Solutions to CSEC Maths P2 June 2017

Question 1(a)(i)

$\left(4\frac{1}{3} - 1\frac{2}{5}\right) \div \frac{4}{15} = \left(\frac{13}{3} - \frac{7}{5}\right) \div \frac{4}{15}$

$= \left(\frac{(5\times13)-(7\times3)}{15}\right) \div \frac{4}{15}$

$= \left(\frac{65-21}{15}\right) \div \frac{4}{15}$

$= \frac{44}{15} \div \frac{4}{15}$

$= \frac{44}{15} \times \frac{15}{4}$

$= 11$ (in exact form)

Question 1(a)(ii)

$\frac{(3.1-1.15)^2}{0.005} = \frac{(1.95)^2}{0.005}$

$= \frac{3.8025}{0.005}$

$= 760.5$ (in exact form)

Question 1(b)(i)

Under Plan A,

Total cost of phone = Deposit + (Monthly installments × # of months) + Tax

$= \$400 + (\$65 \times 12) + \$0$

$= \$400 + \$780$

$= \$1180$

Question 1(b)(ii)

Required to determine which plan is the better deal.

Under Plan B,

Total cost of phone $=$ Deposit $+$ (Monthly installments $\times$ # of months) $+$ Tax

$$= \$600 + (\$80 \times 6) + 0.05\big(600 + (80 \times 6)\big)$$

$$= \$600 + \$480 + 0.05(600 + 480)$$

$$= \$1134$$

Plan B is the better deal as it has an overall cost ($\$1180 - \$1134 = \$46$) cheaper than that of Plan A.

Question 1(c)(i)

Number of kWh used $=$ Final reading on 31st March – Initial reading on 1st March

$$= 0\,3\,3\,0\,7 - 0\,3\,0\,1\,1$$

$$= 296\ kWh$$

If 1 kWh costs $5.10

Then 296 kWh cost $5.10 × 296

$$= \$1509.60$$

Therefore, John pays $1509.60 for electricity consumption for the month of March 2016.

Question 1(c)(ii)

Number of kWh used on April $= \frac{\$2351.10}{\$5.10}$

$$= 461\ kWh$$

At the end of April, the meter reading should read $= 0\,3\,3\,0\,7 + 4\,6\,1$

$$= 0\,3\,7\,6\,8$$

Question 2(a)(i)

$6y^2 - 18xy$

$= 6y(y - 3x)$

Question 2(a)(ii)

$4m^2 - 1$

$= (2m + 1)(2m - 1)$ (difference of two squares)

Question 2(a)(iii)

$2t^2 - 3t - 2$

$= 2t^2 - 4t + t - 2$

$= 2t(t - 2) + 1(t - 2)$

$= (2t + 1)(t - 2)$

Question 2(b)

$\frac{5p+2}{3} - \frac{3p-1}{4}$

$= \frac{4(5p+2)-3(3p-1)}{12}$

$= \frac{20p+8-9p+3}{12}$

$= \frac{20p-9p+8+3}{12}$

$= \frac{11p+11}{12}$

$= \frac{11(p+1)}{12}$

Question 2(c)(i)

$d = \sqrt{\frac{4h}{5}}$

When $h = 29$,

$d = \sqrt{\frac{4(29)}{5}}$

$= \sqrt{\frac{116}{5}}$

$= \sqrt{23.2}$

$= 4.82$ (to 3 significant figures)

Question 2(c)(ii)

Required to make h the subject of the formula.

$d = \sqrt{\frac{4h}{5}}$

Squaring both sides gives:

$d^2 = \frac{4h}{5}$

$5d^2 = 4h$

$h = \frac{5d^2}{4}$

Question 3(a)(i)

