^{10} - 1$

$= 1024 - 1$

$= 1023$

For Figure 10, number of strings $= 2^{10+1} - 1$

$= 2048 - 1$

$= 2047$

Question 9(a)(i)

$f(x) = 2x - 7$

Let $y = f(x)$.

$y = 2x - 7$

Interchange the variables $x$ and $y$.

$x = 2y - 7$

Make $y$ the subject.

$2y = x + 7$

$y = \frac{x+7}{2}$

$\therefore f^{-1}(x) = \frac{x+7}{2}$

$g(x) = x^2 + 1$

Let $y = g(x)$.

$y = x^2 + 1$

Interchange the variables $x$ and $y$.

$x = y^2 + 1$

Make $y$ the subject.

$y^2 = x - 1$

$y = \sqrt{x - 1}$

$\therefore g^{-1}(x) = \sqrt{x - 1}$

Now,

$fg(x) = f[g(x)]$

$= 2(x^2 + 1) - 7$

$= 2x^2 + 2 - 7$

$= 2x^2 - 5$

Let $y = fg(x)$.

$y = 2x^2 - 5$

Interchange the variables $x$ and $y$.

$x = 2y^2 - 5$

Make $y$ the subject.

$2y^2 = x + 5$

$y^2 = \frac{x+5}{2}$

$y = \sqrt{\frac{x+5}{2}}$

$\therefore (fg)^{-1}(x) = \sqrt{\frac{x+5}{2}}$

Question 9(a)(ii)

$(fg)^{-1}(x) = \sqrt{\frac{x+5}{2}}$

$(fg)^{-1}(5) = \sqrt{\frac{5+5}{2}}$

$= \sqrt{\frac{10}{2}}$

$= \sqrt{5}$

$f^{-1}(5) = \frac{5+7}{2}$

$= \frac{12}{2}$

$= 6$

$g^{-1}f^{-1}(5) = g^{-1}(6)$

$= \sqrt{6-1}$

$= \sqrt{5}$

$\therefore (fg)^{-1}(5) = g^{-1}f^{-1}(5)$

## Question 9(b)(i)

The car arrived at Town C at 08:30.

## Question 9(b)(ii)

The car was at a stop from 07:00 to 07:30 which is 30 minutes and from 08:30 to 08:45 which is 15 minutes.

The total time the car stopped $= 30 + 15$

$= 45$ minutes

## Question 9(b)(iii)

Constant speed $= \frac{Distance}{Time}$

$= \frac{100-40}{1\ hr}$

$= 60\ kmh^{-1}$

Question 9(b)(iv)

Total distance $= 40 + 60 + 100$

$= 200\ km$

Total time taken $= 1 + 1 + 1\frac{1}{4}$

$= 3\frac{1}{4}$ hours

Average speed $= \frac{Total\ distance\ covered}{Total\ time\ taken}$

$= \frac{200}{3\frac{1}{4}}$

$= \frac{800}{12}$

$= 61.54\ kmh^{-1}$

## Question 10(a)(i)

The two tangents that can be drawn to a circle from a point outside the circle are equal in length.

Hence, $EH = EF$ and triangle $EHF$ is isosceles.

$\therefore E\hat{H}F = E\hat{F}H$ since the base angles of an isosceles triangle are equal

$$\angle EFH = \frac{180° - 44°}{2}$$

$$= \frac{136°}{2}$$

$$= 68°$$

## Question 10(a)(ii)

The angle made by a tangent to a circle and a radius, at the point of contact, is a right angle. So, $O\hat{H}E$ and $O\hat{F}E = 90°$.

The sum of the angles of a quadrilateral is 360°.

$$F\hat{O}H = 360° - (90° + 90° + 44°)$$

$$= 136°$$

The angle subtended by a chord at the center of a circle, in this case, $F\hat{O}H$, is twice the angle that the chord subtends at the circumference, standing on the same arc.

$$F\hat{G}H = \frac{1}{2}(136°)$$

$$= 68°$$

## Question 10(a)(iii)

The angle made by a tangent to a circle and a radius, at the point of contact, is a right angle.

$\therefore O\widehat{H}E = 90°$.

## Question 10(a)(iv)

The angle in a semi-circle is a right angle.

$\therefore J\widehat{G}H = 90°$.