The universal set is $U = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$

The members of set $M = \{odd\ numbers\}$

$$M = \{3, 5, 7, 9, 11\}$$

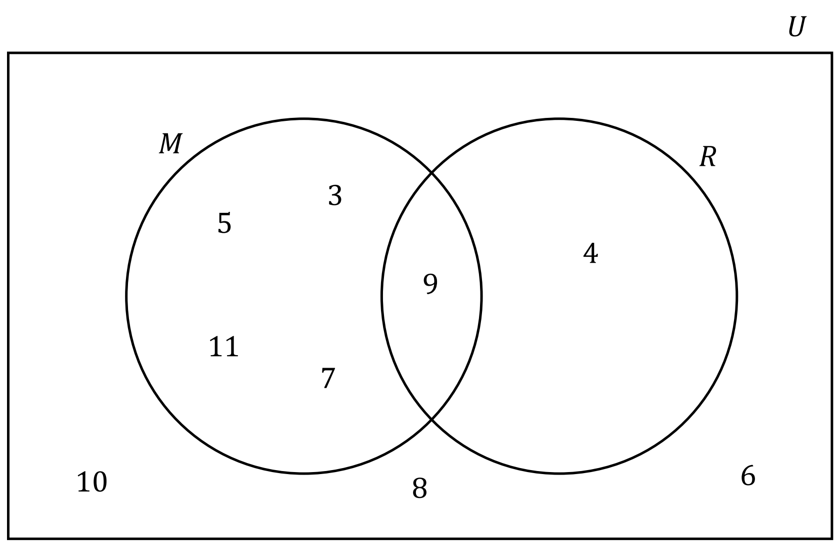
Question 3(a)(ii)

The members of set $R = \{square\ numbers\}$

$$R = \{4, 9\}$$

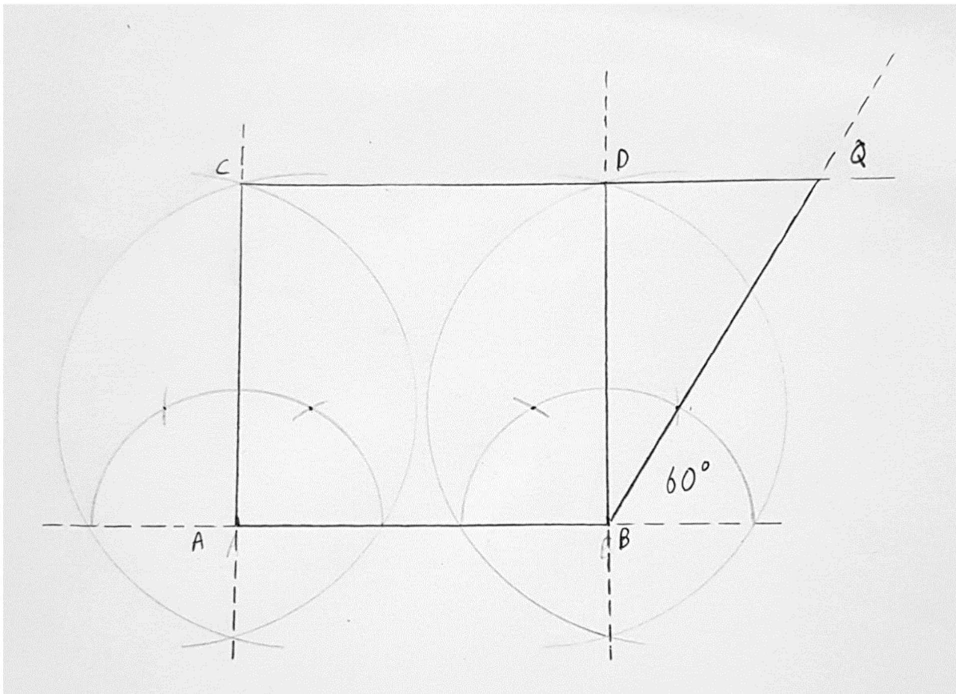
Question 3(a)(iii)

The Venn diagram is as follows:



Question 3(b)(i) and (ii)

Required to construct the square $ABCD$, with sides $6\ cm$.



Question 3(b)(iii)

Required to measure and state the length of BQ.

The length of $BQ = 6.9\ cm$.

Question 4(a)(i)

$f(3) = \frac{1}{3}(3) - 2$

$= 1 - 2$

$= -1$

$f(-3) = \frac{1}{3}(-3) - 2$

$= -1 - 2$

$= -3$

$\therefore f(3) + f(-3) = -1 + (-3)$

$= -1 - 3$

$= -4$

Question 4(a)(ii)

$f(x) = 5$

$\therefore \frac{1}{3}x - 2 = 5$

$\frac{1}{3}x = 5 + 2$

$\frac{1}{3}x = 7$

$x = 7 \times 3$

$x = 21$

Question 4(a)(iii)

$f(x) = \frac{1}{3}x - 5$

Let $y = f(x)$.

$y = \frac{1}{3}x - 5$

Interchanging the variables x and y.

$x = \frac{1}{3}y - 5$

Making y the subject of the formula.

$x + 5 = \frac{1}{3}y$

$y = 3(x + 2)$

$y = 3x + 6$

$\therefore f^{-1}(x) = 3x + 6$

Question 4(b)(i)

For the line l_1, two points are $(0, 1)$ and $(2, 5)$.

Gradient of line $l_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5-1}{2-0}$

$= \frac{4}{2}$

$= 2$

For the line l_2, two points are $(12, 0)$ and $(0, 6)$.

Gradient of line $l_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-0}{0-12}$

$= \frac{6}{-12}$

$= -\frac{1}{2}$

Question 4(b)(ii)

The general equation of a straight line is of the form $y = mx + c$.

Gradient of line $l_1 = 2$

y-intercept of line $l_1 = 1$

$\therefore$ The equation of the line l_1 is $y = 2x + 1$.

Question 4(b)(iii)

Gradient of line $l_1 = 2$

Gradient of line $l_2 = -\frac{1}{2}$

The gradient of line l_2 is the negative reciprocal of the gradient of line l_1.

Therefore, line l_1 is perpendicular to line l_2.

Question 5(a)(i)

The base angles of the isosceles triangle RQT are equal.

$\therefore$ Angle $RTQ = 76°$

The sum of the interior angles in a triangle add up to 180°.

$\therefore$ Angle $RQT = 180° - (76° + 76°)$

$= 28°$

Question 5(a)(ii)

The base angles of the isosceles triangle RQP are equal.

$\therefore$ Angle $QPR =$ Angle QRP

The exterior angle of a triangle is equal to the sum of the interior opposite angles.

Angle $QPR +$ Angle $QRP = 28°$

Angle $QRP = \frac{28°}{2}$

$= 14°$

Hence,

Angle $PRT =$ Angle $QRP +$ Angle QRT

$= 14° + 76°$

$= 90°$

Question 5(a)(iii)

Angle $SRP = 145° - 90°$

$= 55°$

The sum of the interior angles in a triangle is 180°.

Angle $SPR = 180° - (100° + 55°)$

$= 25°$

Hence,

Angle $SPT = 25° + 14°$

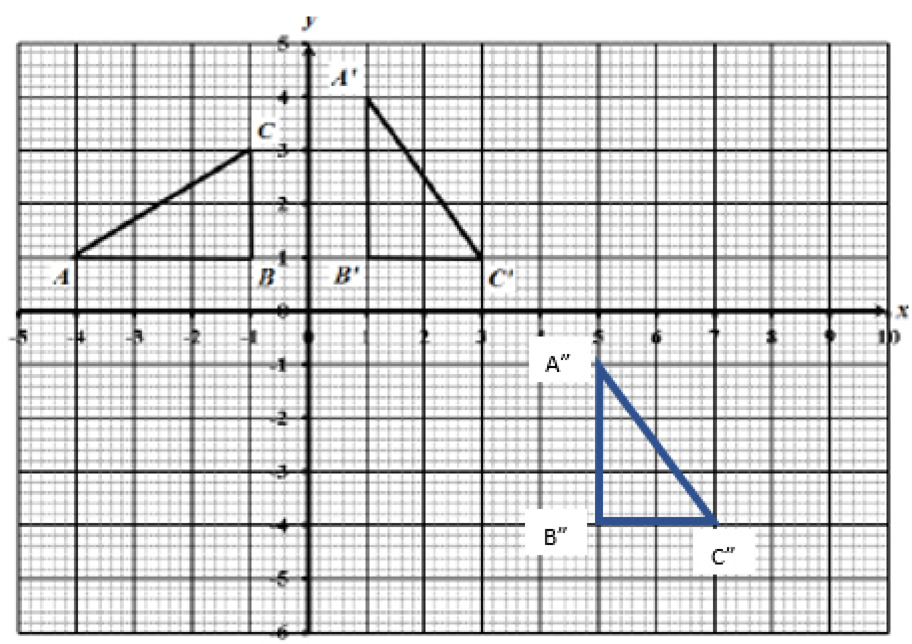
$= 39°$

Question 5(b)(i)

$A'B'C'$ is a 90° clockwise rotation of ABC about the origin, O.