## Question 10(b)(i)

The sum of the angles in a triangle is equal to 180°.

$F\widehat{T}S = 180° - (90° + 40°)$

$= 50°$

Angles in a straight line $= 180°$.

$T\widehat{S}R = 180° - 40°$

$= 140°$

The sum of the angles in a triangle is equal to 180°.

$R\widehat{T}S = 180° - (140° + 22°)$

$= 18°$

## Question 10(b)(ii)

Using the sine rule,

$$\frac{ST}{\sin 22°} = \frac{150}{\sin 18°}$$

$$ST = \frac{150 \times \sin 22°}{\sin 18°}$$

$$ST = 181.8\ m \qquad \text{(to 1 decimal place)}$$

## Question 10(b)(iii)

$$\frac{TF}{181.83} = \sin 40°$$

$$TF = 181.83 \times \sin 40°$$

$$TF = 116.9\ m \qquad \text{(to 1 decimal place)}$$

## Question 11(a)(i)

The position vectors of points $A$, $B$ and $C$, relative to the origin $O$, are $\overrightarrow{OA} = \binom{2}{-2}$,

$\overrightarrow{OB} = \binom{6}{1}$ and $\overrightarrow{OC} = \binom{10}{4}$ respectively.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\overrightarrow{OA} + \overrightarrow{OB}$

$= -\binom{2}{-2} + \binom{6}{1}$

$= \binom{4}{3}$

$\overrightarrow{AB} = \binom{4}{3}$ is of the form $\binom{x}{y}$, where $x = 4$ and $y = 3$

Now,

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -\overrightarrow{OA} + \overrightarrow{OC}$

$= -\binom{2}{-2} + \binom{10}{4}$

$= \binom{8}{6}$

$\overrightarrow{AC} = \binom{8}{6}$ is of the form $\binom{x}{y}$, where $x = 8$ and $y = 6$

## Question 11(a)(ii)

Required to determine whether $A$, $B$ and $C$ are collinear.

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

$= 2\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{AC}$ is a scalar multiple (=2) of $\overrightarrow{AB}$.

Hence, $\overrightarrow{AC}$ is parallel to $\overrightarrow{AB}$.

$A$ is a common point. Hence, $A$ and $B$ both lie on $\overrightarrow{AC}$ and $A$, $B$ and $C$ are collinear.

## Question 11(b)

Required to determine the value of $x$ for which $\begin{pmatrix} 3 & x \\ 2 & 4 \end{pmatrix}$ is a singular matrix

Let $A = \begin{pmatrix} 3 & x \\ 2 & 4 \end{pmatrix}$.

$|A| = (3 \times 4) - (x \times 2)$

$= 12 - 2x$

If $A$ is a singular matrix, then $|A|$ or $\det A = 0$.

$\therefore 12 - 2x = 0$

$2x = 12$

$x = \frac{12}{2}$

$x = 6$

## Question 11(c)(i)

Required to determine $NP$.

$N = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 5 \\ 2 & 1 \end{pmatrix}$

$$NP = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (4 \times 1) + (1 \times 2) & (4 \times 5) + (1 \times 1) \\ (3 \times 1) + (2 \times 2) & (3 \times 5) + (2 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 4 + 2 & 20 + 1 \\ 3 + 4 & 15 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 21 \\ 7 & 17 \end{pmatrix}$$

## Question 11(c)(ii)

Required to determine whether matrix multiplication is commutative.

We are given that $PN = \begin{pmatrix} 19 & 11 \\ 11 & 4 \end{pmatrix}$.

It was found that $NP = \begin{pmatrix} 6 & 21 \\ 7 & 17 \end{pmatrix}$.

Since $PN \neq NP$, then matrix multiplication is not commutative.

## Question 11(c)(iii)

Required to determine the inverse of $N$.

$N = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

$$\det N = (4 \times 2) - (1 \times 3)$$

$$= 8 - 3$$

$$= 5$$

$$\therefore N^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -(1) \\ -(3) & 4 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix}$$

## Question 11(c)(iv)

Required to calculate the values of $x$ and $y$ for which $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = N^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} (2 \times 1) + (-1 \times 2) \\ (-3 \times 1) + (4 \times 2) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 - 2 \\ -3 + 8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\therefore x = 0$ and $y = 1$