Question 5(b)(ii)

Translating $\Delta A'B'C'$ through the vector $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$.



Question 6(a)(i)

Radius $= \frac{Diameter}{2}$

$= \frac{28}{2}$

$= 14\ m$

Area of the field $= \frac{\theta}{360°}\pi r^2$

$= \frac{90°}{360°} \times \frac{22}{7} \times (14)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154\ m^2$

Question 6(a)(ii)

Perimeter of the field $= (2 \times radius) + \left(\frac{90}{360} \times \pi \times d\right)$

$= (2 \times 14) + \left(\frac{1}{4} \times \frac{22}{7} \times 28\right)$

$= 50\ m$

Question 6(b)(i)

Using Pythagoras' Theorem,

$BC^2 = AC^2 + AB^2$

$(10)^2 = AC^2 + (6)^2$

$100 = AC^2 + 36$

$AC^2 = 100 - 36$

$AC^2 = 64$

$AC = 8\ cm$

Area of $\Delta ABC = \frac{bh}{2}$

$= \frac{6 \times 8}{2}$

$= \frac{48}{2}$

$= 24\ cm^2$

Question 6(b)(ii)

Volume of the prism = Area of cross-section × Length of prism

$540 = 24 \times \text{Length of prism}$

Length of prism $= \frac{540}{24}$

Length of prism $= 22.5\ cm$

Question 6(b)(iii)

Surface area of the prism = Sum of the area of all five sides

Area of $\Delta ABC = \frac{bh}{2}$

$= \frac{6 \times 8}{2}$

$= \frac{48}{2}$

$= 24\ cm^2$

Area of $\Delta DEF = \frac{bh}{2}$

$= \frac{6 \times 8}{2}$

$= \frac{48}{2}$

$= 24\ cm^2$

Area of rectangle $ABED = 6 \times 22.5$

$= 135\ cm^2$

Area of rectangle $ADFE = 8 \times 22.5$

$= 180\ cm^2$

Area of rectangle $BEFC = 10 \times 22.5$

$= 225\ cm^2$

Hence, the surface area of the prism $= 24 + 24 + 135 + 180 + 225$

$= 588\ cm^2$

Question 7(a)(i)

The upper class limit is 39.

Question 7(a)(ii)

The class width is $(39.5 - 19.5) = 20$.

Question 7(a)(iii)

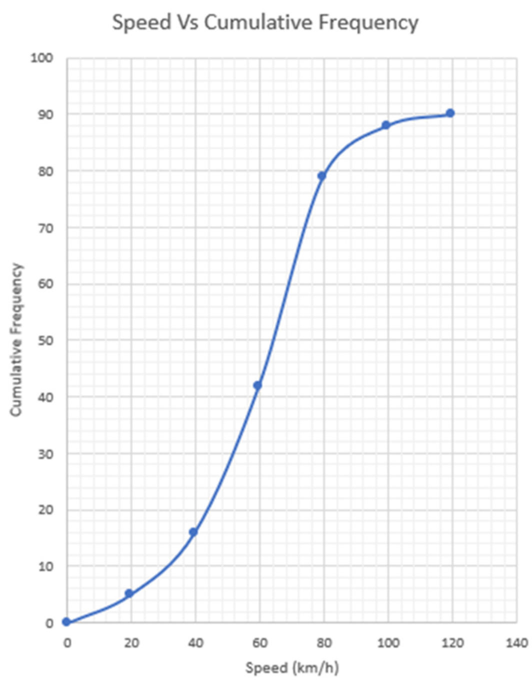
Sixteen vehicles passed a checkpoint at no more than 39.5 kmh^{-1}.

Question 7(b)

Speed (kmh^{-1})	**Frequency**	**Cumulative Frequency**
0-19	5	5
20-39	11	16
40-59	26	42
60-79	37	79
80-99	9	88
100-119	2	90

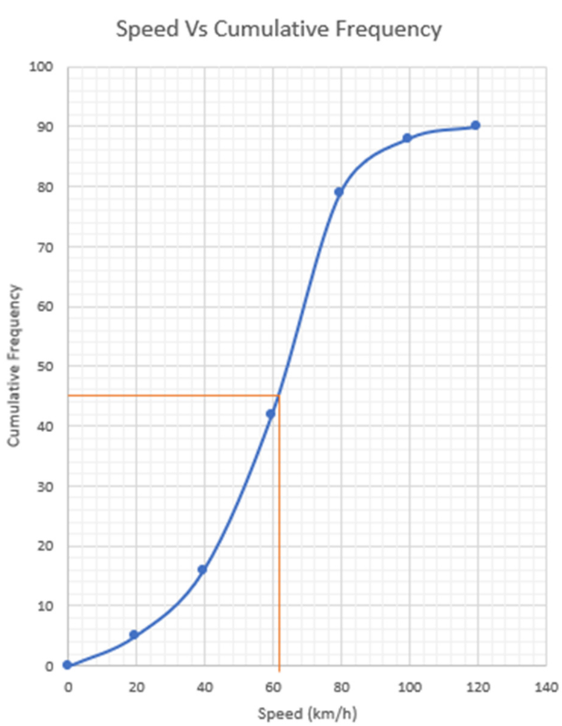
Question 7(c)

The cumulative frequency curve is shown below.



Question 7(d)(i)

50% of the 90 vehicles = 45 vehicles.



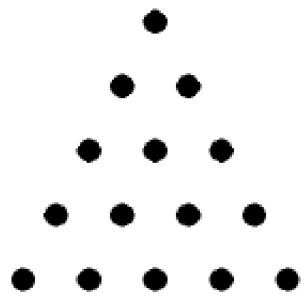
Question 7(d)(ii)

The estimated speed is 62 kmh^{-1}.

WhatsApp +1868 472 4221 to register for premium online class @ The Student Hub

Question 8(a)

Figure 4 of the sequence is shown below:



Question 8(b)

The number of dots in Figure 6 $= 1 + 2 + 3 + 4 + 5 + 6$

$= 21$

Question 8(c)

Figure, n	**Number of Dots, d, in terms of n**	**Number of Dots Used, d**
1	$\frac{1}{2} \times 1 \times (1 + 1)$	1
2	$\frac{1}{2} \times 2 \times (2 + 1)$	3
3	$\frac{1}{2} \times 3 \times (3 + 1)$	6
11	$\frac{1}{2} \times 11 \times (11 + 1)$	66
n		

Question 8(d)

Required to determine which figure in the sequence has 210 dots.

$\frac{1}{2}n(n+1)=210$

$n(n+1)=420$

$n(n+1)=20(20+1)$

$\therefore n=20$

So, Figure 20 has 210 dots.

Question 8(e)

Required to write a simplified algebraic expression for the number of dots, d, in the Figure n.

Figure, n	**Number of Dots, d, in terms of n**	**Number of Dots Used, d**
1	$\frac{1}{2}\times 1\times(1+1)$	1
2	$\frac{1}{2}\times 2\times(2+1)$	3
3	$\frac{1}{2}\times 3\times(3+1)$	6
11	$\frac{1}{2}\times 11\times(11+1)$	66
n	$\frac{1}{2}\times n\times(n+1)$	$\frac{1}{2}n(n+1)$

Question 8(f)

Let $\frac{1}{2}n(n+1) = 1000$

$$n(n+1) = 2000$$

There are no two consecutive integers, n and $n+1$ being consecutive integers, whose product is exactly 2000.

Therefore, no diagram has 1000 dots.

Question 9(a)(i)(a)

Points are: $O(0,0)$ and $A(25,10)$

Gradient of $OA = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{10-0}{25-0}$

$= \frac{10}{25}$

$= \frac{2}{5}$

Question 9(a)(i)(b)

Points are: $A(25,10)$ and $B(40,10)$

Gradient of $AB = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{10-10}{40-25}$

$= \frac{0}{15}$

$= 0$

Question 9(a)(ii)

The cyclist started from rest, where his velocity was ……….0………ms^{-1}, and steadily increased his velocity by …… $\frac{2}{5}$ ……ms^{-1} each second during the first 25 seconds. During the next 15 seconds, his velocity remained constant, that is, his acceleration was ……. 0 ……..ms^{-2}.

Question 9(a)(iii)

Total distance = Area under the graph

$$= \frac{1}{2}[(40 - 25) + (40 - 0)] \times 10$$

$$= \frac{1}{2}(15 + 40) \times 10$$

$$= 275\ m$$

Average speed $= \frac{Total\ distance\ covered}{Total\ time\ taken}$

$$= \frac{275}{40}$$

$$= 6.875\ ms^{-1}$$

Question 9(b)(i)

$x^2 + 2xy = 5$ → Equation 1

$x + y = 3$ → Equation 2

Substituting $(1, 2)$ into Equation 1 gives:

$(1)^2 + 2(1)(2) = 1 + 4$

$= 5$

Therefore, $(1, 2)$ is a solution for Equation 1.

Substituting $(1, 2)$ into Equation 2 gives:

$1 + 2 = 3$

Therefore, $(1, 2)$ is a solution for Equation 2.

Hence, $(1, 2)$ is a solution for the pair of simultaneous equations.

Question 9(b)(ii)

$x^2 + 2xy = 5$ → Equation 1

$x + y = 3$ → Equation 2

Rearranging Equation 2 gives:

$y = 3 - x$ → Equation 3

Substituting Equation 3 into Equation 1 gives:

$$x^2 + 2x(3 - x) = 5$$

$$x^2 + 6x - 2x^2 = 5$$

$$-x^2 + 6x = 5$$

$$x^2 - 6x + 5 = 0$$

$$x^2 - 5x - x + 5 = 0$$

$$x(x - 5) - 1(x - 5) = 0$$

$$(x - 1)(x - 5) = 0$$

Either $x - 1 = 0$ or $x - 5 = 0$

$x = 1$ $x = 5$

We already know that $(1, 2)$ is one solution.

When $x = 5$,

$y = 3 - 5$

$y = -2$

∴ The other solution is $(5, -2)$.

Question 10(a)(i)

The opposite angles of a cyclic quadrilateral are supplementary.

$\angle SPQ = 180° - 58°$

$= 122°$

Question 10(a)(ii)

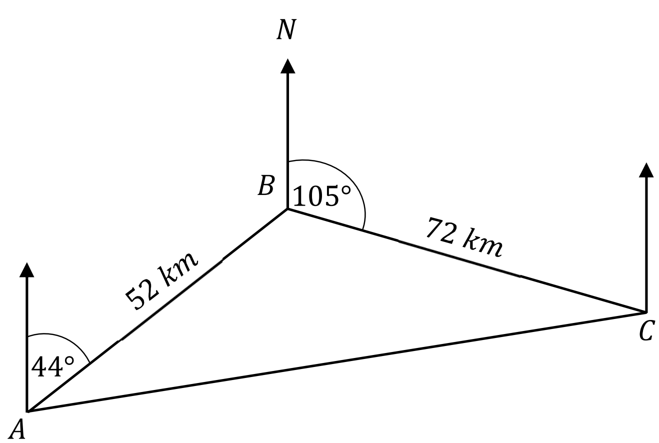
The angle subtended by a chord at the center of a circle, is twice the angle that the chord subtends at the circumference, standing on the same arc.

$\angle SOQ = 2(58°)$

$= 116°$

Question 10(b)(i)

The diagram is as follows:



Question 10(b)(ii)

Alternate angles are equal.

$\angle ABS = 44°$

Two angles which make a straight line are supplementary.

$\angle CBS = 180° - 105°$

$= 75°$

$\therefore \angle ABC = 44° + 75°$

$= 119°$

Question 10(b)(iii)

By the cosine rule:

$AC^2 = (52)^2 + (72)^2 - 2(52)(72)\cos 119°$

$= 2704 + 5184 - (-3630.25)$

$= 11518.25$

$AC = \sqrt{11518.25}$

$= 107\ km$ (to the nearest km)

Question 10(b)(iv)

Co-interior angles are supplementary.

$\angle BCN = 75°$

By the sine rule:

$$\frac{52}{\sin A\hat{C}B} = \frac{107.3}{\sin 119°}$$

$$\sin A\hat{C}B = \frac{52 \sin 119°}{107.3}$$

$$\sin A\hat{C}B = 0.434$$

$$A\hat{C}B = \sin^{-1}(0.434)$$

$$A\hat{C}B = 25.07°$$

The bearing of A from $C = 360° - (75° + 25.07°)$

$= 260°$ (to the nearest degree)

Question 11(a)(i)

$$AB = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \times 4) + (2 \times 3) & (3 \times 0) + (2 \times -1) \\ (5 \times 4) + (4 \times 3) & (5 \times 0) + (4 \times -1) \end{pmatrix}$$

$$= \begin{pmatrix} 12 + 6 & 0 - 2 \\ 20 + 12 & 0 - 4 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & -2 \\ 32 & -4 \end{pmatrix}$$

$$BA = \begin{pmatrix} 4 & 0 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} (4 \times 3) + (0 \times 5) & (4 \times 2) + (0 \times 4) \\ (3 \times 3) + (-1 \times 5) & (3 \times 2) + (-1 \times 4) \end{pmatrix}$$

$$= \begin{pmatrix} 12 + 0 & 8 + 0 \\ 9 - 5 & 6 - 4 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 8 \\ 4 & 2 \end{pmatrix}$$

Since $\begin{pmatrix} 18 & -2 \\ 32 & -4 \end{pmatrix} \neq \begin{pmatrix} 12 & 8 \\ 4 & 2 \end{pmatrix}$, then $AB \neq BA$.

Question 11(a)(ii)

$$\det A = ad - bc$$

$$= (3)(4) - (2)(5)$$

$$= 12 - 10$$

$$= 2$$

$$adj\ A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$$

$A^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$

$= \begin{pmatrix} \frac{4}{2} & \frac{-2}{2} \\ \frac{-5}{2} & \frac{3}{2} \end{pmatrix}$

$= \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}$

Question 11(a)(iii)

$AA^{-1} = I$

$\therefore I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Question 11(b)(i)

$3x + 2y = 1$

$5x + 4y = 5$

In matrix form,

$\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

Question 11(b)(ii)

$\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

Question 11(c)(i)

$\overrightarrow{OQ} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\overrightarrow{QS} = 3\begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 15 \\ 0 \end{pmatrix}$

$\overrightarrow{OS} = \overrightarrow{OQ} + \overrightarrow{QS}$

$= \begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} 15 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 20 \\ 0 \end{pmatrix}$ which is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 20$ and $y = 0$

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= -\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ which is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 1$ and $y = -3$

$\overrightarrow{PR} = 3\overrightarrow{OP}$

$= 3\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 12 \\ 9 \end{pmatrix}$

$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 12 \\ 9 \end{pmatrix}$

$= \begin{pmatrix} 16 \\ 12 \end{pmatrix}$

$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$

$= -\begin{pmatrix} 16 \\ 12 \end{pmatrix} + \begin{pmatrix} 20 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -12 \end{pmatrix}$ which is of the form $\begin{pmatrix} x \\ y \end{pmatrix}$, where $x = 4$ and $y = -12$

Question 11(c)(ii)

$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\overrightarrow{RS} = \begin{pmatrix} 4 \\ -12 \end{pmatrix}$

$= 4\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$= 4\,\overrightarrow{PQ}$

Hence, $\overrightarrow{RS}$ is a scalar multiple of $\overrightarrow{PQ}$.

Therefore, $\overrightarrow{RS}$ and $\overrightarrow{PQ}$ are parallel.